$$\frac{\partial^2 \ell}{\partial a^2} = -\frac{N}{\sigma^2} s^2$$

$$\frac{\partial^2 \ell}{\partial a \partial b} = \frac{\partial^2 \ell}{\partial b \partial a} = -\frac{N}{\sigma^2}\mu$$

$$\frac{\partial^2 \ell}{\partial b^2} = -\frac{N}{\sigma^2}$$

$$\underline{\underline{M}} = \frac{N}{\sigma^2}\begin{bmatrix} s^2 & \mu \\ \mu & 1 \end{bmatrix}$$

$$CR(a,b) = \underline{\underline{M}}^{-1} = \frac{\sigma^2}{N}\frac{1}{s^2-\mu^2}\begin{bmatrix} 1 & -\mu \\ -\mu & s^2 \end{bmatrix}$$

- IF $\sigma^2 \uparrow$, THEN $CR \uparrow$
- IF $N \uparrow$, THEN $CR \downarrow$ $\sim \frac{1}{\sqrt{N}}$ ERROR DECAY
- CR DEPENDS ON $\{t_n\}$ → OPTIMAL EXPERIMENT DESIGN

- $\sigma_a^2(a,b) = \frac{\sigma^2}{N(s^2-\mu^2)} \geq \sigma_a^2(a) = \frac{\sigma^2}{N s^2}$

  MORE PARAMETERS → LESS ACCURACY (HIGHER CR BOUND)

- OVERFITTING
  - REGULARIZED ML:

$$\underline{\hat{\theta}}_{ML,R} = \arg\min_{\underline{\theta}} \left[\ell(\underline{\theta}) + \lambda \|\underline{\theta}\|^2 \right]$$

WHAT IS A GOOD VALUE ?

7. BAYES APPROACH – MAXIMUM A POSTERIORI ESTIMATE

RANDOM
- $\theta \rightarrow p(\theta)$ — PRIOR DISTRIBUTION
- $e(n) \rightarrow y(n) \rightarrow p(D_N|\theta)$ — CONDITIONAL DISTR. LIKELIHOOD

BAYES THEOREM:

DERIVATION

$$p(\theta|D_N) = \frac{p(D_N|\theta)\, p(\theta)}{p(D_N)} \sim p(D_N|\theta)\, p(\theta)$$

POSTERIOR DISTRIBUTION $\sim$ LIKELIHOOD $*$ PRIOR