5. ESTIMATION FROM FREQUENCY-DOMAIN DATA

(MEASURED AT ARBITRARY FREQUENCIES)

$$Z^N = \{U(\omega_k), Y(\omega_k)\}_{k=1}^N$$

PARSEVAL: $V_N(\theta, Z^N)\Big|_{\text{TIME DOMAIN}} = V_N(\theta, Z^N)\Big|_{\text{FREQUENCY DOMAIN}}$

$$V_N(\theta) = \sum_{k=1}^{N} |Y(\omega_k) - G(\omega_k, \theta) U(\omega_k)|^2 \cdot \frac{1}{|H(e^{i\omega_k}, \theta)|^2}$$

- H DOES NOT DEPEND ON θ — KNOWN NOISE MODEL
- H DOES DEPEND ON $\theta \rightarrow \hat{\theta}_N$ MAY BE INCONSISTENT

MAXIMUM LIKELIHOOD IN FREQUENCY-DOMAIN

$$Y(k) = G(\Omega_k, \theta) U(k) + H(\Omega_k, \theta) E(k)$$

$$E(k) \sim N_c(0, \sigma^2)$$

$$\downarrow$$

$$N_c\left(G(\Omega_k, \theta) U(k),\ \sigma^2 |H(\Omega_k, \theta)|^2\right)$$

(TRANSIENTS LEAKAGE OMITTED OR PERIODIC INPUTS)

$$V_N(\theta) \rightarrow \frac{1}{2} N \log \sigma^2 + \sum \log |H(\Omega_k, \theta)|^2 + \text{CONSTANTS}$$
$$+ \frac{1}{2} \sum \frac{1}{\sigma^2} |Y(\omega_k) - G(\Omega_k, \theta) U(\omega_k)|^2 \frac{1}{|H(\Omega_k, \theta)|^2}$$

$$\hat{\theta}_N^{ML} = \arg\min_\theta \Big[\underbrace{N\left(\frac{1}{N} \sum \frac{1}{\sigma^2} |Y - GU|^2 \frac{1}{|H|^2 \sigma^2}\right)}_{\frac{N}{\sigma^2} W_N(\theta)} + \underbrace{2 \sum \log |H|}_{(*)} \Big]$$

(*) EXTRA TERM: — NOISE MODEL GIVEN, FIXED, NOT θ DEPENDENT

$$-\int_{-\pi}^{\pi} \log |H|^2 \, d\omega = \phi \longrightarrow \sum_{1}^{N} \log |H|^2 \quad \Omega_k = \frac{2\pi k}{N} \rightarrow \phi$$

REGULAR SPACING

$$\neq \phi \text{ IN GENERAL}$$

$$W_N(\theta) = \frac{1}{N} \sum \left|\hat{G} - G\right|^2 \left|\frac{U}{H}\right|^2$$

LIKE ETFE LS WITH WEIGHTING