ERROR ANALYSIS

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$$G(j\omega_k) = \frac{Y_0(k) + N_Y(k)}{U_0(k) + N_U(k)} = G_0(j\omega_k)\,\frac{1 + \frac{N_Y(k)}{Y_0(k)}}{1 + \frac{N_U(k)}{U_0(k)}}$$

$$= G_0(j\omega_k)\left(1 + \frac{N_Y(k)}{Y_0(k)}\right)\underbrace{\left(1 - \frac{N_U(k)}{U_0(k)} + \left(\frac{N_U}{U_0}\right)^2\right) + \text{HIGHER ORDER TERMS}}_{(*)}$$

BIAS: $-(N1, N2, *)$

$$\mathcal{E}\{G(j\omega_k)\} = G_0(j\omega_k)$$

ALSO FOR FULL TAYLOR SERIES, IF SERIES CONVERGES

$$\frac{1}{1+x} \longrightarrow |x| < 1$$

CONDITION: $\left|\frac{N_U(k)}{U_0(k)}\right| < 1$ (ALWAYS VIOLATED FOR GAUSSIAN)

- HIGH SNR $\sigma_U(k) < |U_0(k)|$ GOOD APPROXIMATION
- LOW SNR ?

FOR $U_0(k)$ FIXED

$\sigma_{YU} = \phi$

RELATIVE BIAS:

$$b(k) = \frac{\mathcal{E}\{G(j\omega_k)\}}{G_0(j\omega_k)} - 1$$

$$= -\mathrm{EXP}\left(-|U_0(k)|^2/\sigma_U^2(k)\right)$$

N_U, N_Y CORRELATED (FEEDBACK)

$$\rho(k) = \sigma_{YU}^2(k)/\sigma_U(k)\sigma_Y(k)$$

$$b(k) = -\mathrm{EXP}\left(-|U_0(k)|^2/\sigma_U^2(k)\right)\left(1 - \rho(k)\,\frac{U_0(k)/\sigma_U(k)}{Y_0(k)/\sigma_Y(k)}\right)$$

VARIANCE (FIRST ORDER TERMS ONLY)

$$G(j\omega_k) \simeq G_0(j\omega_k)\left(1 + \frac{N_Y(k)}{Y_0(k)} - \frac{N_U(k)}{U_0(k)}\right) = G_0(j\omega_k) + N_G(j\omega_k)$$

$$N_G(j\omega_k) = G_0(j\omega_k)\left(\frac{N_Y}{Y_0} - \frac{N_U}{U_0}\right) \qquad \mathcal{E}\{N_G\} = \phi$$