OPTIMAL REGULARIZATION

OPTIMAL $\underline{\underline{P}}$ FOR A CERTAIN TRUE SYSTEM $\underline{\theta}_0$?

BEST CHOICE OF $\underline{\underline{P}}$ FOR A GIVEN $\underline{\theta}_0$

$$MSE\left(\hat{\underline{\theta}}_N^{REG}(\underline{\underline{P}})\right) \geq MSE\left(\hat{\underline{\theta}}_N^{REG}(\underline{\theta}_0\underline{\theta}_0^T)\right)$$

FOR ANY $\underline{\underline{P}} \geq \phi$

OPTIMAL CHOICE IS INDEPENDENT FROM $N, U(t)$

$$\hat{\underline{\theta}}_N^{OPT} = \left(\underline{\theta}_0\underline{\theta}_0^T \underline{\Phi}_N^T\underline{\Phi}_N + \sigma^2\underline{\underline{I}}\right)^{-1}\underline{\theta}_0\underline{\theta}_0^T\underline{\Phi}_N^T\underline{Y}_N$$

- $\underline{\theta}_0$: UNKNOWN
- $\underline{\theta}_0$: ADAPTIVE CHOICE / ROBUST CHOICE

EXPERIMENTS

FINEXP1.M ORD = 1, 3 ; K = 200, K = 500 ; IMPULSE RESP. SHAPE, LENGTH

KROUT – 0, V.LOW, HIGH
- λ – LOW — VARIANCE?
- λ – HIGH — BIAS?

KROUT = 0, KRIN = LOW, HIGH → BIAS!

KROUT, KRIN – BOTH ↑ – BIASES ↙ NOISE AT INPUT / REG.

FINEXP2.M

DC NOT OPTIMALLY COMPUTED!

KROUT – 0.5, KRIN = 0

$\lambda_2 = 10 \rightarrow 1000$ BIAS

λ_1 (DC) = 1.5 – 10.5 OPTIMAL REG. (COVARIANCE SHAPE!)

ORD: 1 ↗ 3	λ_1	1.5	1.5	10.5
K: 200 ↗ 500	λ_2	1000	10	10

+ KRIN : EFFECTS (ORD 3)
0.1 ↗ 0.2

FINEXP3.M

- NOISY LS
- REG LS λ ↓
- REG LS λ ↑
- REG LS – BAYES (DC)

K =	200	500
ORD =	1	3