$$MSE(\hat{\theta}) = \mathcal{E}\{(\hat{\theta}-\theta)^2\} \geq \frac{|1+b'(\theta)|^2}{M(\theta)} + b(\theta)^2$$

$\left((1+b')^2\right.$ MAYBE < 1

BOUND FOR BIASED MAYBE LESS, THEN FOR UNBIASED

MULTIVARIABLE:

UNBIASED: $\quad cov(\underline{\hat{\theta}}) - \underline{\underline{M}}^{-1} \geq \phi \quad$ POSITIVE SEMI DEFINITE

BIASED:

$$cov(\underline{\hat{\theta}}) \geq \frac{\partial \psi(\underline{\theta})}{\partial \underline{\theta}} \underline{\underline{M}}^{-1}(\underline{\theta}) \left(\frac{\partial \psi(\underline{\theta})}{\partial \underline{\theta}}\right)^T$$

$$\mathcal{E}\{(\underline{\hat{\theta}}-\underline{\theta})(\underline{\hat{\theta}}-\underline{\theta})^T\}$$

$$\left[\underline{\underline{M}}\right]_{mk} = \mathcal{E}\left\{\frac{\partial}{\partial\theta_m} \ln p(\,) \frac{\partial}{\partial\theta_k} \ln p\right\} = -\mathcal{E}\left\{\frac{\partial^2}{\partial\theta_m \partial\theta_k} \ln p\right\}$$

MINIMUM VARIANCE UNBIASED ESTIMATE:

$$\left(cov(\underline{\hat{\theta}}) \overset{!?}{=} \underline{\underline{M}}^{-1}(\theta) \right) \qquad \frac{\partial \ln p(\underline{x},\theta)}{\partial \theta} = \underline{\underline{M}}(\theta)\left(\underline{g}(\underline{x}) - \underline{\theta}\right)$$

FOR SOME $g(\cdot)$

EXAMPLE 3: $\quad y(n) = a + e(n) \qquad n = 0, 1, \ldots, N-1$

$$p(\underline{y}, a) \sim \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left(-\frac{1}{2\sigma^2}\sum_{n=0}^{N-1}(y(n)-a)^2\right)$$

$$\frac{\partial \ln p}{\partial a} = \frac{N}{\sigma^2}\left(\frac{1}{N}\sum_n y(n) - a\right)$$

$$\frac{\partial^2 \ln p}{\partial a^2} = -\frac{N}{\sigma^2} \qquad var(\hat{a}) \geq \frac{\sigma^2}{N}$$