3. ML – GAUSSIAN CASE

$$e(n) \sim N(\emptyset, \sigma_e^2) \qquad y(n) \sim N(\underline{\Phi}^T \underline{\theta}, \sigma_e^2)$$

$$L(\underline{\theta}) = p(\underline{y}|\underline{\theta}) \sim \left(\frac{1}{\sqrt{2\pi}\,\sigma_e}\right)^N e^{-\frac{1}{2\sigma^2}\underbrace{\sum (y - y_M)^2}_{\ell(\theta)}}$$

$$\boxed{\left.\hat{\underline{\theta}}_{ML}\right|_{\text{GAUSSIAN PDF}} = \hat{\underline{\theta}}_{LS}}$$

EXAMPLE 1.:

$$y(n) = a + e(n)$$

$$\quad \llcorner\; y_M(n) = \underline{\Phi}^T \underline{\theta} = [1][a]$$

$$e(n) \sim N(0, \sigma^2)$$

IDENTICALLY DISTRIBUTED INDEPENDENT (i.i.d) (DISCRETE WHITE NOISE)

$$p(y(n)|a) \sim N(a, \sigma^2)$$

$$p(y(1), y(2), \dots, y(N)) = \prod_{n=1}^{N} N(a, \sigma^2)$$

$$\downarrow$$

$$L(a) = \left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^N \prod_{n=1}^{N} e^{-\frac{(y(n)-a)^2}{2\sigma^2}} = \left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^N e^{-\frac{\sum_{n=1}^{N}(y(n)-a)^2}{2\sigma^2}}$$

$$\ell(a) = \frac{N}{2}\ln(2\pi) + N\ln\sigma + \frac{1}{2\sigma^2}\sum_{n=1}^{N}(y(n)-a)^2$$

$$\frac{\partial \ell(a)}{\partial a} = \frac{1}{\sigma^2}\sum_{n=1}^{N}(y(n)-a) = \emptyset \longrightarrow \left[\hat{a}_{ML} = \frac{1}{N}\sum_{n=1}^{N} y(n)\right] \quad (= \hat{a}_{LS})$$

CONCLUSION: IF WE KNOW THE PDF LEADING TO L AND CAN HANDLE IT, ML IS BETTER.

$$\left(\left.\hat{\underline{\theta}}_{ML}\right|_{\substack{\text{NOT}\\\text{GAUSSIAN}}} \neq \hat{\underline{\theta}}_{LS}\right)$$

OTHERWISE