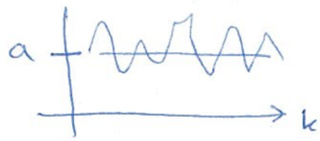
IX. EXERCISES

EX. 1 ESTIMATING CONSTANT LEVEL IN NOISE



NOISE INDEPENDENT FROM MEASUREMENT TO MEASUREMENT

$$y(k) = a + n_y(k)$$

$$y_M(k) = a = \underbrace{[1]}_{\underline{\phi}^T}\ \underbrace{[a]}_{\underline{\theta}}$$

N MEASUREMENTS

$$\underline{\Phi} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \qquad \underline{\Phi}^T\underline{\Phi} = N \qquad \underline{\Phi}^T \underline{y} = \sum_{k=1}^{N} y(k)$$

$$\boxed{\hat{\theta} = \hat{a} = \frac{1}{N}\sum_{k=1}^{N} y(k)}$$

EX. 2 LINEAR PHENOMENON IN NOISE

$$y(k) = a + b\,u(k) + n_y(k) \qquad y_M(k) = a + b\,u(k)$$

$$= [1\ \ u(k)]\begin{bmatrix} a \\ b \end{bmatrix} = \underline{\phi}^T \cdot \underline{\theta}$$

$$\underline{\Phi} = \begin{bmatrix} 1 & u(1) \\ 1 & u(2) \\ \vdots & \vdots \\ 1 & u(N) \end{bmatrix} \qquad \underline{\Phi}^T\underline{\Phi} = \begin{bmatrix} N & \Sigma u(k) \\ \Sigma u(k) & \Sigma u^2(k) \end{bmatrix}$$

$$\underline{\Phi}^T \underline{y} = \begin{bmatrix} \Sigma y(k) \\ \Sigma y(k) u(k) \end{bmatrix}$$

$$\underline{\hat{\theta}} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} N & \Sigma u(k) \\ \Sigma u(k) & \Sigma u^2(k) \end{bmatrix}^{-1} \cdot \begin{bmatrix} \Sigma y(k) \\ \Sigma y(k) u(k) \end{bmatrix} = \frac{1}{N\Sigma u^2(k) - (\Sigma u(k))^2}\begin{bmatrix} \Sigma u^2(k) & -\Sigma u(k) \\ -\Sigma u(k) & N \end{bmatrix}\begin{bmatrix} \Sigma y(k) \\ \Sigma y(k) u(k) \end{bmatrix}$$

$$= \dots$$

EX. 3

$$y_M(k) = a\,u(k) + b\,u(k-1) + c\,u(k-2)$$

$$\underline{\Phi} = \begin{bmatrix} u(1) & 0 & 0 \\ u(2) & u(1) & 0 \\ u(3) & u(2) & u(1) \\ \dots & \dots & \dots \\ u(N) & u(N-1) & u(N-2) \end{bmatrix} = [\ \ ]$$