MISO: $\underset{1 \times n_e}{\underline{Y}_0} = \underset{1 \times n_u}{\underline{G}_0} \; \underset{n_u \times n_e}{\underline{U}_0}$ $\qquad \underline{G}_0 = \underline{Y}_0 \underline{U}_0^{-1}$

$n_e = n_u$, BUT $n_y < n_u$, $n_y = n_u$, $\underbrace{n_y > n_u}_{\text{TYPICAL}}$

WHAT IF $\underline{U}_0^{-1}$ SINGULAR?

$$\underline{G}_0 = \underline{Y}_0 \underbrace{\underline{U}^* (\underline{U}_0 \underline{U}_0^*)^{-1}}_{\underline{U}_0^+}$$

$\underline{U}_0^+$ MOORE-PENROSE PSEUDO-INVERSE

NOISY CASE: – MORE EXPERIMENTS

$n_e > n_u$ $\quad$ └ $\operatorname{var}(\hat{\underline{G}}) \downarrow$

$$\hat{\underline{G}} = \underline{Y}\, \underline{U}^+ \qquad \hat{\underline{G}} = \underline{Y} \cdot \underline{U}^+$$

(IF A FULL RANK; LINEARLY INDEPENDENT COLUMNS $\rightarrow A^*A$ INVERTIBLE $\rightarrow$

– " – – " – ROWS $\rightarrow AA^*$ – " – $\rightarrow$

$\rightarrow A^+A = I$ LEFT INVERSE

$\rightarrow AA^+ = I$ RIGHT – " –)

COVARIANCE OF $\underline{G}$

$$\operatorname{vec} \underline{A} = \operatorname{vec}\left(\left[[\,]\,[\,] \cdots [\,]\right]\right) = \begin{bmatrix} [\,] \\ [\,] \\ \vdots \\ [\,] \end{bmatrix} \qquad \underline{A} \otimes \underline{B} = \begin{bmatrix} a_{11}\underline{B} & \cdots & a_{1q}\underline{B} \\ \vdots & & \\ a_{p1}\underline{B} & \cdots & a_{pq}\underline{B} \end{bmatrix}$$

$$\operatorname{Cov}\{\underline{X}\} = \mathcal{E}\{\underline{X} \cdot \underline{X}^*\} = \mathcal{E}\{[\, x_i \bar{x}_j \,]\} = [\, \mathcal{E}\{x_i \bar{x}_j\} \,]$$