AND

$$p(\underline{x}|\underline{y}) = \mathcal{N}\left(\underline{x} \,\middle|\, \underline{\underline{\Sigma}}_N \{\underline{\underline{A}}^T \underline{\underline{L}}(\underline{y}-\underline{b}) + \underline{\underline{\Lambda}}\,\mu\}, \underline{\underline{\Sigma}}_N\right)$$

$$= \mathcal{N}(\underline{x} \,|\, \underline{m}_N, \underline{\underline{\Sigma}}_N)$$

$$\underline{\underline{\Sigma}}_N = (\underline{\underline{\Lambda}} + \underline{\underline{A}}^T \underline{\underline{L}}\,\underline{\underline{A}})^{-1}$$

EXAMPLE 7:

$$y(n) = \theta_0 + e(n) = y_M(n) + e(n)$$

$N = 1$ ONE MEASUREMENT

$$y_M = \underline{\Phi}^T \underline{\theta} \qquad [1][\theta_0]$$

$$p(\theta) = \frac{1}{\sqrt{2\pi}\,\sigma_\theta} \exp\left[-\frac{1}{2\sigma_\theta^2}(\theta-\theta_0)^2\right] \quad \leftarrow \text{PRIOR}$$

$$p(y|\theta) = \frac{1}{\sqrt{2\pi}\,\sigma_e} \exp\left[-\frac{1}{2\sigma_e^2}(y-\theta)^2\right] \quad \leftarrow \text{MEASUREMENT}$$

$$\ell(\theta) \Rightarrow \frac{\partial}{\partial\theta}\left[-\frac{1}{2\sigma_e^2}(y-\theta)^2 - \frac{1}{2\sigma_\theta^2}(\theta-\theta_0)^2\right]$$

$$\frac{(y-\theta)}{\sigma_e^2} - \frac{(\theta-\theta_0)}{\sigma_\theta^2} = \phi$$

$$\hat{\theta}_{MAP} = \frac{\frac{1}{\sigma_\theta^2}\theta_0 + \frac{1}{\sigma_e^2} y}{\frac{1}{\sigma_e^2} + \frac{1}{\sigma_\theta^2}}$$

EXAMPLE 8: MORE MEASUREMENTS

$$p(y(n)|\underline{\theta}) \sim \mathcal{N}(\underbrace{\underline{\Phi}\,\underline{\theta}}_{[1]\cdot\theta}, \sigma_e^2) = \frac{1}{\sqrt{2\pi}\,\sigma_e} \exp\left[-\frac{1}{2\sigma_e^2}(y(n)-\theta)^2\right]$$

$$p(\underline{y}|\theta) \sim \mathcal{N}(\underbrace{\underline{\Phi}\,\underline{\theta}}_{\begin{bmatrix}1\\1\\\vdots\\1\end{bmatrix}\theta}, \sigma_e^2 \underline{\underline{I}}) = \left(\frac{1}{\sqrt{2\pi}}\right)^N \underbrace{\frac{1}{\sigma_e^N}}_{|\sigma_e^2 \underline{\underline{I}}|^{1/2}} \exp\left[\underbrace{-\frac{1}{2\sigma_e^2}\sum(y(n)-\theta)^2}_{-\frac{1}{2}(\underline{y}-\underline{\Phi}\,\underline{\theta})^T(\sigma_e^2 \underline{\underline{I}})^{-1}(\underline{y}-\underline{\Phi}\,\underline{\theta})}\right]$$

$$\frac{\partial}{\partial\theta} \rightarrow \frac{1}{\sigma_e^2}\left(\sum_n (y(n)-\theta)\right) - \frac{1}{\sigma_\theta^2}(\theta-\theta_0) = \phi$$

$\frac{1}{\sigma_e^2} = \beta$ (NOISY MEASUREMENT PRECISION)

$\frac{1}{\sigma_\theta^2} = \alpha$ (A PRIORI KNOWLEDGE PRECISION)

$$\boxed{\hat{\theta}_{MAP} = \frac{\alpha\theta_0 + N\beta\underbrace{\left(\frac{1}{N}\sum y(n)\right)}_{\hat{\theta}_{ML}}}{\alpha + N\beta}}$$