Exercise 5.

Time Domain analysis

Required knowledge

- Time-domain description of first- and second-order systems.
- Measurement of phase shift between periodic signals.
- Transmission line theory: reflection calculation, wave propagation.
- Theory of averaging of noisy signals.

Introduction

Time domain investigation of signals and systems is one of the most essential tool of electrical engineering. When a physical phenomenon is investigated, its time domain behavior is one of the most important property which should be observed. In infocommunication often the shape of the received signal carries the information (e.g., its amplitude, phase, rate of change…). Even if a signal is stored or transmitted in digital form, most essential building blocks of digital signals (bits) are represented by analogue signals in the physical layer. In order to establish a high quality digital communication, the analogue signals must be well-conditioned: high signal-to-noise ratio should be achieved, the state transitions should be sharp enough, oscillation and reflections should be avoided.

Simple first- and second-order systems and transmission lines that will be investigated in the measurement are basic building blocks of several complex systems, so it is crucial to be familiar with the time-domain behavior and measurement technique of these systems.

Aim of the measurement

Students will perform the following task: (1) time- and phase measurement, (2) frequency dependent transfer of linear systems, investigation in time domain, (3) signal shaping in distributed parameter systems, (4) averaging as noise suppression. They will get acquainted with time domain reflectometry, and practice the time and phase measurement with oscilloscope, and failure diagnosis by means of investigation of time domain waveforms.

Web links

http://en.wikipedia.org/wiki/Lissajous_curve

http://en.wikipedia.org/wiki/Time-domain_reflectometry

Measurement instruments

Power supply	Agilent E3630A
Function generator	Agilent 33220A
Oscilloscope	Agilent 54622A

Test board

The test board consist the following circuits:

- first-order, passive low-pass filter (R-C filter), 10 different resistors, resistors can be selected with switch,
- first-order, passive high-pass filter (R-C filter), 10 different resistors, resistors can be selected with switch,
- second-order, active low-pass filter (Sallen-Key). Cutoff frequency and quality factor can be adjusted by changing the resistor and the capacitor in 10-10 different settings.

The test board (VIK-05-01) is the same as in the exercise Frequency domain analysis, please check the figures there.

Theoretical background

Measurement of pulse parameters

The definition of pulse parameters are given in Figure 5-1):

- Rise-time: the time during which the signal increases from the 10% to 90% of the final value. Care should be taken, since the base point is at the low level of the signal. For example, if U_{low} =1 V and U_{high} =10 V, then threshold values are: $U_{10\%}$ = 1.9 V and $U_{90\%}$ = 9.1 V.
- Fall time: the time during which the signal decreases from 90% to 10% of the initial value. 90% and 10% again refers to the difference between U_{low} and U_{high}.
- Overshoot: the difference between the peak value and the final value of the signal. It is often given relative to the final value in percent.
- Undershoot: the difference between the negative peak value and the final value of the signal at the falling edge.
- Droop: the decrease of the amplitude of the pulse from the beginning to the end.
- Impulse width: the time difference between the 50% threshold levels of the positive and negative edges.
- Settling time (ringing time): the time during which the signal settles after the level transition at its input within a specified interval around the final value of the signal (and doesn’t leaves this interval any more). The typical values of specified interval are, e.g., ±0.1%, ±1%, ±5% around the final value.

These methods are based on graphical evaluation, hence the measurement of the parameters is sometimes not obvious and not well-defined (e.g., at wrong signal-to-noise ratio, spurious oscillations occur…). In these cases, the measurement report should contain the detailed description of the measurement.

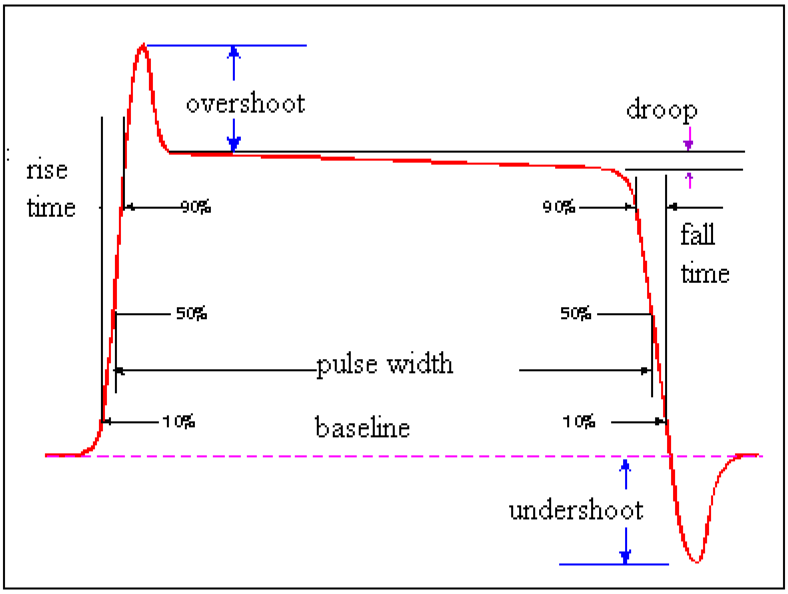


Figure 5-1. Definition of pulse parameters

It is a general rule that either of the previous parameters are measured, the range (both time/div and volt/div) should be as tight as possible, i.e., we should zoom on the measured part of the signal as close as possible in order to minimize the measurement error.

Modern digital oscilloscopes are able to measure these parameters automatically (Quick Measure button on the oscilloscopes used in the laboratory). However, the ranges should be set manually before these functionalities are used, and measurements should be verified visually, since these automatic measurements are based on the data which are displayed on the screen of the oscilloscope. For example, if rise time is measured and the time/div setting is too high, then the rising edge may be seen as 1-2 pixels on the screen. In this case even the oscilloscope can not do precise measurement. Contrary, if the time/div is too fine, and the steady-state high and low levels can not seen on the screen (we zoom too close to the edge, and other parts of the signal can not be seen), then the oscilloscope cannot correctly calculate the 10% and 90% threshold levels, so the measurement will be incorrect. Quick Measure function is a useful tool, however, it is recommended to make some measurements manually, otherwise we won't know how to set up the oscilloscope for the measurements.

Measurement of the transfer function

It is well known that a linear time-invariant system can change only the phase and amplitude of a sine wave applied to its input. Hence, the system can be characterized at each frequency by a complex number (complex gain) whose phase is the phase shift of the system, and its magnitude is the gain of the system. The transfer function is the complex gain of the linear system as function of frequency.

Several methods are known which allow the measurement of the transfer function of linear systems. In the following, some of these methods are summarized (the emphasis is put on the measurement of magnitude characteristics).

Measurement of amplitude characteristics with stepped sine

A well-known method of measurement of amplitude characteristics is performed using a sine wave generator and an AC multimeter (Figure 5–2). The measurement doesn't require expensive special instruments if high precision is not crucial. Its disadvantage is that the measurement is relatively time consuming, since the amplitude characteristics should be measured point-by-point along the whole frequency range. The frequency resolution of the measurement is determined by the frequency resolution of the sine wave generator. When only the bandwidth is to be measured, it can be done by setting the frequency to the center frequency where the gain is nominal, and than the frequency should be changed until the output signal decreases by 3 dB. The multimeter can often be exchanged with an oscilloscope, but the precision of an oscilloscope is generally worst than that of a multimeter.

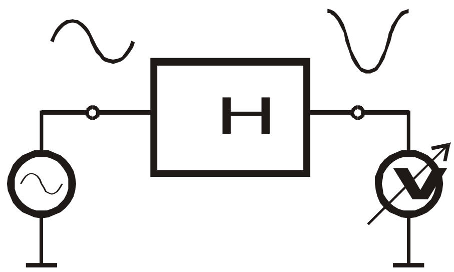


Figure 5–2. Measurement of transfer function with sine wave generator and multimeter

The amplitude reference point has to be set before beginning the measurement. Every subsequent measurement result is compared to this reference point. The reference point is set according to the type of the amplitude characteristics (e.g., high-pass, low-pass, band-pass…). For example, if a system has low-pass characteristics as shown inFigure 5–3, the reference point should be set at low frequency, at least one or two decades below the cutoff (corner) frequency. If the multimeter has fixed 0 dB point, it is recommended to set the input signal such that 0 dB appears at the output. Some of the modern multimeters allow us to set the 0 dB point to an arbitrary value. In this case, the input signal should be set as high as possible in order to ensure good signal-to-noise ratio. Care should be taken when setting the level of input signal! A common mistake is that the output signal becomes distorted, e.g., due to saturation, or the measured values are out of the range of the instruments. Except of some special cases, neither the input nor the output signals can exceed the supply voltage. If a passive circuit is measured (e.g., first-order RC network), no power supply is required. The level of input signal shouldn’t be changed during the whole measurement. It is generally recommended to check the shapes of the signals with an oscilloscope during the measurement.

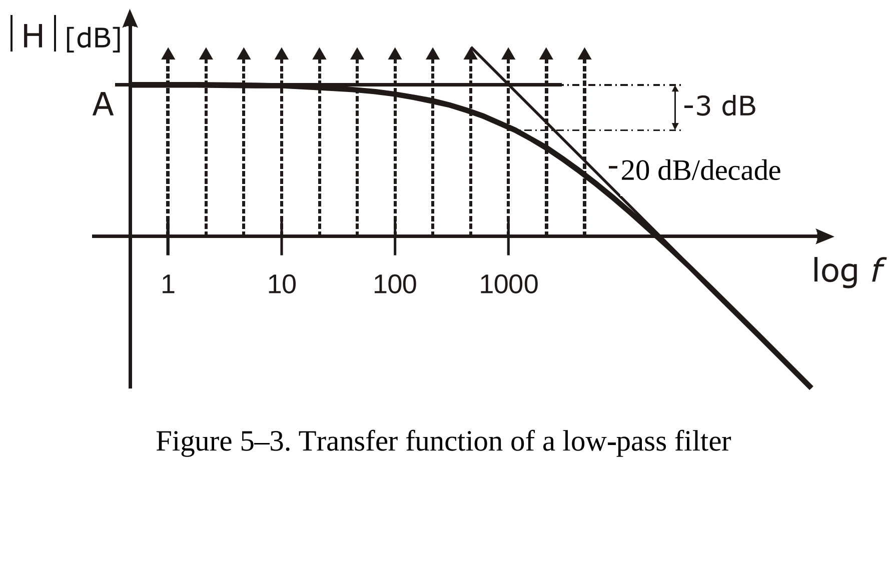


Figure 5–3. Transfer function of a low-pass filter

During the course of the measurement, the frequency is often changed logarithmically (see Figure 5–3), e.g., with steps 1-2-5-10-..., but it is recommended to measure with finer steps in the vicinity of the cutoff frequency. The cutoff frequency is often defined as the frequency where the amplitude characteristics decreases by 3 dB below the nominal value. (E.g., if the nominal gain is 9 dB, the gain is 6 dB at the cutoff frequency.)

The stepped sine wave method has the advantage that it offers a good signal-to-noise ratio. However, the measurement of the whole amplitude characteristics requires considerable time, since the frequency should be changed after each measurement, and we should wait until the transient vanishes after each time the frequency is changed.

Measurement of phase difference

Two methods will be introduced how the phase difference between two signals of the same frequency can be measured.

The first method is traced back to the measurement of ratio of time intervals, more precisely on the ratio of the time delay between two signals and the period of the signal. The method is illustrated in Figure 5-4. The two signals are fed to the two different channels of the oscilloscope. Then we should search the same reference points on the two signals. Practically, the positive or negative zero crossing points are used as reference points. Let Δt denote the time delay between these two reference points. Furthermore, let T denote the period which can be measured as the time difference between two consecutive positive or negative zero crossing of the signal. The phase difference can be calculated as $\varphi = \frac{\Delta t}{T} \cdot 360°$. The advantage of the method is that it is not sensitive to the time base error of the oscilloscope, only the linearity of the time base is required. However, it is true only until the time/div setting remains the same when Δt and T are measured. If Δt is considerably smaller than T, then Δt should be measured with smaller time/div setting (finer time resolution). In this case the error of time base can not be neglected when measurement error is calculated. It depends on the specification of the oscilloscope whether Δt and T should be measured with the same or different time base setup.

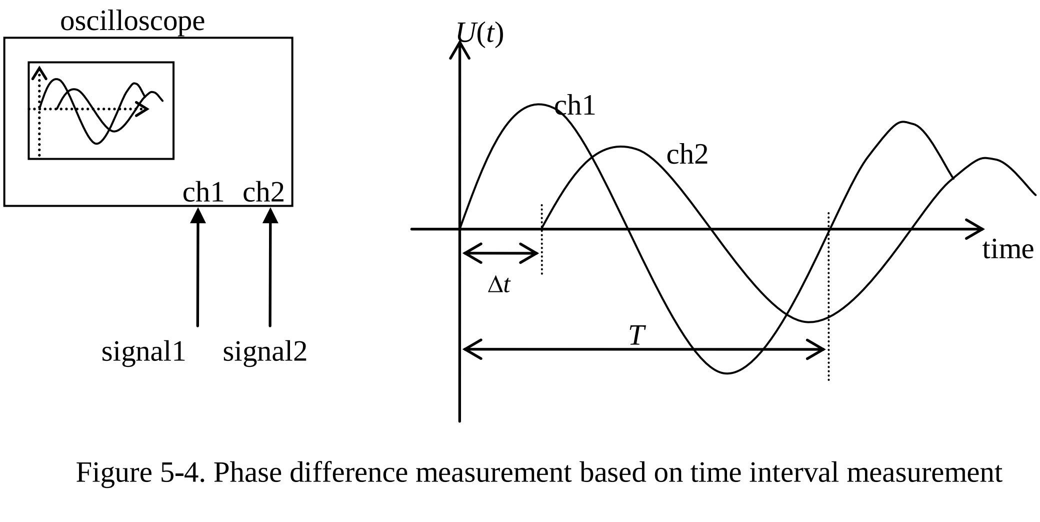


Figure 5-4. Phase difference measurement based on time interval measurement

The oscilloscope used in the laboratories have built-in phase shift measurement functionality which is based on the previous method. This tool works properly only when at least one whole period of the observed signal can be seen on the display of the oscilloscope. This constraint limits the accuracy when small phase difference is measured, since in this case we can not zoom of the time difference (Δt) between the signals which would required to make a precise measurement. This example shows that it is highly recommended to be able to perform manual measurement since automatic functionalities may fail in some cases.

The second method of phase shift measurement is based on the so called Lissajous plot. In this case, the oscilloscope should be used in X-Y mode, and the signals should be connected to the two different channels of the oscilloscope. We consider now sinusoidal signals of the same frequency. First, the channels should be grounded (it can be set in the menu where coupling is set), and the oscilloscope ray should be set to the origin of the display. Then, grounding should be turned off, and the vertical and horizontal dimension of the ellipse appearing on the display should be measured as shown in Figure 5-5. The phase shift can be calculated from the measured values as $\varphi = \arcsin\frac{a}{b}$. Let's note that the measurement is based on the ratio of two quantities that are measured on the same axis, so the gain and calibration errors are eliminated as far as both *a* and *b* are measured with the same volt/div setting.

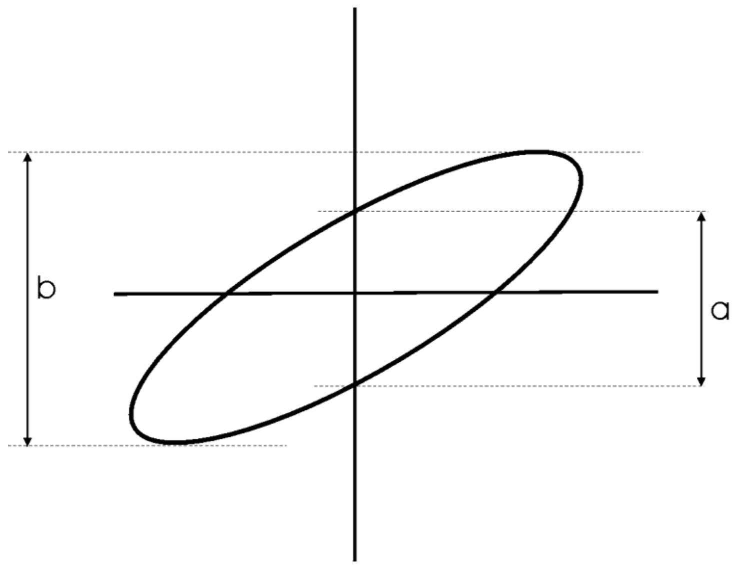


Figure 5-5. Phase difference measurement based on Lissajous-plot

First-order RC circuits

During the course of laboratory measurement first-order, low- and high-pass filters will be performed. One of the simplest measurement method is the measurement of the step response of the systems which can be performed with a simple square wave generator. The analytical form of the step responses of general first-order, low- and high-pass systems are:

$$v_{\mathrm{LP}}(t) = A(1 - e^{-\frac{t}{\tau}}), \qquad v_{\mathrm{HP}}(t) = Ae^{-\frac{t}{\tau}}, \tag{5-1}$$

where A is the amplitude gain and τ stands for the time constant of the system. In the laboratory, first-order RC filters will be investigated whose schematic diagrams are shown below:

Figure 5-6. Schematic diagrams of first-order, low- and high-pass RC networks.

The time constant of such systems is $\tau = RC$, and their gain is unity: $A = 1$. The responses of these systems on a step function of amplitude U_{peak} are:

$$v_{\text{LP,RC}}(t) = U_{\text{peak}}(1 - e^{-\frac{t}{RC}}), \qquad v_{\text{HP,RC}}(t) = U_{\text{peak}} e^{-\frac{t}{RC}}, \tag{5-2}$$

Step response of the first order RC networks are shown in the figures below:

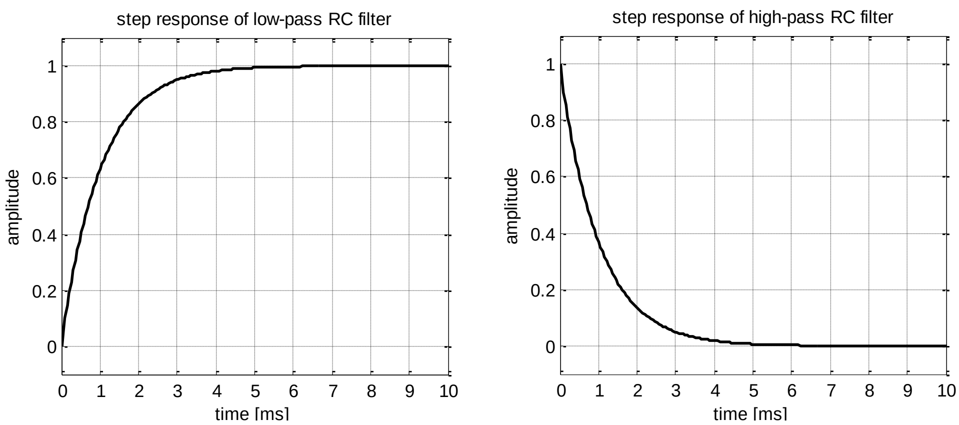


Figure 5-7. Step responses of first-order, low- and high-pass RC networks. Time constant is: $\tau = 1/\omega_0 = RC = 1$ msec.

Measurement of time constant of first-order systems

The time constant of the systems will be measured using square wave input signals. If the half of the period of the square wave used as excitation signal is considerably longer (at least 5 or 10 times) than the time constant of the system to be measured, the square wave can be regarded as a periodic step function, and the output of the system can be regarded as the step response of the system. The measurement arrangement is found in the figure below. First-order RC circuits contain only passive components so they do not require supply voltage.

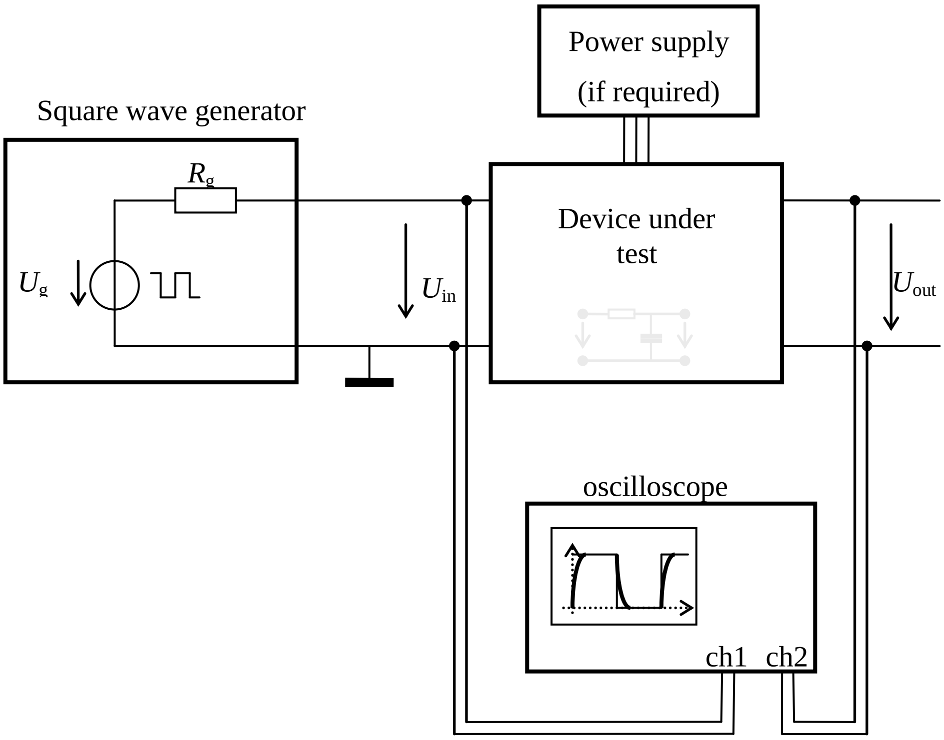


Figure 5-8. Measurement arrangement of step response and time constant.

In the figure, R_g denotes the output impedance of the function generator (R_g = 50 Ω). This resistance has practical significance if the input impedance of the DUT is not considerably higher than R_g. In this case, the input signal can be less than the value set on the function generator since the input impedance of the DUT and R_g form a voltage divider. R_g can also influence the time constant of the system, since it is added to the resistance of the RC network.

Exercise 5.

Three methods will be introduced to measure the time constant based on the step response. These methods are summarized in the figures below:

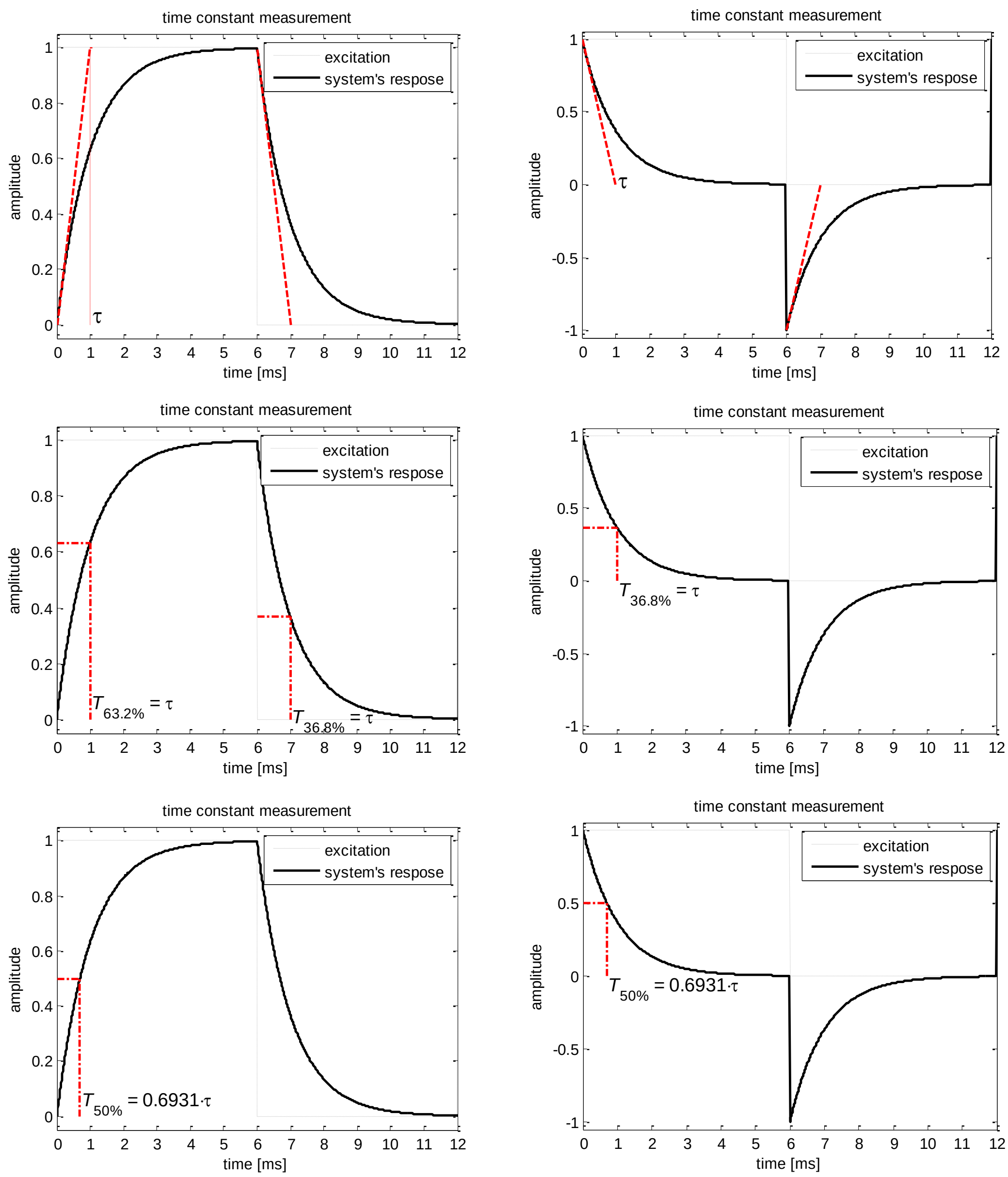


Figure 5-9. Illustrations of time constant measurement methods based on step response. Left column: low-pass filter; right column: high-pass filter. First row: tangent at zero point; second row: measurement at the 63.2% and 36.8% of the maximum value; last row: measurement at the 50% of the maximum. The time constant in this example is τ = 1 ms.

In the examples, the half of the square wave is more than five times the time constant so the system's response achieves the steady state before each new edge of the excitation signal.

The methods of time constant measurement are:

1. Time constant measurement based on tangent at zero point:
   - At the falling edge: draw the tangential of the step response at the beginning of the falling edge. The tangential crosses the time axis at the time constant.

  - At the rising edge: draw the tangential of the step response at the beginning of the rising edge. The tangential reaches the final value of the step response at the time constant.
2. Measurement at the 63.2% and 36.8% of the maximum value:
  - low-pass filter: the step function reaches the 63.2% of the final value after rising edge, and it reaches the 36.8% of the initial value at the falling edge after the time constant. Note that 63.2% = 1–1/*e* and 36.8% = 1/*e*.
  - high-pass filter: the step function reaches the 36.8% of the initial value after the time constant (i.e., it decreases to the *e*-th part during the time constant).
3. Measurement at the 50% of the maximum value: the step function reaches its 50% after 0.6931 times the time constant. Note that 0.6931 = –log(1–1/2).

All of these methods can be proved according to the step response of first-order systems given in equation (5-1).

To prove the first method, the derivative of the step response should be calculated, that is $v'_{\mathrm{LP}}(t=0) = \frac{A}{\tau} e^{-\frac{t}{\tau}} = \frac{A}{\tau}$ for low-pass filter. Since $v_{\mathrm{LP}}(t=0) = 0$, the tangential reaches the amplitude A after at $t = \tau$. Proof is similar for high-pass filter (it should be solved as homework).

The proof the second and third method differs only in the last step. The final value of the step response of low-pass filter is A. In order to calculate how many time it takes to reach a value aA, we should solve the equation $v_{\mathrm{LP}}(t) = A(1 - e^{-\frac{t}{\tau}}) = aA$. One obtains that it is true for $t = -\tau \cdot \ln(1-a)$. For a = 63.2% = 1–1/e one obtains $t = -\tau \cdot \ln(1-(1-1/e)) = \tau$, and for a = 50% = 0.5 one obtains $t = -\tau \cdot \ln(1-0.5) = 0.6931 \cdot \tau$. Proof is similar for high-pass filter (it should be solved as homework).

Step response of second-order, low-pass filter

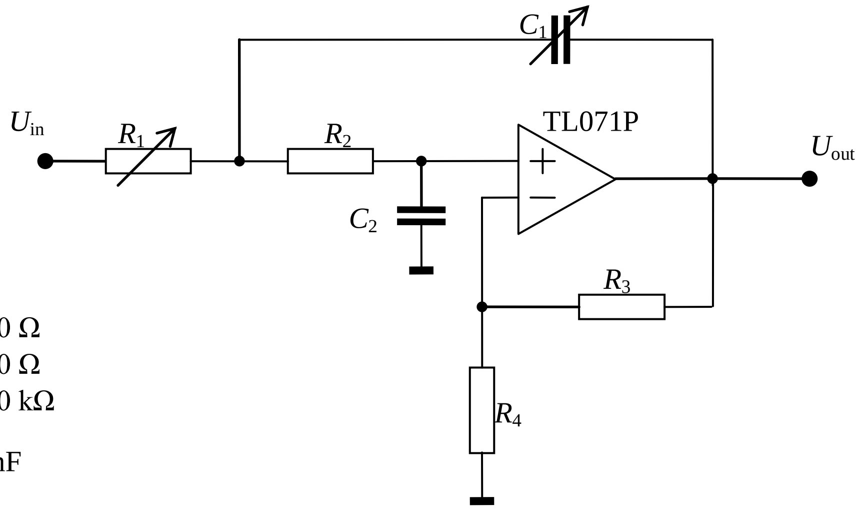
The general transfer function of second-order, low-pass filters is of the form (both forms are often used):

$$W(s) = \frac{A}{1 + \frac{1}{Q}\frac{s}{\omega_0} + \frac{s^2}{\omega_0^2}} = \frac{A}{1 + 2\xi\frac{s}{\omega_0} + \frac{s^2}{\omega_0^2}}, \tag{5-3}$$

where Q denotes the quality factor, ξ denotes the damping factor, ω_0 is the natural frequency (undamped resonance frequency) and A is the gain.

In the laboratory the so-called Sallen-Key topology is used to realize the filters. Its schematic is shown in Figure 5-10. R_1 and C_1 can be set with code switches. They have influence on the resonance frequency and on the quality factor. The detailed analysis of the circuit is out of the scope of this guide, it can be found in several text books and on the Internet. In the measurement, only the behavior of the circuit as a typical second-order system is investigated. The measurement of the step response of the filter is performed according to Figure 5-8. Since the circuit contains an operational amplifier, it requires supply voltage!

Exercise 5.



R_2: 100 Ω
R_3: 100 Ω
R_4: 100 kΩ

C_2: 1 nF

R_1: 100 Ω, 180 Ω, 330 Ω, 560 Ω, 1 kΩ, 1.8 kΩ, 3.3 kΩ, 5.6 kΩ, 10 kΩ, 18 kΩ
C_1: 4.7 nF, 6.8 nF, 10 nF, 65 nF, 33 nF, 68 nF, 100 nF, 330 nF, 680 nF, 1 μF

Figure 5-10. Schematic diagram of the second-order, low-pass filter.

The expected step responses of an ideal second-order, low-pass filter are shown in Figure 5-11. with different Q and ω_0 parameters.

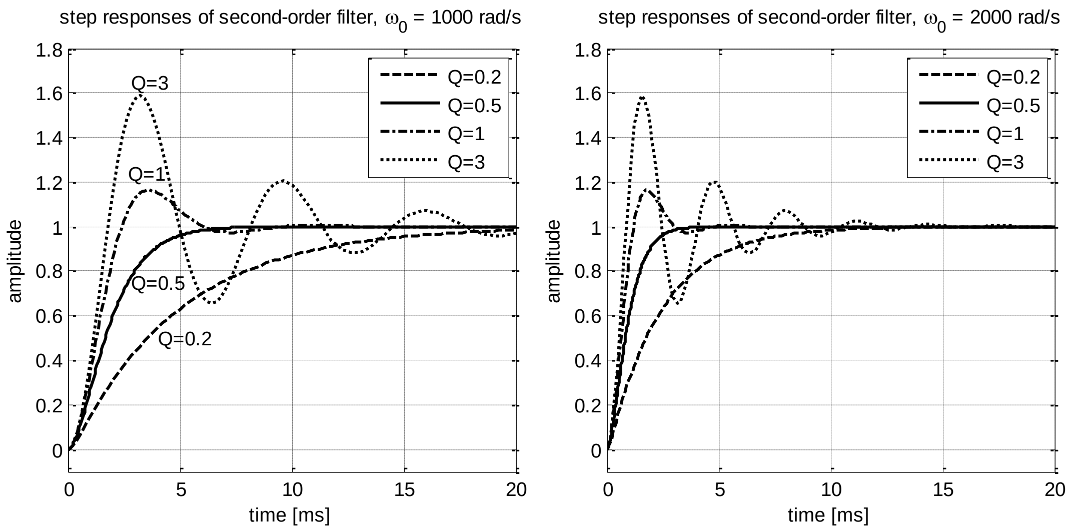


Figure 5-11. Step responses of the second-order, low-pass filter with ω_0 = 1 krad/s (left figure) and ω_0 = 2 krad/s (right figure), and with different quality factors.

The step response is determined by parameters Q and ω_0. ω_0 determines the frequency of oscillation (if the system is underdamped), and Q determines the ability of the system to oscillation. Five important cases are distinguished according to the value of Q:

- $Q < ½$: the system is overdamped (exponential settling)
- $Q = ½$: the system is critically damped (fastest settling without oscillation)

- $\infty > Q > ½$: the system is underdamped (settling with exponentially decreasing oscillation)
- $Q = \infty$: the system is undamped (oscillation with constant amplitude)
- $Q < 0$: the system is unstable (oscillation with exponentially increasing oscillation)

Two parasitic effect emerging during the step response measurement should be noted. When the input (U_{in}) is observed in the arrangement Figure 5-8, an oscillation may be observed on the input even if the function generator is set correctly (it generates a square wave). The oscillation is present due to the finite output impedance of the generator ($R_g = 50\ \Omega$), which causes that the voltage on the output is coupled back to the input through capacitor C_1 and resistor R_1. The coupling ratio is especially high if resistance R_1 has low value, and capacitance C_1 has high value. In this case, the output impedance of the function generator is comparable with the impedance of R_1 and C_1 (e.g., if $R_1 = 100\ \Omega$, then the voltage division ratio between the generator and the common junction of R_1 and C_1 is $R_1/(R_1+R_g)$=1/3!).

The second effect that can be observed is that the measured step response may contain an abrupt jump right after the rising edge of the input signal although ideally it is a continuous function. The reason is the finite output impedance of the operational amplifier (OPA): the step function is coupled from the input to the output through R_1 and C_1 (C_1 behaves like a short circuit for a short time interval at the edges of a square signal). In ideal case, the OPA would have zero output impedance (it behaves as a voltage generator), so the coupling from the input would be zero. However, the output impedance of a real OPA may be even some ten ohms which is comparable with the minimal value of R_1.

Although these parasitic effect cause slight difference between the theory and practice, but show good examples what kind of aspects should be taken into account in a practical measurement.

Time-domain reflectometry (TDR)

Transmission lines are often used in micro-wave circuits, impulse technique and high-speed digital systems. Every conductor can be regarded as transmission line if its length is at least approximately the tenth of the wavelength of the signal to be transmitted. If high frequency signal is transmitted through a conductor, the nature of the propagation of electromagnetic waves in transmission lines should be considered.

Figure 5-12 shows a block diagram where time-domain reflections in transmission lines can be investigated. A voltage generator is used which is able to provide a step function at its output (in practice a square wave generator is used with long period such that steady state is achieved between level transitions). The output impedance of the generator equals to the wave impedance of the transmission line, i.e. $Z_g = Z_o$ (matched termination on the input). The voltage $e_x(t)$ is measured with an oscilloscope at the output of the generator that is connected to the input of the transmission line. The transmission line is terminated with a real valued load with impedance of value Z_L.

Exercise 5.

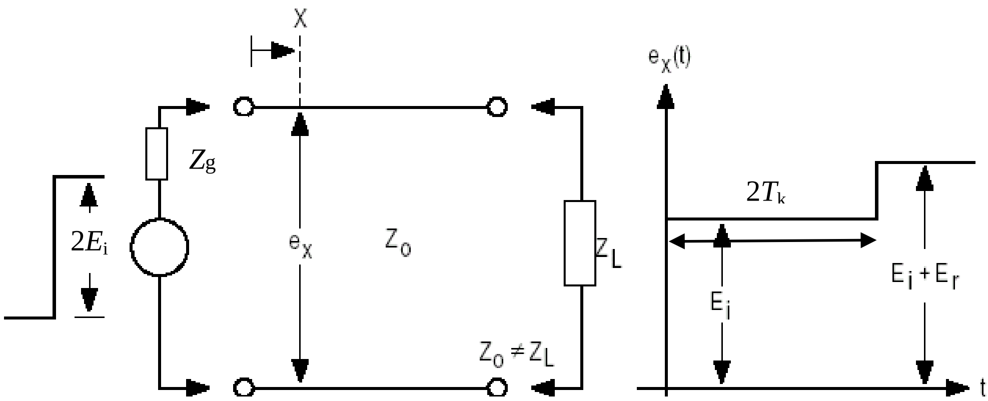


Figure 5-12. Block diagram of time-domain reflectometry

Let estimate the signal shapes with simple physical considerations. The system is in idle state before time instant $t = 0$. When the step function of amplitude $2E_\mathrm{i}$ appears at the input, the transmission line shows a wave impedance of Z_o independently on the load impedance. The reason is that the signal has finite propagation speed, so the signal "doesn't know" when it is appeared on the input what are the load conditions; it "sees" only the wave impedance of the cable. Hence, a voltage divider is formed from the generator impedance Z_g and wave impedance Z_o, and a step wave of amplitude E_i propagates towards the end of the end of the cable. When the wave reaches the load impedance (after a time T_k), a reflection occurs. The reflection coefficient (γ) depends on the load impedance and wave impedance of the cable as the following equation:

$$\gamma = \frac{E_r}{E_i} = \frac{Z_L - Z_0}{Z_L + Z_0}\,. \qquad (5\text{-}4)$$

The reflected wave is γ times of the incident wave, i.e., it is $E_r = \gamma E_i$. The voltage on the load is zero until a wave reaches this point (until time instant T_k), and after the time instant T_k the sum of the incident and the reflected voltage ($E_i+\gamma E_i$) is measured. The voltage observed on the input remains E_i until the reflected wave arrives back to the input, and than it becomes ($E_i+\gamma E_i$). Since we investigate the case when $Z_\mathrm{g} = Z_\mathrm{o}$, so no more reflection occurs on the input, hence the steady state has been achieved.

The propagation time from the input to the end of the cable is denoted by T_k. The round-trip delay during which the first reflection (E_r) arrives at the input of the cable is $2T_k$. If the input is matched, i.e., $Z_\mathrm{g} = Z_\mathrm{o}$, then the steady state has been achieved, and the steady-state input voltage is $U_\mathrm{ss} = E_i+\gamma E_i$. Substituting γ into this equation, and using that $Z_\mathrm{g} = Z_\mathrm{o}$, one obtains:

$$U_\mathrm{ss} = 2E_i\,\frac{Z_\mathrm{L}}{Z_\mathrm{L} + Z_\mathrm{g}}\,, \qquad (5\text{-}5)$$

which means that the steady-state voltage can be calculated as if the load impedance were directly connected to the generator (note that the amplitude of the input signal is $2E_i$).

The above described measurement is called TDR (Time-Domain Reflectometry). This kind of measurement can be used to detect whether a cable is terminated correctly (no reflection occurs.

Some important case is illustrated in the following figure:

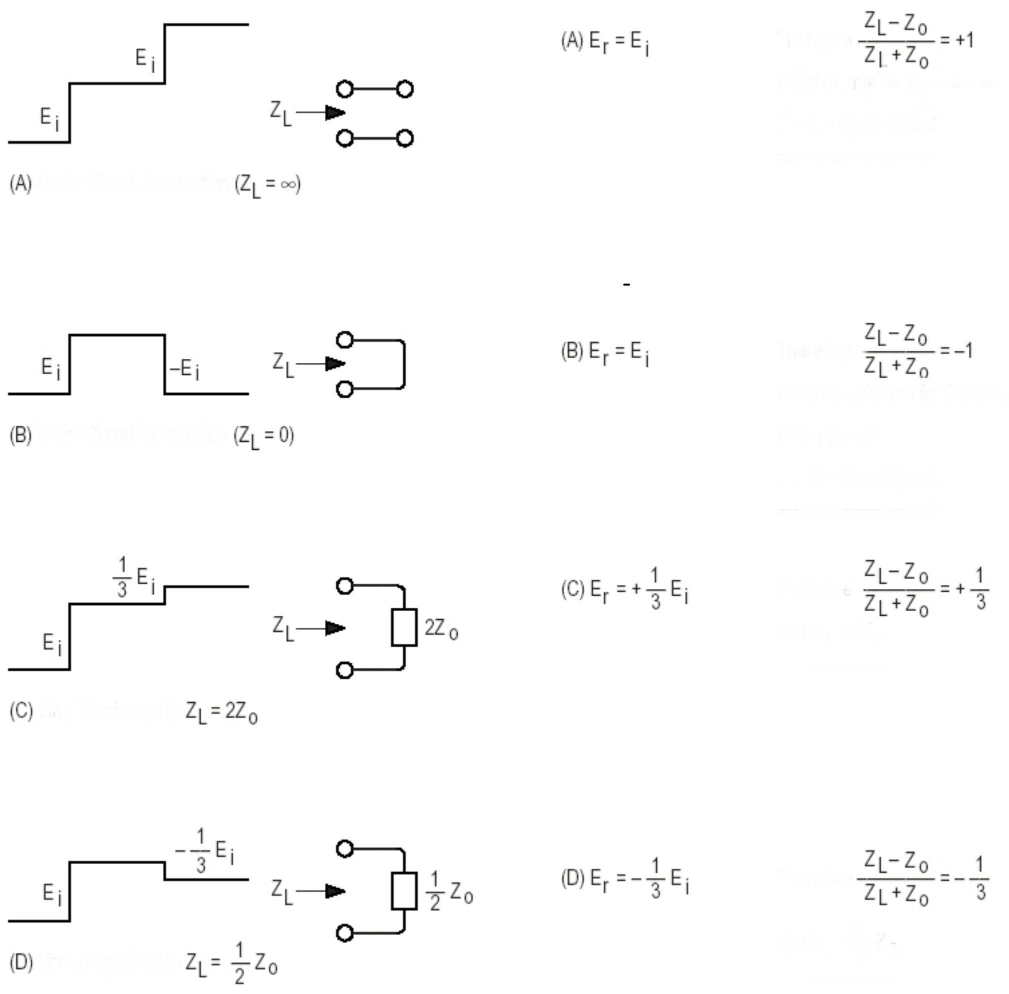


Figure 5-13. Waveforms in the case of different load impedance

Case B is important when short pulses have to be generated, since the pulse duration can be tuned with the length of the cable.

Note that if the generator impedance isn't matched ($Z_g \neq Z_o$) too, reflection happens also on the input, and all of the previously described rules can be applied to calculate reflection on the input.

If the load impedance is not a real-valued one, then the waveforms are more complex. The waveforms in initial state ($t = 0$) can be approximated by substituting $Z_C \to 0$; $Z_L \to \infty$, and in steady state ($t = \infty$) conditions $Z_L \to 0$; $Z_C \to \infty$ can be used. In the intermediate states the waveforms are exponential depending on the nature of the load. The typical waveforms are illustrated in the following figures.