(2) $\lim \hat{R}_{LS} = \lim \dfrac{\Sigma \cdots}{\Sigma \cdots} = \dfrac{\lim \Sigma \cdots}{\lim \Sigma \cdots}$

$$= \frac{\lim \sum (u_0 + n_u)(i_0 + n_i)}{\lim \sum (i_0 + n_i)^2}$$

$$= \frac{\lim \sum \left(u_0 i_0 + \underbrace{\cancel{u_0 n_i}}_{\phi} + \underbrace{\cancel{i_0 n_u}}_{\phi} + \underbrace{\cancel{n_u n_i}}_{\phi} \right)}{\lim \sum \left(i_0^2 + \underbrace{\cancel{2 i_0 n_i}}_{\phi} + n_i^2 \right)}$$

$$= \frac{u_0 i_0}{i_0^2 + \lim \frac{1}{N} \sum n_i^2} = \left[R_0 \frac{1}{1 + \frac{\sigma_i^2}{i_0^2}} \right]$$

$$\frac{\sigma_i^2}{i_0^2} = \frac{1}{SNR_{INPUT}}$$

CONCLUSION: NO BIAS, IF NO INPUT NOISE!

VARIANCE ANALYSIS: $\hat{R} = R_0 + \Delta R$

$$\mathrm{var}\{\hat{R}\} = \mathcal{E}\{(\Delta R)^2\}$$

$$\hat{R}_{EV} = \frac{\sum (u_0 + n_u)}{\sum (i_0 + n_i)} \simeq R_0 \left(1 + \frac{1}{N} \sum \frac{n_u}{u_0} - \frac{1}{N} \sum \frac{n_i}{i_0} \right)$$

$\frac{1}{1+x} \simeq 1 - x$ IF n_u, n_i INDEPENDENT & $\left| \frac{n_i}{i_0} \right| < 1$ (*)

$$\Delta R = R_0 \left(\frac{1}{N} \sum \frac{n_u}{u_0} - \frac{1}{N} \sum \frac{n_i}{i_0} \right)$$

$$\Delta R^2 = R_0^2 \left(\frac{1}{N^2} \sum \frac{n_u^2}{u_0^2} + \frac{1}{N^2} \sum \frac{n_i^2}{i_0^2} + \underbrace{\cdots}_{\phi} \right)$$

$$\lim \longrightarrow \frac{1}{N} \sigma_u^2 / u_0^2 \qquad \frac{1}{N} \sigma_i^2 / i_0^2$$

$$\mathrm{var}\{\hat{R}_{EV}\} = \frac{R_0^2}{N} \left(\frac{\sigma_u^2}{u_0^2} + \frac{\sigma_i^2}{i_0^2} \right) = \mathrm{var}\{\hat{R}_{SA}\} = \mathrm{var}\{\hat{R}_{LS}\}$$

- IF EXPECTATION EXISTS (*)
- FIRST ORDER APPROXIMATION!

READINGS: BOOK 1-2, Ch 1; BOOK 3, Ch 1, Ex 1.5, 1.7, 1.8, 1.9, 1.10, Ch 2.1-2.3, 2.4.1

EXPERIMENTS: BOOK 5, Ch 1, Fig 1.1-1.3, 1.7, 1.9, Ex: 1a, 1b, 2, 3, 4