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BIAS OF ETFE:

$$\mathcal{E}\{\hat{G}_N(e^{j\omega})\} = G_0(e^{j\omega}) + \frac{\rho_1(N)}{U_N(\omega)}$$

$$|\rho_1(N)| \le \frac{C_1}{\sqrt{N}} \qquad \left(\text{OR } \rho_1 = \emptyset \text{ IF } u(t) \text{ PERIODIC}\right)!$$

$$C_1 = \left(2 \sum_{1}^{\infty} |k g_0(k)|\right) \max |u(t)|$$

VARIANCE OF ETFE: (MSE)

$$\mathcal{E}\left\{\left(\hat{G}_N(e^{j\omega}) - G_0(e^{j\omega})\right)\left(\hat{G}_N(e^{-j\xi}) - G_0(e^{-j\xi})\right)\right\} =$$

$$= \begin{cases} \dfrac{\Phi_v(\omega) + \rho_2(N)}{|U_N(\omega)|^2} & \xi = \omega \\[2ex] \dfrac{\rho_2(N)}{U_N(\omega)\, U_N(-\xi)} & |\xi - \omega| = \dfrac{2\pi k}{N} \quad k = 1, 2, \dots, N-1 \end{cases}$$

$$|\rho^2(N)| \le \frac{C_2}{N}$$

$$C_2 = C_1^2 + \sum_{-\infty}^{\infty} |\tau R(\tau)|$$

$$\left(C_1 = \emptyset \text{ IF } u(t) \text{ PERIODIC}\right)$$

CASE 1 – u(t) REALIZATION OF RANDOM PROCESS

$|U_N(\omega)|^2$ ERRATIC FUNCTION OF ω, FLUCTUATION AROUND $\Phi_u(\omega)$, CLOSE TO $\emptyset$ IN NOMINATOR $\rightarrow$
$\rightarrow$ EXCESSIVE VARIANCE (APPROX.!)

ETFE ASYMPTOTICALLY UNBIASED (OUTPUT NOISE ONLY) AT INCREASINGLY MANY (WITH N) FREQUENCIES

ETFE VARIANCE DOES NOT DECAY, AS N $\nearrow$
└ NOISE-TO-SIGNAL RATIO AT ω

ESTIMATES AT DIFFERENT FREQUENCIES ARE ASYMPTOTICALLY UNCORRELATED