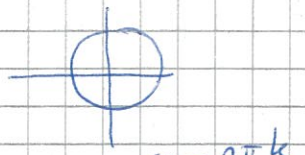
FREQUENCY RESPONSE ANALYSIS IN FREQUENCY DOMAIN

$u(t)$ SAMPLED $\{u(t)\}_{t=1}^{N}$ $\quad U_N(\omega) = \frac{1}{\sqrt{N}} \sum_{t=1}^{N} u(t)\, e^{-j\omega t}$ (DFT)

$$u(t) = \frac{1}{\sqrt{N}} \sum_{k=-N/2+1}^{N/2} U_N\left(\frac{2\pi k}{N}\right) e^{j\frac{2\pi k t}{N}}$$

$$= \frac{1}{\sqrt{N}} \sum_{k=1}^{N} U_N\left(\frac{2\pi k}{N}\right) e^{j\frac{2\pi k t}{N}}$$



$\omega = 2\pi \frac{k}{N}$

$$U_N(\omega) = U_N(\omega + 2\pi\ell)$$

$$\overline{U_N(\omega)} = U_N(-\omega)$$

PERIODOGRAM: $|U_N(\omega)|^2 \sim u(t)$ ENERGY AT ω

PARSEVAL EQU.: $\sum_{k=1}^{N} \left|U_N\left(\frac{2\pi k}{N}\right)\right|^2 = \sum_{t=1}^{N} u^2(t)$ $\quad \left((x,y) = (\underline{F}x, \underline{F}y)\right)$

$|U_N(\omega)|^2 \to \phi_u(\omega)$

ESTIMATE OF SPECTRUM

$$\left(\frac{1}{N} \sum_{k=1}^{N} e^{j\frac{2\pi k v}{N}} = \begin{cases} 1 & v = \phi \\ 0 & v \neq \phi \end{cases} \quad 1 \le v < N \right)$$

$$\underline{\underline{F}}^{*} \cdot \underline{\underline{F}} = N \cdot \underline{\underline{I}}$$

EXAMPLE:

$u(t) = A \cos \omega_0 t \qquad \omega_0 = \frac{2\pi}{N_0} \qquad N = S \cdot N_0$

$$U_N(\omega) = \frac{1}{\sqrt{N}} \frac{A}{2} \sum_{t=1}^{N} \left(e^{j(\omega_0 - \omega)t} + e^{-j(\omega_0 + \omega)t} \right)$$

$$|U_N(\omega)|^2 = \begin{cases} N \cdot \frac{A^2}{4} & \omega = \pm\omega_0 \\ \phi & \omega = \frac{2\pi k}{N} \quad k \neq S \end{cases}$$

IF $u(t) = u(t + N_0)$ PERIODIC MULTI

$$|U_N(\omega_r)|^2 = \begin{cases} S \cdot |A_r|^2 & r = 0, \pm 1, \pm 2, \dots, \pm \frac{N_0}{2} \\ & \text{AT HARMONIC FREQUENCIES} \\ \phi & \text{IN BETEEN} \end{cases}$$