

ASSUME = $\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N x(k) = E\{x(k)\}$

$E\{n_u \cdot n_v\} = \phi$ NOISES INDEPENDENT

$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_k n_i(k) n_v(k) = \phi$

$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N (x(k) - \mu)^2 = \sigma_x^2$ VARIANCE

ZERO MEAN
NOISES

$$\begin{cases} u(k) = U_0 + n_u(k) \\ \hat{r}(k) = \hat{r}_0 + n_r(k) \end{cases}$$

BIAS ANALYSIS

$$\begin{aligned} (3) \quad \lim_{N \rightarrow \infty} \hat{R}_{EV} &= \lim_{N \rightarrow \infty} \frac{\sum \dots}{\sum \dots} = \frac{\lim \sum \dots}{\lim \sum \dots} \\ &= \frac{\lim \sum (U_0 + n_u(k))}{\lim \sum (\hat{r}_0 + n_r(k))} = \frac{U_0 + \lim \frac{1}{N} \sum n_u}{\hat{r}_0 + \lim \frac{1}{N} \sum n_r} = \frac{U_0}{\hat{r}_0} = R_0 \end{aligned}$$

BIAS $\equiv \phi$

$$(1) \quad \lim_{N \rightarrow \infty} \hat{R}_{SA} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum \frac{u(k)}{\hat{r}(k)} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum (U_0 + n_u(k)) \times \frac{1}{\hat{r}_0} \sum_{l=0}^{\infty} (-1)^l \left(\frac{n_r(k)}{\hat{r}_0} \right)^l$$

$\frac{1}{1+x} = 1 - x + x^2 - \dots, |x| < 1$

IF $\left| \frac{n_r(k)}{\hat{r}_0} \right| < 1$ (*)

ODD TERMS \rightarrow YIELD ϕ
EVEN \rightarrow ONLY (2)

$$\approx \frac{1}{N} \sum_{k=1}^N (U_0 + n_u(k)) \frac{1}{\hat{r}_0} \left(1 + \left(\frac{n_r(k)}{\hat{r}_0} \right)^2 \right)$$

$$= \frac{U_0}{\hat{r}_0} \frac{1}{N} \sum \left(1 + \frac{n_u}{U_0} \right) \left(1 + \left(\frac{n_r(k)}{\hat{r}_0} \right)^2 \right)$$

$$= \frac{U_0}{\hat{r}_0} \left(1 + \frac{1}{N} \sum n_u + \frac{1}{N} \sum \frac{n_r^2}{\hat{r}_0^2} \right) \rightarrow R_0 \left(1 + \frac{\sigma_r^2}{\hat{r}_0^2} \right)$$

- IF $|*| < 1$ ONLY

AND IF INPUT NOISE = GAUSSIAN?
WHAT IS THE PROBLEM?

IS GAUSSIAN IMPORTANT? (CENTRAL LIMIT THEOREM)