LEAST-SQUARES OPTIMIZATION & LINEAR REGRESSION MODELS

GENERAL MODEL $y(n) = y_M(n) + e(n)$

$$y_M(n) = \theta_1 \phi_1(u(n)) + \theta_2 \phi_2(u(n)) + \dots + \theta_M \phi_M(u(n))$$

SCALAR CASE

$$= \underline{\theta}^T \underline{\phi}(u(n)) = \underline{\phi}^T \underline{\theta}$$

$$\underline{y}_M = \begin{bmatrix} y_{M,1}(1) \\ y_{M,2}(2) \\ \dots \\ y_{M,N}(n) \end{bmatrix} = \begin{bmatrix} \underline{\phi}^T(u(1)) \\ \underline{\phi}^T(u(2)) \\ \dots \\ \underline{\phi}^T(u(n)) \end{bmatrix} \underline{\theta} = \underline{\underline{\Phi}}(u)\, \underline{\theta}$$

$$[\underline{\underline{\Phi}}]_{ij} = \phi_j(u(i))$$

ith data

jTH ORDER IN REGRESSION

BASIS FUNCTION FOR

$$\theta = \Phi^{-1} y$$

$$V(\underline{\underline{\Phi}}, \underline{\theta}) = J(\underline{\theta}) = \|\underline{y} - \underline{\underline{\Phi}}\,\underline{\theta}\|_2^2 = (\underline{y} - \underline{\underline{\Phi}}\,\underline{\theta})^T(\underline{y} - \underline{\underline{\Phi}}\,\underline{\theta})$$

$$\left(\sum_{n=1}^{N} e^2(n) = \sum_{n=1}^{N} \left(y(n) - \underline{\phi}^T(u(n))\,\underline{\theta}\right)^2 \right)$$

$$\underline{a}^T \underline{b} = \sum a_k b_k = \underline{b}^T \underline{a}$$

$$\underline{a}^T \rightarrow \underline{a}^H \text{ IF COMPLEX}$$

$$\|\cdot\|_2^2 \begin{cases} \sum (\ )^2 \\ \sum |\ |^2 \end{cases}$$

$$(\underline{\underline{A}}\,\underline{\underline{B}})^T = \underline{\underline{B}}^T \underline{\underline{A}}^T$$

1. STANDARD LEAST-SQUARES

$\underline{\underline{\Phi}}$ = #R > #C NO SOLUTION

$$V = \underline{y}^T\underline{y} - \underline{\theta}^T\underline{\underline{\Phi}}^T\underline{y} - \underline{y}^T\underline{\underline{\Phi}}\,\underline{\theta} + \underline{\theta}^T\underline{\underline{\Phi}}^T\underline{\underline{\Phi}}\,\underline{\theta}$$

$$= \underline{y}^T\underline{y} - 2\underline{\theta}^T\underline{\underline{\Phi}}^T\underline{y} + \underline{\theta}^T(\underline{\underline{\Phi}}^T\underline{\underline{\Phi}})\underline{\theta}$$

$$\frac{\partial V}{\partial \underline{\theta}} = -2\underline{\underline{\Phi}}^T\underline{y} + 2(\underline{\underline{\Phi}}^T\underline{\underline{\Phi}})\underline{\theta} = \emptyset \qquad \boxed{\hat{\underline{\theta}} = (\underline{\underline{\Phi}}^T\underline{\underline{\Phi}})^{-1}\underline{\underline{\Phi}}^T\underline{y}}$$

$$\left(\frac{\partial(\underline{x}^T\underline{a})}{\partial \underline{x}} = \underline{a}\right)$$

IF INVERSE EXISTS