$$\hat{g}_*(\tau) = \frac{R_{yu}(\tau)}{\alpha} \cong \frac{1}{\alpha}\frac{1}{N}\sum_{t=\tau}^{N} y(t)\,u(t-\tau)$$

IF INPUT **NOT** WHITE NOISE

$$\hat{R}_{yu}^{N}(\tau) = \sum_{0}^{M} \hat{g}(k)\,\hat{R}_{u}^{N}(k-\tau)$$

$$\hat{R}_{u}^{N}(\tau) = \frac{1}{N}\sum_{t=\tau}^{N} u(t)\,u(t-\tau)$$

IMPULSE RESPONCE AS LS ESTIMATE

$$y_0(t) = \sum g(k)\,u(t-k) = \underline{\Phi}^T(u)\,\underline{\theta}$$

BASIS FUNCTIONS $u(t-k)$

PARAMETERS $g(k)$

N MEASUREMENTS

$$\underline{Y}_N = \underline{\underline{\Phi}}_N \underline{\theta} + \underline{E} \qquad y(k) = y_0(k) + e(k)$$

$$V_{LS} = \| \underline{Y}_N - \underline{\underline{\Phi}}_N \underline{\theta} \|_2^2 \qquad \boxed{\underline{\hat{\theta}}_{LS} = (\underline{\underline{\Phi}}_N^T \underline{\underline{\Phi}}_N)^{-1} \underline{\underline{\Phi}}_N^T \underline{Y}_N}$$

REGULARIZED LS (BETTER DEAL IN BIAS/VARIANCE TRADE-OFF)

$$V_{LS,R} = \| \underline{Y}_N - \underline{\underline{\Phi}}_N \underline{\theta} \|_2^2 + \underline{\theta}^T \underline{\underline{D}}\, \underline{\theta} \sim \| \underline{\underline{D}}^{1/2} \underline{\theta} \|_2^2$$

$$\boxed{\underline{\hat{\theta}}_{LS}^{REG} = (\underline{\underline{\Phi}}_N^T \underline{\underline{\Phi}}_N + \underline{\underline{D}})^{-1} \underline{\underline{\Phi}}_N \cdot \underline{Y}_N}$$

BIAS: $b(\underline{\hat{\theta}}_N^{REG}) = -(\underline{\underline{\Phi}}_N^T \underline{\underline{\Phi}} + \underline{\underline{D}})^{-1} \underline{\underline{D}}\, \theta_0$

(CO)VARIANCE $P(\underline{\hat{\theta}}_N^{REG}) = (\underline{\underline{\Phi}}_N^T \underline{\underline{\Phi}} + \underline{\underline{D}})^{-1} \sigma^2 \underline{\underline{\Phi}}_N^T \underline{\underline{\Phi}}_N (\underline{\underline{\Phi}}_N^T \underline{\underline{\Phi}}_N + \underline{\underline{D}})^{-1}$

$$MSE(\underline{\hat{\theta}}_N^{REG}) = (\underline{\underline{\Phi}}_N^T \underline{\underline{\Phi}} + \underline{\underline{D}})^{-1} (\sigma^2 \underline{\underline{\Phi}}_N^T \underline{\underline{\Phi}}_N + \underline{\underline{D}}\,\underline{\theta}_0 \underline{\theta}_0^T \underline{\underline{D}}^T)(\underline{\underline{\Phi}}_N^T \underline{\underline{\Phi}}_N + \underline{\underline{D}})^{-1}$$