VIII. CONSTRAINED LEAST-SQUARES

$$\min_{\theta} \| y - \Phi \theta \|_2^2 \quad \text{UNDER CONDITION}$$

$$C \theta = b$$

$$\mathcal{L} = \| y - \Phi \theta \|_2^2 + \lambda^T (C \theta - b)$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = 2 \Phi^T (\Phi \theta - y) + C^T \lambda = \phi$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = C \theta - b = \phi$$

$$\hat{\theta} = (\Phi^T \Phi)^{-1} \left(\Phi^T y - \frac{1}{2} C^T \lambda \right)$$

$$b = C (\Phi^T \Phi)^{-1} \left(\Phi^T y - \frac{1}{2} C^T \lambda \right)$$

$$\lambda = 2 \left[C (\Phi^T \Phi)^{-1} C^T \right]^{-1} \left[C (\Phi^T \Phi)^{-1} \Phi^T y - b \right]$$

$$\boxed{\hat{\theta} = (\Phi^T \Phi)^{-1} \left(\Phi^T y - C^T (C (\Phi^T \Phi)^{-1} C^T)^{-1} (C (\Phi^T \Phi)^{-1} \Phi^T y - b) \right)}$$