$\hat{G}_N = G(\cdot,\theta) + V(\omega)$ — VARIANCE — $\frac{\Phi_v(2\pi k/N)}{|U_N(2\pi k/N)|}$

$Q_N(\omega,\theta) = \frac{1}{\text{VARIANCE}}$ ~ SNR AT THE FREQUENCY IN QUESTION → OPTIMAL WEIGHTING FOR LINEAR REGRESSION

4. LTI PARAMETER IDENTIFICATION IN TIME DOMAIN

LINEAR REGRESSION & LS

$\hat{y}(t|\theta) = \underline{\varphi}^T(t)\underline{\theta} + \varepsilon(t)$

$V_N(\theta, Z^N) = \frac{1}{N}\sum_{t=1}^{N} \frac{1}{2}\left[y(t) - \underline{\varphi}^T(t)\underline{\theta}\right]^2$

$\hat{\theta}_N^{LS} = \underset{\theta\in D_M}{\arg\min}\ V_N(\theta, Z^N) = \left[\underbrace{\frac{1}{N}\sum_{t=1}^{N}\underline{\varphi}(t)\underline{\varphi}^T(t)}_{R_N(N)}\right]^{-1}\underbrace{\frac{1}{N}\sum_{t=1}^{N}\underline{\varphi}(t)y(t)}_{f_N(N)}$

$\underline{\varphi}(t)$ – LAGGED $y(t)$, $u(t)$

$R(N)$ – AUTO & CROSS COVARIANCES (CORRELATION ANALYSIS)

PROPERTIES OF THE SOLUTION $\quad y(t) = \underline{\varphi}^T(t)\theta_0 + v_0(t)$

$\hat{\theta}_N^{LS} - \theta_0 \Rightarrow \lim_{N\to\infty} \bar{R}(N)\ \frac{1}{N}\sum \underline{\varphi}(t)v_0(t) = (\bar{R}^*)^{-1} f^* \to \phi\ (?)$

R^* – NON SINGULAR

$f^* \simeq \phi$ – $v_0(t)$ WHITE; $v(t)$ & $u(t)$ INDEPENDENT; NO COMMON DYNAMICS; NO CORRELATION

} ESTIMATE CONSISTENT

STOCHASTIC FRAMEWORK

$f_y(\theta, X^N)$: PDF OF y

$y^N \in \mathbb{R}^N$ RANDOM OBSERVATIONS

$\hat{\theta}(y^N) : \mathbb{R}^N \to \mathbb{R}^d$