6 15

ESTIMATING VOLTERRA KERNELS IN TIME DOMAIN

$$y(t) = \sum^{M_1} g^1(k)\, u(k-t) + \sum\sum^{M_2} g^2(k_1,k_2)\, u(k_1-t)\, u(k_2-t) + \dots$$

$$\underline{\theta} = [g^1(0)\ g^1(1) \dots g^1(M_1)\ g^2(0,0)\ g^2(0,1) \dots g^2(0,M_2)\ g^2(1,1), \dots]$$

$$\underline{\varphi}(t) = [u(-t) \dots u(M_1-t)\ u(-t)u(-t) \dots \quad \dots \quad u(M_2-t)u(M_2-t)]$$

$$\underline{y} = \underline{\varphi}^T \underline{\theta} + \underline{e} \qquad LS: \ \hat{\underline{\theta}} = (\underline{\varphi}\,\underline{\varphi}^T)^{-1} \underline{\varphi}^T \underline{y}$$

– CURSE OF DIMENSIONALITY:

ORDER / MEMORY

m $\quad$ n_m

$$n_{\theta_m} = \frac{\prod_{i=0}^{m-1} (n_m + i)}{m!}$$

(1) M_1

(2) $M_2(M_2+1)/2$

... ...

↓

PLENTY OF DATA

↓

REGULARIZATION
(FOR NL KERNELS)

(APPROXIMATIONS: KERNEL SURFACE ≃ SPLINES)

REGULARIZATION
FOR LINEAR KERNEL

$$\hat{\underline{\theta}}_N^{REG} = \arg\min_{\underline{\theta}} \|\underline{Y}_N - \underline{\Phi}_N^T \underline{\theta}\|^2 + \underline{\theta}^T \underline{D}\,\underline{\theta} = (\underline{\Phi}_N \underline{\Phi}_N^T + \underline{D})^{-1} (\underline{\Phi}_N \underline{\Phi}_N^T)\, \hat{\underline{\theta}}_N^{LS}$$

$$(\underline{\Phi}_N \underline{\Phi}_N^T)^{-1} \underline{\Phi}^T \underline{Y}_N$$

BAYES APPROACH: $\underline{D} = \sigma^2 \underline{\underline{P}}_N^{-1}$

A PRIORI COVARIANCE OF $\underline{\theta}$

E.G.: $\underline{P}_{DC}(i,j) = c \cdot \rho^{|i-j|}\, \lambda^{(i+j)/2}$

DIAGONAL / CORRELATED

PHYSICAL A PRIORI
– IRF DECAYING
SMOOTH

↓ NL

$$\hat{\underline{\theta}}_N^{REG} = \arg\min_{\theta} \|\underline{Y}_N - \underline{\Phi}_N^T \theta\|^2 + [\underline{\theta}_1^T\ \underline{\theta}_2^T] \begin{pmatrix} \underline{D}_1 & 0 \\ 0 & \underline{\underline{D}}_2 \end{pmatrix} \begin{bmatrix} \underline{\theta}_1 \\ \underline{\theta}_2 \end{bmatrix}$$

$\sigma^2 \underline{\underline{P}}_{n,2}^{-1}$

EXTENSIVE RESEARCH

ALSO HIGHER ORDERS