LTI MODELS & PREDICTION

$$y(t) = G(q)u(t) + v(t)$$

$$v(t) = H(q)e(t)$$ — WHITE NOISE

$$G(q) = \sum_{k=1}^{\infty} g(k) q^{-k}$$

$$H(q) = 1 + \sum_{k=1}^{\infty} h(k) q^{-k}$$

PARAMETRIC MODELS:

$$y(t) = G(q,\theta)u(t) + H(q,\theta)e(t)$$

DISTURBANCE PREDICTION

$$v(t) = H(q)e(t) = \sum_{k=0}^{\infty} h(k) e(t-k)$$

Ass: $H(q), \bar{H}(q)$ STABLE

$\sum |h| < \infty$

$$e(t) = \bar{H}(q)v(t) = \sum_{k=0}^{\infty} \bar{h}(k) v(t-k)$$

$\bar{H}(q) = H^{-1}(q)$ NOISE MODEL INVERTIBLE!

ONE-STEP-AHEAD PREDICTOR OF $v(t)$

$\{v(s) \quad s \le t-1\}$ GIVEN → ESTIMATE OF $v(t)$

→ KNOWLEDGE OF $e(s)$ AND $e(t-1)$

$$v(t) = \sum_{k=0}^{\infty} h(k) e(t-k) = e(t) + \underbrace{\sum_{k=1}^{\infty} h(k) e(t-k)}_{m(t-1)}$$

CONDITIONAL EXPECTATION

THE MOST PROBABLE VALUE OF $v(t)$ A POSTERIOR:

$$\hat{v}(t|t-1) = m(t-1) = [H(q) - 1]e(t) = (1 - \bar{H}(q))v(t)$$

$$H(q)\hat{v}(t|t-1) = [H(q) - 1]v(t)$$

OUT PUT PREDICTION OF $y(t)$

$$v(s) = y(s) - G(q)u(s) \quad s \le t-1$$

$$\hat{y}(t|t-1) = G(q)u(t) + \hat{v}(t|t-1)$$

$$= G(q)u(t) + [1 - \bar{H}(q)]v(t)$$

$$= G(q)u(t) + [1 - \bar{H}(q)][y(t) - G(q)u(t)]$$