

EX. 6 → EX. 3. LINEAR PREDICTION  
(SYSTEM IDENTIFICATION)

(27)

$$y_H(k) = \sum_{n=0}^{N-1} h(n) u(n-k) = [u(n) \ u(n-1) \ \dots \ u(n-N+1)] \begin{bmatrix} h(0) \\ h(1) \\ \vdots \\ h(N-1) \end{bmatrix}$$

~ AT START OF MEASUREMENT

$$y_H(0) = h(0) u(0)$$

$$y_H(1) = h(0) u(1) + h(1) u(0)$$

----- ETC.

$$\underline{\Phi} = \begin{bmatrix} \text{triangle} \end{bmatrix}$$

~ AFTER WAITING OUT TRANSIENTS:

$$y_H(n) = h(0) u(n) + h(1) u(n-1) + \dots + h(N-1) u(n-N+1)$$

(CONT.)

$$\underline{\Phi} = \begin{bmatrix} \text{rectangle} \end{bmatrix}$$

EX. 7 SIGNAL SMOOTHING

(WEIGHTED REGULARIZATION)

$$y(k) = x(k) + n_y(k)$$

SMOOTHING: ENERGY OF DERIVATIVE ↑ ↓

$$x'(k) = x(k) - x(k-1)$$

$$x''(k) = x'(k) - x'(k-1) = x(k) - 2x(k-1) + x(k-2)$$

$$\min_{\underline{u}} \|\underline{y} - \underline{u}\|_2^2 + \lambda \|\underline{D} \underline{u}\|_2^2$$

$$\underline{D} = \begin{bmatrix} 1 & -2 & 1 & 0 & \dots & 0 \\ 0 & 1 & -2 & 1 & & \\ \vdots & 0 & 1 & -2 & 1 & \vdots \\ 0 & \dots & 0 & 1 & -2 & 1 \end{bmatrix}$$

$$\hat{\underline{u}} = (\underline{I} + \lambda \underline{D}^T \underline{D})^{-1} \underline{y} \rightarrow \text{Mathlab exercises}$$

EX. 8 DECONVOLUTION

$$y(n) = \sum_{k=0}^{N-1} h(k) u(n-k) + n_y(n)$$

↑ MEASURED      ↑ KNOWN      ↓ ESTIMATED

$$\underline{\Phi} = \begin{bmatrix} h(0) & 0 & \dots & 0 \\ h(1) & h(0) & & \\ \vdots & h(1) & \ddots & 0 \\ h(N-1) & \dots & h(1) & h(0) \end{bmatrix}$$

$$\underline{\theta} = \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(N-1) \end{bmatrix} \quad (\text{CONT.})$$