(4) (10)

$$P_{SE}(i,j) = c \cdot e^{-\frac{(i-j)^2}{2\lambda^2}}$$ SQUARED EXPONENTIAL

$$P_{SS}(i,j) = \begin{cases} c\,\frac{\lambda^{2i}}{2}\left(\lambda^j - \frac{\lambda^i}{3}\right) & i \geq j \\ c\,\frac{\lambda^{2j}}{2}\left(\lambda^i - \frac{\lambda^j}{3}\right) & i < j \end{cases}$$ STABLE SPLINE

ETC.

PARAMETERS OF COVARIANCE $\rightarrow$ <u>HYPER PARAMETERS</u>
(PARAMETERS OF PARAMETERS)

$$P_{DC}(i,j) = c \cdot e^{-\alpha|i-j|}\, e^{-\beta\frac{(i+j)}{2}}$$

$$H.P.\ (c, \alpha, \beta) \sim \underline{\theta}_{HYP}$$

<u>MARGINAL LIKELIHOOD:</u> $\underline{Y}_N = \underline{\Phi}\,\underline{\theta} + \underline{E}$

$e(k) \sim N(0, \sigma^2)$

$N(\underline{0}, \underline{P})$

$$p(\underline{Y}_N) \sim N(\underline{0}, \underbrace{\underline{\Phi}_N^T \underline{P}\,\underline{\Phi}_N + \sigma^2 \underline{I}}_{\underline{\Sigma}_Y(\underline{P})})$$

$$\left(\leftarrow p(\underline{Y}_N \mid \underline{\theta}_{HYP})\right)$$

$$p_{\underline{Y}}(\underline{P}) = \frac{1}{\sqrt{2\pi\,|\underline{\Sigma}_Y(\underline{P})|}} \exp\left[-\frac{1}{2}\underline{Y}_N^T \underline{\Sigma}_Y^{-1} \underline{Y}_N\right]$$

$$\hat{\underline{\theta}}_{HYP} = \underset{\underline{\theta}_H}{\mathrm{ARGMAX}}\ p_{\underline{Y}}(\underline{P}) =$$

$$= \underset{\underline{\theta}_H}{\mathrm{ARGMIN}} \left[\underline{Y}_N^T \underline{\Sigma}_N^{-1} \underline{Y}_N + \ln\left(\det(\underline{\Sigma}_Y)\right)\right]$$

$\frac{\partial}{\partial}$ $\frac{\partial}{\partial}$

LIFTING: PROBLEM OF ESTIMATING $\underline{\theta}$ TO A PROBLEM OF ESTIMATING PARAMETERS IN $\underline{P}$ (DESCRIBING DISTRIBUTION OF $\underline{\theta}$)

$$\left(\frac{\partial \underline{A}^{-1}}{\partial \alpha} = -\underline{A}^{-1}\frac{\partial \underline{A}}{\partial \alpha}\underline{A}, \quad \frac{\partial |\underline{A}|}{\partial \alpha} = |\underline{A}|\,\mathrm{Tr}\left[\underline{A}^{-1}\frac{\partial \underline{A}}{\partial \alpha}\right]\right)$$