

(5) (19)

$$\text{cov}(\text{vec}(\hat{\underline{G}}(j\omega_k))) = \mathbb{E} \left\{ \begin{bmatrix} 1 & 2 & \dots & n \\ \vdots & \vdots & \vdots & \vdots \\ n & n & \dots & n \end{bmatrix} \begin{bmatrix} \hat{G}_{11} & \hat{G}_{12} & \dots & \hat{G}_{1n} \\ \hat{G}_{21} & \hat{G}_{22} & \dots & \hat{G}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{G}_{n1} & \hat{G}_{n2} & \dots & \hat{G}_{nn} \end{bmatrix} \begin{bmatrix} \hat{G}_{11}^* & \hat{G}_{12}^* & \dots & \hat{G}_{1n}^* \\ \hat{G}_{21}^* & \hat{G}_{22}^* & \dots & \hat{G}_{2n}^* \\ \vdots & \vdots & \ddots & \vdots \\ \hat{G}_{n1}^* & \hat{G}_{n2}^* & \dots & \hat{G}_{nn}^* \end{bmatrix} \right\}$$

$\mathbb{E} \begin{bmatrix} \hat{G}_{11} \\ \hat{G}_{21} \\ \vdots \\ \hat{G}_{n1} \end{bmatrix} \begin{bmatrix} \hat{G}_{11}^* & \hat{G}_{21}^* & \dots & \hat{G}_{n1}^* \end{bmatrix}$   
 $\mathbb{E} \{ \hat{G}_{k1} \hat{G}_{l1}^* \}$   
 $n=1$   
 DIFFERENT OUTPUTS

$\mathbb{E} \{ \hat{G}_{k2} \hat{G}_{l1}^* \}$   
 DIFFERENT INPUTS  
 - - - OUTPUTS

$\begin{bmatrix} \text{var} & \text{cov} \\ \text{cov} & \text{var} \end{bmatrix}$

$n_u = n_y = 2$  CASE

$$\mathbb{E} \left\{ \begin{bmatrix} \hat{G}_{11} \\ \hat{G}_{21} \\ \hat{G}_{12} \\ \hat{G}_{22} \end{bmatrix} \begin{bmatrix} \hat{G}_{11}^* & \hat{G}_{21}^* \\ \hat{G}_{12}^* & \hat{G}_{22}^* \end{bmatrix} \right\} \rightarrow \mathbb{E} \left\{ \begin{bmatrix} |\hat{G}_{11}|^2 & \hat{G}_{11} \hat{G}_{21}^* & \hat{G}_{11} \hat{G}_{12}^* & \hat{G}_{11} \hat{G}_{22}^* \\ \hat{G}_{21} \hat{G}_{11}^* & |\hat{G}_{21}|^2 & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & |\hat{G}_{22}|^2 \end{bmatrix} \right\}$$

$$\text{cov}(\text{vec}(\hat{\underline{G}}(j\omega_k))) = \left( \underline{U}_o(k) \underline{U}_o^*(k) \right)^{-1} \otimes \left( \underline{V}(k) \underline{C}_z(k) \underline{V}^*(k) \right)$$

$$\underline{V}(k) = \begin{bmatrix} \underline{I}_{n_y} & -\underline{G}_o(j\omega_k) \end{bmatrix} \quad \underline{C}_z(k) = \begin{bmatrix} \underline{C}_u(k) & \underline{C}_{y_u}(k) \\ \underline{C}_{y_u}^*(k) & \underline{C}_y(k) \end{bmatrix}$$

SENSITIVE TO CONDITION NUMBER

$$\text{COND}(\underline{U}_o \underline{U}_o^*) = \left( \text{COND}(\underline{U}_o) \right)^2$$

NOISE COVARIANCE  
MATRICES FOR 1  
EXPERIMENT

INPUT SIGNALS  $\begin{cases} \text{(RANDOM) MULTISINES} & \rightarrow \text{LESS EFFORT (PERIODIC)} \\ \text{RANDOM SIMULTANEOUS SIGNALS} & \rightarrow \text{LEAKAGE (WINDOWING)} \\ & \rightarrow \text{INCREASED AVERAGING} \end{cases}$

$$\hat{\underline{G}} = \frac{\hat{\Phi}_{yu}}{\hat{\Phi}_{uu}} \quad \begin{cases} \text{PERIODIC} \equiv \text{CONSTANT} \\ \text{RANDOM} \equiv \text{RANDOM} \end{cases}$$