- $\alpha \downarrow \phi$ $\quad \hat{\theta}_{MAP} \rightarrow \hat{\theta}_{ML}$ (ONLY DATA)

- $\alpha \uparrow \infty$ $\quad \hat{\theta}_{MAP} \rightarrow \theta_0$ (ONLY A PRIORI INFORMATION)

$$\hat{\theta}_{MAP} = \left(\frac{1}{1+\frac{N\beta}{\alpha}}\right)\theta_0 + \left(\frac{\frac{N\beta}{\alpha}}{1+\frac{N\beta}{\alpha}}\right)\left[\frac{1}{N}\sum_n y(n)\right]$$

- IF $\alpha \gg \beta$ (NOISY MEASUREMENTS)
  - N SMALL $\rightarrow \theta_0$ BETTER INFORMATION
  - $N \gg 1 \rightarrow$ ML — " — " —

- IF $\theta_0 \equiv 0$ $\quad \hat{\theta}_{MAP} \simeq \hat{\theta}_{ML}$ AS $N \rightarrow \infty$

- IF $\theta_0$ TRUE VALUE: $\mathcal{E}\{\hat{\theta}_{MAP}\} = \theta_0$ NO BIAS

  IF $p(\theta) \sim N(\theta_1 \neq \theta_0, \sigma_\theta^2)$, E.G. $\theta_1 = \phi$ $\quad$ BIASED!

$$\mathcal{E}\{\hat{\theta}_{MAP}\} = \frac{1}{1+\frac{N\beta}{\alpha}}\theta_1 + \frac{\frac{N\beta}{\alpha}}{1+\frac{N\beta}{\alpha}}\theta_0 \rightarrow \theta_0 \text{ AS } N \rightarrow \infty$$

ASYMPTOTICALLY UNBIASED

## EXAMPLE 9: GENERAL CASE

$$\underline{x} = \underline{\theta} \qquad \underline{\mu} = \underline{m}_0 \qquad \underline{\underline{\Lambda}}^{-1} = \underline{\underline{S}}_0 = \underline{\underline{\Sigma}}_\theta \left(= \frac{1}{\alpha}\underline{\underline{I}}\right)$$

$$\underline{\underline{A}} = \underline{\underline{\Phi}} \qquad \underline{b} = \underline{\phi} \qquad \underline{\underline{L}}^{-1} = \underline{\underline{\Sigma}}_e \left(= \frac{1}{\beta}\underline{\underline{I}}\right)$$

$$\underline{\hat{\theta}}_{MAP} \rightarrow \underline{m}_N = \underline{\underline{S}}_N\left(\underline{\underline{S}}_0^{-1}\underline{m}_0 + \beta\underline{\underline{\Phi}}^T\underline{y}\right)$$

$$\underline{\underline{S}}_N^{-1} = \underline{\underline{S}}_0^{-1} + \beta\underline{\underline{\Phi}}^T\underline{\underline{\Phi}}$$

$\underline{\underline{S}}_0 = \frac{1}{\alpha}\underline{\underline{I}} \qquad \alpha \rightarrow \phi$

$$\underline{m}_N = (\underline{\underline{\Phi}}^T\underline{\underline{\Phi}})^{-1}\underline{\underline{\Phi}}^T\underline{y}$$

(ML!)

$$\underline{m}_N\Big|_{N=\phi} = \underline{m}_0$$