

BAYES - APPROACH

(99)

$$\underline{\theta} \sim \mathcal{N}(\underline{\theta}, \underline{P}) \quad \underline{y}_N = \underline{\Phi}_N \underline{\theta} + \underline{e} \quad e(k) \sim \mathcal{N}(0, \sigma^2)$$

$$p(\underline{\theta} | \underline{y}_N) \sim \mathcal{N}(\underline{\hat{\theta}}_N^{\text{MAP}}, \underline{P}^{\text{MAP}})$$

$$\underline{\hat{\theta}}_N^{\text{MAP}} = (\underline{\Phi}_N^T \underline{\Phi}_N + \sigma^2 \underline{P}^{-1})^{-1} \underline{\Phi}_N^T \underline{y}_N$$


$$\underline{P}^{\text{MAP}} = \sigma^2 (\underline{\Phi}_N^T \underline{\Phi}_N + \sigma^2 \underline{P}^{-1})^{-1}$$

$$\left[\underline{\hat{\theta}}_N^{\text{REG}} = \underline{\hat{\theta}}_N^{\text{MAP}} \leftrightarrow \underline{D} = \sigma^2 \underline{P}^{-1} \right] \quad (\text{GAZI } \underline{P})?$$

COVARIANCE MATRIX FOR STABILITY & SMOOTHNESS

(A PRIORI INFORMATION INTO THE PRIOR)

IF SYSTEM STABLE — FIR $g(t)$ DECAYS EXPONENTIALLY 

IF IR SMOOTH \Rightarrow NEIGHBOURING VALUES HAVE POSITIVE CORRELATIONS 

TYPOICAL REGULARIZATION (COVARIANCE) MATRIX (PILOMETTO, NICOLAO)
 ~ 2010

DIAGONAL / CORRELATED (DC)

$$P_{DC}(i, j) = c \cdot \underset{\text{DECAY}}{\rho^{i-j}} \underset{\text{CORREL.}}{\lambda^{\frac{|i-j|}{2}}}$$

$$\begin{aligned} c &\geq 0 \\ 0 &\leq \lambda < 1 \\ |\rho| &\leq 1 \end{aligned}$$

TUNED / CORRELATED (TC)

$$P_{TC}(i, j) = c \cdot \underset{\text{DECAY + CORREL.}}{\min(\lambda^i, \lambda^j)}$$

$$\rho = \sqrt{\lambda}$$

OTHER KERNELS

$$P_{CS}(i, j) = \begin{cases} c \frac{i^2}{2} (j - i/3) & i \geq j \\ c \frac{j^2}{2} (i - j/3) & i < j \end{cases} \quad \text{CUBIC SPLINE}$$