$$= -\frac{A}{\sigma^2} \sum \left[A\cos^2(2\pi f_0 n + \phi) - A\cos(4\pi f_0 n + 2\phi) \right]$$

$$= -\frac{A^2}{\sigma^2} \sum \left[\frac{1}{2} - \frac{1}{2}\cos(4\pi f_0 n + 2\phi) \right]$$

$$= -\frac{NA^2}{2\sigma^2} + \frac{A^2}{2\sigma^2} \sum \cos(4\pi f_0 n + 2\phi)$$

IF $N \uparrow$ $\gg$

$$\mathrm{var}\{\hat{\phi}\} \underset{\text{APPROX.}}{\geq} \frac{2\sigma^2}{NA^2}$$

6.2 ML ESTIMATE – PROPERTIES

- INVARIANCE: IF $\underline{\hat{\theta}}_{ML}$ OF $\underline{\theta}$, THEN $\underline{\hat{\theta}}_1 = g(\hat{\theta}_{ML})$ ML ESTIMATE OF $g(\underline{\theta})$

- CONSISTENCY: $\lim_{N \to \infty} \underline{\hat{\theta}}_{ML}(N) = \underline{\theta}_0$ (IN PROBABILITY)

  $\left\{ \begin{array}{l} \underline{\hat{\theta}}_{ML} \text{ FROM I.I.D. DATA} \\ n_\theta \not\leftrightarrow N \end{array} \right.$!

- ASYMPTOTIC NORMALITY:

  $$\underline{\hat{\theta}}_{ML} \to N(\cdot, \cdot)$$

  N I.I.D. DATA
  $n_\theta \not\leftrightarrow N$

- ASYMPTOTIC EFFICIENCY:

  $$\underline{\hat{\theta}}_{ML} : \mathrm{cov}(\underline{\hat{\theta}}_{ML}(N)) \xrightarrow[N \to \infty]{} \text{CR}$$

  LOWER BOUND

- INFLUENCE OF THE NUMBER OF PARAMETERS:

$$y(n) = a t_n + b + e(n)$$

$e(n)$ └ $N(0, \sigma^2)$ I.I.D.

$$\ell(\underline{y}; [a,b]) = \ldots + \frac{1}{2\sigma^2} \sum (y(n) - a t_n - b)^2$$

$$\frac{\partial \ell}{\partial a} = \frac{1}{\sigma^2} \sum (\; - \; - \;) t_n = \frac{N}{\sigma^2}\left(\frac{1}{N}\sum y(n) t_n - a s^2 - b\mu\right)$$

$$\frac{\partial \ell}{\partial b} = \ldots = \frac{N}{\sigma^2}\left(\frac{1}{N}\sum y(n) t_n - a\mu - b\right)$$

$$s^2 = \frac{1}{N}\sum t_n^2$$

$$\mu = \frac{1}{N}\sum t_n$$