$$G_B(l) = \sum_{\alpha=2}^{\infty} G_B^{2\alpha-1}(l) + O(M^{-1})$$

$$\downarrow$$

$$\frac{C_\alpha}{M^{\alpha-1}} \sum_{k_1,\dots,k_{2\alpha-1}=1}^{M/2} G^{\alpha}(l, k_1, -k_1, \dots, k_{2\alpha-1}, -k_{2\alpha-1}) \prod_{i=1}^{2\alpha-1} |U(k_i)|^2$$

$$C_\alpha = 2^{\alpha-1}(\alpha-1)!!$$

BLA DERIVATION – AP EXAMPLE

$$\text{LET: } y(l) = G^1(l)U(l) + \sum_{k=-\frac{M}{2},\, k\neq 0}^{M/2} G^2(k, l-k)\,U(k)\,U(l-k) + \sum_{k_1,k_2=-M/2}^{M/2} G^3(k_1,k_2,l-k_1-k_2)\,U(k_1)U(k_2)\,U(l-k_1-k_2)$$

$$G_{BLA}(l) = \frac{\mathcal{E}\{y(l)\bar{U}(l)\}}{\mathcal{E}\{|U(l)|^2\}}$$

$$\hat{U}_l^2$$

$$= \frac{\mathcal{E}\{G^1(l)U(l)\bar{U}(l) + \sum G^2(k,l-k)U(k)U(l-k)\bar{U}(l) + \sum\sum G^3(k_1,k_2,l-k_1-k_2)U(k_1)U(k_2)U(l-k_1-k_2)\times\bar{U}(l)\}}{|U(l)|^2}$$

$$= \mathcal{E}\{G^1(l)\frac{|U(l)|^2}{|U(l)|^2} + \sum G^2(k,l-k)\frac{\hat{U}(k)\hat{U}(l-k)\hat{U}(l)}{|\hat{U}(l)|^2}e^{j(\varphi_k+\varphi_{l-k}-\varphi_l)} + \sum\sum G^3(k_1,k_2,l-k_1-k_2)\frac{\hat{U}\hat{U}\hat{U}\hat{U}}{|\hat{U}(l)|^2}e^{j(\varphi_{k_1}+\varphi_{k_2}+\varphi_{l-k_1-k_2}-\varphi_l)}\}$$

$\mathcal{E}$ $\mathcal{E}$

$$\mathcal{E}\{e^{j\sum\varphi_i}\} = \begin{cases} = \phi & \text{ODD NUMBER} \to \text{EVEN NL} \\ \neq \phi & \text{EVEN NUMBER} \to \text{ODD NL} \end{cases} \to \mathcal{E}\{e^{j\sum\varphi}\} = 1$$

φ_{k_1}	φ_{k_2}	$\varphi_{l-k_1-k_2}$	φ_{-l}
k_1	k_2	$l-k_1-k_2$	$-l$
k_1	$-k_1$	l	$-l$
l	k_2	$-k_2$	$-l$
k_1	l	$-k_1$	$-l$

FREQUENCY (AND PHASE) PAIRINGS

$k_1 = k_2$ 3×

$k_1 = l$

$k_2 = l$