

$$\underline{\Phi}^T \underline{\Phi} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

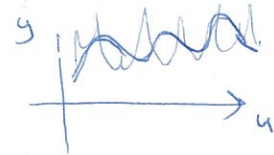
$$\underline{\Phi}^T \underline{y} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \end{matrix}$$

$$\hat{\underline{\theta}} = \left[\begin{matrix} \vdots \\ \vdots \\ \vdots \end{matrix} \right]^{-1} \left[\begin{matrix} \vdots \\ \vdots \\ \vdots \end{matrix} \right] \quad (\text{CONT.})$$

EX. 4 POLYNOMIAL REGRESSION

$$y_H(u) = \begin{bmatrix} 1 & u(u) & u^2(u) & \dots & u^M(u) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_M \end{bmatrix} = \underline{\Phi}^T(u(u)) \underline{\theta}$$

$$\underline{\Phi} = \begin{bmatrix} 1 & u(1) & u^2(1) & \dots & u^M(1) \\ 1 & u(2) & u^2(2) & \dots & u^M(2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & u(N) & u^2(N) & \dots & u^M(N) \end{bmatrix} \quad (\text{CONT.})$$



$$y(u) = NL[u(u)] + n_y(u)$$

EX. 5 ORTHOGONAL POLYNOMIAL REGRESSION

$$\phi_e(u(u)) = \text{Polynom}_e(u(u)) \begin{cases} u^e(u) & \text{POLYNOMIAL} \\ T_e(u(u)) & \text{ORTHOGONAL POLYNOMIAL} \end{cases}$$

$$y_H(u) = \underline{\Phi}^T(u(u)) \underline{\theta}$$

$$\sum_{u=1}^N T_e(u(u)) T_m(u(u)) = \delta_{em}$$

$$= \begin{bmatrix} p_1(u(u)) & p_2(u(u)) & \dots & p_M(u(u)) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_M \end{bmatrix}$$

$$[\underline{\Phi}^T \underline{\Phi}]_{em} = \sum_{u=1}^N \phi_e(u(u)) \phi_m(u(u)) \rightarrow \sum_u^{e+m} u(u) \quad (\text{CONT.})$$

$$(\underline{\Phi}^T \underline{\Phi})^{-1} \rightarrow \begin{bmatrix} N & \sum u & \dots & \sum u^M \\ \sum u & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \sum u^{2M} \end{bmatrix}^{-1}$$

$$\rightarrow \begin{bmatrix} 1 & \dots & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & \dots & 1 \end{bmatrix}$$

$$[\underline{\Phi}^T \underline{y}]_e \rightarrow \sum_k u^e(u) y(u)$$

$$\rightarrow \sum_k T_e(u(u)) y(u)$$