X. RECURSIVE (LEAST-SQUARES) ESTIMATION

EX.: $\hat{\theta}(N) = \frac{1}{N} \sum_{k=1}^{N} y(k)$

$$\hat{\theta}(N+1) = \frac{1}{N+1} \sum_{k=1}^{N+1} y(k) = \frac{N}{N+1} \frac{1}{N} \sum_{k=1}^{N} y(k) + \frac{1}{N+1} y(N+1)$$
$$= \frac{N}{N+1} \hat{\theta}(N) + \frac{1}{N+1} y(N+1)$$
$$= \hat{\theta}(N) + \frac{1}{N+1}\left(y(N+1) - \hat{\theta}(N)\right)$$

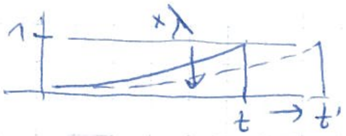
RECURSIVE LS
$$\hat{\theta}_t = \arg\min_{\theta} \sum_{k=1}^{t} \beta(t,k)\left[y(k) - \underline{\Phi}^T(k)\,\underline{\theta}\right]^2$$

$$\hat{\theta}_t = \bar{R}^{-1}(t)\, f(t) \qquad \bar{R}(t)\,\theta(t) = f(t)$$

$$\bar{R}(t) = \sum_{k=1}^{t} \beta(t,k)\, \underline{\Phi}(k)\underline{\Phi}^T(k)$$
$$f(t) = \sum_{k=1}^{t} \beta(t,k)\, \underline{\Phi}(k)\, y(k)$$

ASSUME: $\beta(t,k) = \lambda(t)\, \beta(t-1,k) \qquad 0 \le k \le t-1$

$\beta(t,t) = 1 \qquad \beta(t,k) = \prod_{j=k+1}^{t} \lambda(j)$



$$\bar{R}(t) = \lambda(t)\, \bar{R}(t-1) + \underline{\Phi}(t)\underline{\Phi}^T(t)$$
$$f(t) = \lambda(t)\, f(t-1) + \underline{\Phi}(t)\, y(t)$$

$$\hat{\theta}_t = \bar{R}(t)^{-1} f(t) = \bar{R}(t)^{-1}\left[\lambda(t)\, f(t-1) + \underline{\Phi}(t)\, y(t)\right]$$
$$= \bar{R}(t)^{-1}\left[\lambda(t)\, \bar{R}(t-1)\, \hat{\theta}_{t-1} + \underline{\Phi}(t)\, y(t)\right]$$
$$= \bar{R}(t)^{-1}\left[\left(\bar{R}(t) - \underline{\Phi}(t)\underline{\Phi}^T(t)\right)\hat{\theta}_{t-1} + \underline{\Phi}(t)\, y(t)\right]$$
$$= \hat{\theta}_{t-1} + \bar{R}^{-1}(t)\, \underline{\Phi}(t)\left[y(t) - \underline{\Phi}^T(t)\, \hat{\theta}_{t-1}\right]$$

LET $P(t) = \bar{R}(t)^{-1}$ EFFICIENT MATRIX INVERSE:

$$[A + BCD]^{-1} = A^{-1} - A^{-1}B\left[DA^{-1}B + C^{-1}\right]^{-1} DA^{-1}$$

$A = \lambda(t)\, \bar{R}(t-1) \qquad C \equiv 1 \qquad B = \underline{\Phi}(t) \qquad D = \Phi^T(t)$