

a. STAT. $\phi_s(\omega) = |G(e^{j\omega})|^2 \phi_w(\omega)$

(96)

$\phi_{sw}(\omega) = G(e^{j\omega}) \phi_w(\omega)$

$$\left[y(t) = G(q) u(t) + H(q) e(t) \right]$$

\downarrow
a. STAT. DET
 $\phi_u(\omega)$

\downarrow WHITE NOISE
VARIANCE λ
INDEPENDENT OF $u(t)$

a. STAT.

$$\begin{cases} \phi_y(\omega) = |G(e^{j\omega})|^2 \phi_u(\omega) + \lambda |H(e^{j\omega})|^2 \\ \phi_{yu}(\omega) = G(e^{j\omega}) \phi_u(\omega) \end{cases}$$

SPECTRAL FACTORIZATION

$\phi_u(\omega)$ - REAL, AND IF $\phi_u(\omega) > 0$

RATIONAL FUNCTION OF $\cos \omega$
($e^{j\omega}$)

THEN THERE EXIST A MONIC RATIONAL FUNCTION OF z , $R(z)$

$R(z) = 1 + \dots + z^n$ $\begin{cases} \text{NO POLES} & \text{ON/OUTSIDE} \\ \text{NO ZEROS} & \text{UNIT CIRCLE} \end{cases}$

THAT: $\phi_u(\omega) = \lambda |R(e^{j\omega})|^2$

IF STOCH. PROCESS $u(t)$ HAS RATIONAL SPECTRUM,

IT CAN BE REPRESENTED AS $u(t) = R(q) e(t)$

\downarrow
RATIONAL: $R(q) = \frac{C(q)}{A(q)}$

$C(q) = 1 + c_1 q^{-1} + \dots + c_{n_c} q^{-n_c}$

$A(q) = 1 + a_1 q^{-1} + \dots + a_{n_a} q^{-n_a}$

$v(t) + a_1 v(t-1) + \dots + a_{n_a} v(t-n_a) = e(t) + c_1 e(t-1) + \dots$

$\left[v(t) = -a_1 v(t-1) - \dots + e(t) + \dots \right]$ AR/MA process
MODEL

\downarrow (NO PARAMETRIC IDENTIFICATION OF TF) (AR, MA)