IF INVERSE EXISTS:

$$\underline{\lambda} = 2(\underline{\underline{\Phi}}\,\underline{\underline{\Phi}}^T)^{-1}\underline{y}$$

$$\boxed{\hat{\underline{\theta}} = \underline{\underline{\Phi}}^T(\underline{\underline{\Phi}}\,\underline{\underline{\Phi}}^T)^{-1}\underline{y}}$$

V. WEIGHTED, UNDERDETERMINED LEAST-SQUARES

$$\min \| \underline{\underline{W}}^{1/2}\underline{\theta} \|_2^2 \qquad \text{ETC.}$$

$$\boxed{\hat{\underline{\theta}} = \underline{\underline{W}}^{-1}\underline{\underline{\Phi}}^T(\underline{\underline{\Phi}}\,\underline{\underline{W}}^{-1}\underline{\underline{\Phi}}^T)^{-1}\underline{y}}$$

VI. REGULARIZATION

$$V = c_1 \|\underline{y} - \underline{\underline{\Phi}}\,\underline{\theta}\|_2^2 + c_2 \|\underline{\theta}\|_2^2$$

$$\hookrightarrow \quad V = \|\underline{y} - \underline{\underline{\Phi}}\,\underline{\theta}\|_2^2 + \lambda \|\underline{\theta}\|_2^2 \qquad \lambda > \phi$$

REGULARIZATION CONSTANT

$$\frac{\partial V}{\partial \underline{\theta}} = 2\,\underline{\underline{\Phi}}^T(\underline{\underline{\Phi}}\,\underline{\theta} - \underline{y}) + 2\lambda\,\underline{\theta} = \phi$$

$$\underline{\underline{\Phi}}^T\underline{y} = (\underline{\underline{\Phi}}^T\underline{\underline{\Phi}} + \lambda\,\underline{\underline{I}})\,\underline{\theta}$$

$$\boxed{\hat{\underline{\theta}} = (\underline{\underline{\Phi}}^T\underline{\underline{\Phi}} + \lambda\,\underline{\underline{I}})^{-1}\underline{\underline{\Phi}}^T\underline{y}}$$

VII. WEIGHTED REGULARIZATION

$$V = \|\underline{y} - \underline{\underline{\Phi}}\,\underline{\theta}\|_2^2 + \lambda \|\underline{\underline{A}}\,\underline{\theta}\|_2^2$$

$$\frac{\partial V}{\partial \underline{\theta}} = 2\,\underline{\underline{\Phi}}^T(\underline{\underline{\Phi}}\,\underline{\theta} - \underline{y}) + 2\lambda\,\underline{\underline{A}}^T\underline{\underline{A}}\,\underline{\theta} = \phi$$

$$(\underline{\underline{\Phi}}^T\underline{\underline{\Phi}} + \lambda\,\underline{\underline{A}}^T\underline{\underline{A}})\,\underline{\theta} = \underline{\underline{\Phi}}^T\underline{y}$$

$$\boxed{\hat{\underline{\theta}} = (\underline{\underline{\Phi}}^T\underline{\underline{\Phi}} + \lambda\,\underline{\underline{A}}^T\underline{\underline{A}})^{-1}\underline{\underline{\Phi}}^T\underline{y}}$$