EXAMPLE 4:

$$y(n) = s(n,\theta) + \underbrace{e(n)}_{N(0,\sigma^2)}$$

$$\frac{1}{\text{CONST.}}\, e^{-\frac{1}{2\sigma^2}\sum (y(n) - s(n,\theta))^2}$$

$$\frac{\partial \ln p}{\partial \theta} = \frac{1}{\sigma^2} \sum (\bigcirc - \bigcirc) \frac{\partial s}{\partial \theta}$$

$$\frac{\partial^2 \ln p}{\partial \theta^2} = \frac{1}{\sigma^2} \sum (\bigcirc - \bigcirc) \frac{\partial^2 s}{\partial \theta^2} - \left(\frac{\partial s}{\partial \theta}\right)^2 \quad (*)$$

$$= -\mathcal{E}\{(*)\} = \frac{1}{\sigma^2} \sum_{n=0}^{N-1} \left(\frac{\partial s(n,\theta)}{\partial \theta}\right)^2 = M(\theta)$$

EXAMPLE 5:

$$\underline{y} = \underline{\underline{\Phi}}\, \underline{\theta} + \underbrace{\underline{e}}_{N(\underline{0},\, \sigma^2 \underline{\underline{I}})}$$

$$p(\underline{y}, \underline{\theta}) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left[-\frac{1}{2\sigma^2} (\underline{y} - \underline{\underline{\Phi}}\,\underline{\theta})^T (\underline{y} - \underline{\underline{\Phi}}\,\underline{\theta})\right]$$

$$\frac{\partial \ln p}{\partial \underline{\theta}} = \frac{1}{\sigma^2}\left[\underline{\underline{\Phi}}^T \underline{y} - \underline{\underline{\Phi}}^T \underline{\underline{\Phi}}\, \underline{\theta}\right] \qquad \underline{\underline{MI}}(\theta) = -\frac{\partial}{\partial \underline{\theta}}^T \left(\frac{\partial \ln p}{\partial \underline{\theta}}\right)$$

$$= \frac{1}{\sigma^2} \underline{\underline{\Phi}}^T \underline{\underline{\Phi}}$$

$$\boxed{\operatorname{cov}(\hat{\underline{\theta}}) = \underline{\underline{MI}}^{-1}(\theta) = \sigma^2 (\underline{\underline{\Phi}}^T \underline{\underline{\Phi}})^{-1}}$$

EXAMPLE 6:

$$y(n) = A \cos(2\pi f_0 n + \phi) + \underbrace{e(n)}_{N(\underline{0},\, \sigma^2 \underline{\underline{I}})}$$

$$p(\underline{y}, \phi) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left[-\frac{1}{2\sigma^2} \sum_{n=0}^{N-1} [y(n) - A\cos(2\pi f_0 n + \phi)]^2\right]$$

$$\frac{\partial \ln p}{\partial \phi} = -\frac{1}{\sigma^2} \sum (\bigcirc - \bigcirc)\, A \sin(2\pi f_0 n + \phi)$$

$$= -\frac{A}{\sigma^2} \sum \left[y(n) \sin(2\pi f_0 n + \phi) - \frac{A}{2} \sin(4\pi f_0 n + 2\phi)\right]$$

$$\frac{\partial^2 \ln p}{\partial \phi^2} = -\frac{A}{\sigma^2} \sum \left[y(n) \cos(2\pi f_0 n + \phi) - A \cos(4\pi f_0 n + 2\phi)\right]$$

$$\mathcal{E}\{\downarrow\} = -\frac{A}{\sigma^2} \sum \left[\mathcal{E}\{y(n)\} \cos(2\pi f_0 n + \phi) - A \cos(4\pi f_0 n + 2\phi)\right]$$