EXAMPLE 10.

$$MSE(\hat{\theta}_{MAP}) = \mathcal{E}\{(\hat{\theta}_{MAP} - \theta_0)^2\}$$

$$= \mathcal{E}\left\{\left[\underbrace{\left(\frac{N\beta}{\alpha + N\beta}\right)}_{\xi}(\mu - \theta_0)\right]^2\right\} \qquad \mu = \frac{1}{N}\sum_n y(n)$$

$$= \xi^2\, \mathcal{E}\{(\underbrace{\mu - \theta_0}_{\frac{1}{N}\sum_n e(n)})^2\} \qquad = \xi^2 \frac{\sigma_e^2}{N} = \frac{\sigma_e^2}{N}\frac{1}{\left(1 + \frac{\alpha}{\beta N}\right)^2}$$

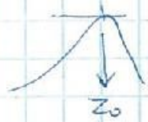
$$\boxed{\begin{array}{l} MSE(\hat{\theta}_{ML}) > MSE(\hat{\theta}_{MAP}) \\ \text{IF } \alpha > \phi \end{array}}$$

EXAMPLE 11: LAPLACE APPROXIMATION

$$p(z) = \frac{1}{Z} f(z) \longrightarrow q(z) \quad \text{GAUSSIAN}$$

Z: $\int f(z)\,dz$ UNKNOWN

- MODE: $p'(z)\big|_{z=z_0} = \phi \qquad \frac{d f(z)}{dz}\Big|_{z_0} = \phi$



- ln GAUSSIAN ~ QUADRATIC

$$\ln f(z) \simeq \ln f(z_0) - \frac{1}{2} A (z - z_0)^2$$

$$A = -\frac{d^2}{dz^2} \ln f(z)\Big|_{z_0}$$

$$\boxed{\begin{array}{l} f(z) \sim f(z_0) \exp\left[-\frac{A}{2}(z - z_0)^2\right] \\ q(z) = \left(\frac{A}{2\pi}\right)^{1/2} \exp\left[-\frac{A}{2}(z - z_0)^2\right] \end{array}}$$