III. FROM LS TO BAYES – ADDING KNOWLEDGE

1. LS – A REVIEW

$$y(n) = y_M(n) + e(n)$$

$$\llcorner\ \underline{\phi}^T(u(n))\,\underline{\theta}$$

LINEAR-IN PARAMETERS (LINEAR REGRESSION)

$$\underline{y} = \underline{\Phi}\,\underline{\theta} + \underline{e}$$

$$V_{LS} = \sum_{n=1}^{N} e^2(n) = \|\underline{y} - \underline{\Phi}\,\underline{\theta}\|_2^2$$

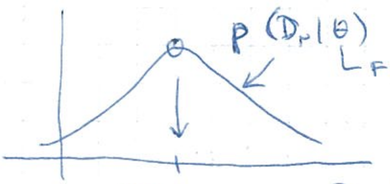
STANDARD SOLUTION $\quad \hat{\underline{\theta}}_{LS} = (\underline{\Phi}^T \underline{\Phi})^{-1} \underline{\Phi}^T \underline{y}$

2. MAXIMUM LIKELIHOOD (ML) APPROACH

THE IDEA: $e(n)$ RANDOM → PDF, ZERO-MEAN

↳ $y(n)$ ALSO RANDOM → PDF, AROUND $y_M(n) = \underline{\phi}^T(n)\,\underline{\theta}$

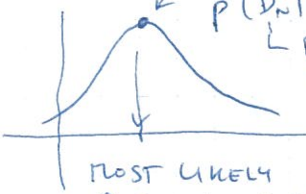
$p(D_N|\underline{\theta})$



$p(D_N|\theta)$ ∟ FIXED

MOST PROBABLE DATA, GIVEN θ $\quad D_N$ (data)

PROBABILITY FUNCTION



$p(D_N|\theta)$ ∟ FIXED MEASURED

θ

MOST LIKELY $\underline{\theta}$, FOR MEASURED DATA

LIKELIHOOD FUNCTION

$$L(D_N|\underline{\theta}) = P(D_N|\underline{\theta}) \qquad \hat{\underline{\theta}}_{ML} = \arg\max_{\underline{\theta}} L(D_N|\underline{\theta})$$

$$\ell(\underline{\theta}) = -\ln L(\underline{\theta}) \qquad \boxed{\hat{\underline{\theta}}_{ML} = \arg\min_{\underline{\theta}} \ell(\underline{\theta})}$$