Exercise 4

Electrical Power Measurement

Introduction

The electrical power is one of the basic electrical quantities which can be defined both on DC and on AC. By measuring the electrical power we can determine the efficiency of electrical circuits or the stress of components, and the power measurement is the base of the measurement of electrical energy or consumption.

The present measurement is governed by the power measurement on different kinds of AC devices and circuits (R-L-C network, lamps, supply unit of a computer). The R-L-C network can be regarded as the model of a fluorescent lighting, where L is the chuckle, R is the substitute for the fluorescent tube and C is the power factor improving tool. On this model, it is possible to measure in two different ways the apparent, the effective and the reactive power. By measuring the power of the lamps, the characteristic features of quasi-linear component can be measured. The example of the power measurement on the supply of a personal computer illustrates the difficulties connected to the power measurement in case of non-linear wave-forms.

The object of the measurement exercise

The exercises complete the knowledge learned about the measurement of electrical power in the preliminary studies. The measurements demonstrate that modern signal processing methods can be effectively applied for traditional measurement tasks. It is also shown how accurate and safe measurements can be carried out on the mains.

Theoretical basics

The detailed theoretical basics can be found in the appropriate textbooks, here only the main equations are recalled. Similarly the power measurement methods are not introduced in detail, only the evaluation of the methods and their comparison are emphasized. In point of the fundamentals please refer to the literature of the core subjects Signals and Systems 1-2, and Measurement Technology.

The main equations

Let the voltage and the current of the impedance to be measured sinusoidal as follows:

$$u(t) = U_p \cos \omega t$$
$$i(t) = I_p \cos(\omega t - \varphi)$$

In this case the phase difference between the voltage and the current equals $+\varphi$. The instantaneous power can be expressed as:

$$p(t) = u(t)i(t) = \frac{U_p I_p}{2}[\cos(2\omega t - \varphi) + \cos\varphi]$$

As in the case of sinusoidal signals the ratio of the peak value and the RMS value is $\sqrt{2}$, the power can be rewritten as follows:

$$p(t) = UI[\cos(2\omega t - \varphi) + \cos\varphi]$$

where U and I are the RMS value of the voltage and the current, respectively. The first term has a double frequency and results in zero after integration, thus the effective power originates from the second term. In the practice the following quantities are used:

Apparent power: $S = UI$ [VA]
Effective power: $P = UI\cos\varphi$ [W]
Reactive power: $Q = UI\sin\varphi$ [var]

Apparent power is also a real number, do not take for the complex power, defined by $\bar{S} = UI^*$. The above quantities have the following relation:

$$S^2 = P^2 + Q^2$$

In the case of periodic but non-sinusoidal voltage and current, the effective power can be calculated for all the Fourier components independently, and they should be summarized:

$$P = \sum_{k=1}^{K} U_k I_k \cos\varphi_k + U_0 I_0$$

where U_k, and I_k denote the RMS value of the voltage and the current of the kth component, respectively, and φ_k is the phase shift of the kth component. For completeness the DC power is added, where U_0 and I_0 stand for the DC component of the voltage and the current, respectively. There are K components calculated, where K is determined by the required accuracy. The reactive power can be defined similarly:

$$Q = \sum_{k=1}^{K} U_k I_k \sin\varphi_k$$

However, now $S^2 \geq P^2 + Q^2$, and the following expression can be written:

$$S^2 = P^2 + Q^2 + D^2, \quad D = \sqrt{S^2 - P^2 - Q^2}$$

where D is the so-called deformed power.

Measurement of effective power

The most important task is the measurement of the effective power. The apparent power can be easily calculated by the measured RMS values of the voltage and the current. Assuming sinusoidal waveforms, the reactive power can be calculated by the effective and the apparent power. In the following some methods are reviewed that are able to measure the effective power.

1. **Electrodynamic wattmeter.** Its operation can be explained by the Permanent Magnet Moving-coil instrument [2]. Instead of the permanent magnet, an additional coil is applied in which the current of the impedance flows. The voltage of the impedance is led to the moving coil. In the case of DC excitation, the resulted torque is proportional to the product of the two quantities. In the case of AC voltage and current, the torque is proportional to the instantaneous power, as well. However, because of the low-pass feature of the mechanical system, the deflection of the meter is proportional to the effective power. The evaluation of the method:
   - **Advantages:** It is a dedicated instrument, the effective power can be read directly, none of the parts of the meter are grounded.
   - **Disadvantages:** The measurement result is biased, as the voltage coil has a finite, and the curent coil nonzero resistivity. To access the current, the circuit of the impedance should be broken. The meter is expensive, assuming high accuracy measurement.
2. **Three voltmeter method.** It is a well-known method, in the case of sinusoidal excitation it can also be used for impedance measurement. The impedance under test

is connected in series by a normal resistor, and the voltages of the impedance, the normal resistor and the excitation is measured. The three voltages can be used for the calculation of the power as follows:

$$P = \frac{U_g^2 - U_z^2 - U_n^2}{2R}$$

where R denotes the normal resistor, U_z, U_n and U_g are the voltage of the impedance, the normal resistor and the excitation, respectively. Detailed description can be found in [2], its error analysis is available in [3]. The evaluation of the method:

- **Advantages:** It is a simple method, it does not require special instrumentation. The result is unbiased even in the case of nonsinusoidal excitation.
- **Disadvantages:** The insertion of the normal resistor is difficult, an accurate resistor is required. The accuracy of the method is limited, as differences are to be calculated.

3. **Measurement of voltage, current, and phase shift separately.** The voltage and the current can be measured easily, the only question is the phase measurement. The phase shift can be measured by a counter or an oscilloscope. The evaluation of the method:
   - **Advantages:** It can be performed by general laboratory instruments.
   - **Disadvantages:** To access the current, the circuit of the impedance should be broken. The measurement of the phase shift is usually not accurate enough, the method cannot be applied in a nonsinusoidal case.
4. **Electronic wattmeter.** The construction of the electronic wattmeters can be different, depending on the age of the meter. If it consists of solely analogue parts, active voltage and current transformers, analogue multiplier and integrator are built in the meter. In modern meters both the voltage and the current are sampled, and the effective power is calculated by the definition. The evaluation of the method:
   - **Advantages:** It is a dedicated instrument, the effective power can be read directly. Modern meters measure by the definition.
   - **Disadvantages:** The accuracy of the analogue implementation is limited. The term „electronic wattmeter” is ambigous, versatile measurement principles and methods are applied.
5. **Measurement by digital storage oscilloscope.** The signal processing capacity of the modern digital storage oscilloscopes allows the measurement by the definition. The voltage is connected to a channel of the oscilloscope, and a voltage proportional to the current is connected to another channel. Among the math functions the product of the two channels is to be selected, then averaing of the product is available, thus the oscilloscope measures the effective power by the definition. RMS value measurement is also a common function, so the apparent power, the reactive power, and the power factor can be calculated, as well. The measurement is accurate if the measurement time is integer multiple of the period time. Usually it can be set easily on the oscilloscope. Current measurement requires a converter, but it is usually an accessory of the oscilloscope. The evaluation of the method:
   - **Advantages:** The measurement is performed by the definition, the current measurement does not require the braking of the circuit. Digital storage oscilloscopes are commonly available in the laboratories.
   - **Disadvantages:** Connection to the circuit should be carefully designed, as the inputs of the oscilloscopes are grounded. High voltage can be connected only by a special probe or using a transformer.

Current measuring tools

Clamp meter (Amprobe DLC-100)

The working principle of the clamp meter is identical with that of the ring current transformer which is laced on a current rail, where the rail plays the role of the primary winding with one number of turns. The clamp meter differs from such current transformers in the construction of the iron core. While the iron core of a transformer is fix, that of the clamp meter can be opened, and it can be placed around the conductor in which the current flows. Using the gap on the iron core, than closing it again, the conductor becomes the primary winding of one number of turns. In the secondary winding placed tight on the iron core a current will flow which is proportional to the current of the primary winding corresponding to the equilibrium of the primary and secondary excitation: $N_1 I_1 = N_2 I_2$ (where $N_1 = 1$).

Measuring the current of the secondary winding means indirectly measuring the current of the conductor in question.

The clamp meter is designed for measurements at industrial frequencies, *basically at 50/60 Hz*, but it can also be used in a wider frequency range (40 Hz – 1 kHz) with less accuracy. It was developed first of all to measure small *leakage currents*. Its most sensitive *measuring range is 40 mA* (with a resolution of 10 μA), *its accuracy is ±1% ±3 LSB* (% of the high end of the range: 4 digits display). The accuracy is similar up to the range of 40 A, above this the accuracy is drastically decreasing (e.g. between 80 and 100 A: ±9 % ±10 LSB).

The measuring unit of the clamp meter can also be used independently of the current transformer for measuring *voltage* (0 – 400 V range) and *resistance* (0 – 400 Ω), but in the actual exercise it is used only for current measurement

Hall-probe current meter (Hameg HZ-56)

The sensor of the current meter is a Hall probe, thus it is capable to measure currents from DC up to 100 kHz. The working principle of the Hall probe is based on the phenomenon that if in a semiconductor chip current flows and perpendicular to the plane of the chip magnetic field is acting, then due to the interaction of the field and the moving charge carriers a voltage develops which is proportional to the induction of the magnetic field and it is perpendicular to the direction of the current („Hall voltage"), and measuring this voltage the magnitude of the magnetic induction can be determined. If the magnetic field (induction) is generated by a current, which flows in a conductor, then the induction is proportional to the current, thus placing the Hall probe in this magnetic field, the Hall voltage will be proportional to the current to be measured after all.

The Hall probe current meter contains an electronic circuit (amplifier) which generates in case of 1 A measured current 100 mV output voltage on the output of the instrument (in other words the *conversion factor of the instrument is 100 mV/A*), and by the measurement of this voltage the value of the current can be determined. *The range of the current measurement extends from 0 to ±30 A.* The accuracy on DC is: *±1 % ±2 mA (% of the measuring range).* The frequency response is given by the operating instructions of the instrument. In the frequency range from *DC up to 100 kHz* the deviation is *smaller than 0,5 dB*, and from DC up to 100 Hz it is practically negligible, so here the DC accuracy can be regarded valid.

Special literature, references

[1] Lecture notes of the subject "Measurement Technology".

[2] István Zoltán dr.: *Méréstechnika („Measurement Technology"),* Műegyetemi Kiadó, 55029, Bp., 1997. (in Hungarian): Calculation of the measurement errors: pp. 20-21. Measurement of power and energy: pp. 105 - 114.

[3] Sujbert – Naszádos – Péceli, "Exercises in Measurement Technology" http://home.mit.bme.hu/~bank/measurement/

Tasks helping the preparation

Do the following self-contained as preliminary preparation for the measurement at home.

1. Read thoroughly the chapters listed in paragraph „Special literature".
2. Read and study the *Measurement tasks*.
3. Answer i.e. solve the Test questions and the Test problems at the end of this document.

The preparation can be checked by the instructor of the measurement taking oral questions.

Applied instruments

Oscilloscope	Agilent 54622A
Function generator	Agilent 33220A
Electronic power meter	Hameg HM8115
AC power supply	Metrel MA-4804
Hall-probe current meter	Hameg HZ-56
Digital multimeter (6½ digit)	Agilent 33401A
Resistor decade	IET Labs RCS500
Transformer	VIK-01-03

Objects to be measured

R-L-C network, light sources, supply unit of a personal computer.

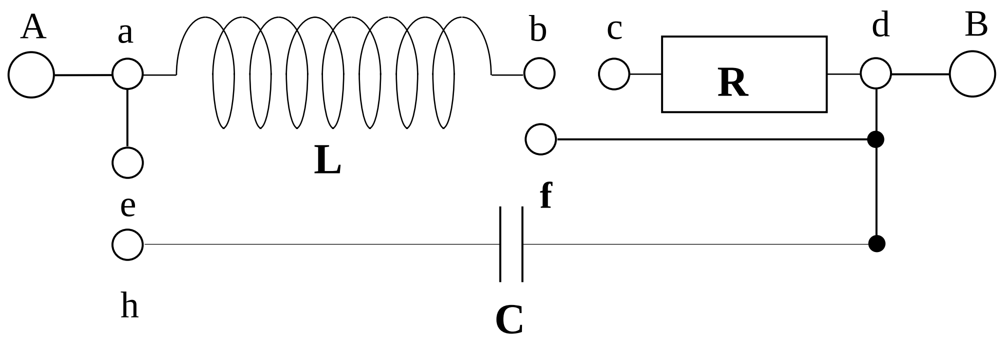

Figure 4-1. The R-L-C network used in the measurement

Measurement tasks

1. Use of a Hall-probe current meter

Consider Figure 4-1. Connect the *R* element of the R-L-C network to the function generator so that the probe can measure the current consumption of the net! Set a 50 Hz sine wave of an amplitude of 10 V_{pp} on the generator! Connect both the voltage and the output of the probe to the oscilloscope and display the waveforms!

Watch the orientation of the probe which ensures the right polarity of the output voltage of the probe! How can the signal to noise ratio of the current signal be improved?

2. Measurement of different power quantities of the R-L-C net excited by sinusoidal voltage

The measurement setup can be seen in Figure 4-2. Build the circuit prescribed by the instructor! Set a 50 Hz sine wave of an amplitude of 10 V_{pp} on the generator and measure the following quantities by the oscilloscope: voltage, current, apparent, effective and reactive power, power factor!

Use the time interval measurement function of the oscilloscope for the determination of the phase shift! Check the voltage and current values by a multimeter!

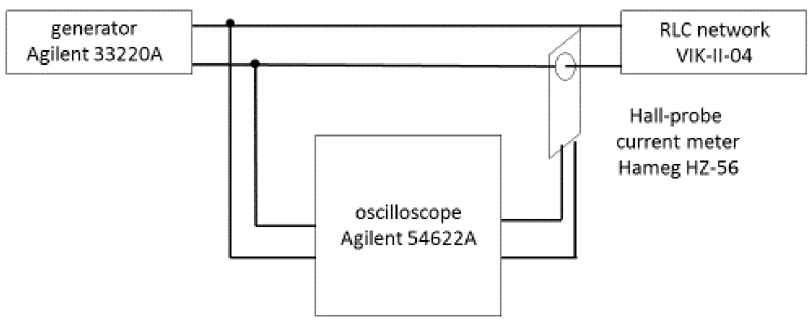

Figure 4-2. Power measurement of the circuit excited by the function generator.

3. Measurement of different power quantities of the R-L-C net excited by general periodic voltage

The measurement setup and the main settings are the same as in the previous task. Set the voltage waveform prescribed by the instructor! Measure again the following quantities by the oscilloscope: voltage, current, apparent, effective and reactive power, power factor!

Now the effective power of the circuit must be measured by the definition, by the application of the „math” and „measure” functions of the oscilloscope!

4. Measurement of different power quantities of the R-L-C net excited by an AC power supply

The measurement setup can be seen in Figure 4-3. The settings of the R-L-C network does not change. For safety reasons the mains is transformed to extra low voltage. Set 230 V voltage on the AC supply! Measure the following quantities: voltage, current, apparent, effective and reactive power, power factor by

a) electronic power meter;

b) oscilloscope!

Apply the signal processing set on the oscilloscope in the previous task! The excitation voltage can be connected to the oscilloscope by the 10:1 probe.

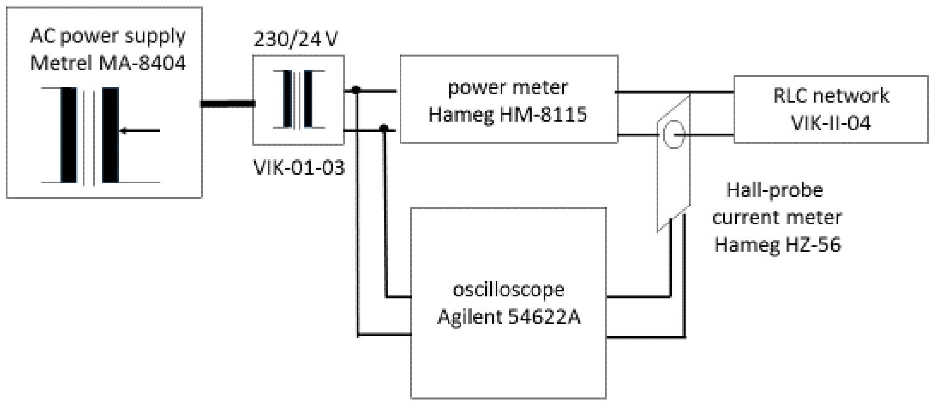

Figure 4-3. Power measurement of the circuit excited by the AC power supply.

5. Measurement of different power quantities of light sources excited by the mains

The measurement setup can be seen in Figure 4-4. The voltage of the circuit is to be reduced by a 230/12 V transformer and a special resistive divider. Only this reduced voltage is allowed to be connected to the oscilloscope! Consider the following light sources:

a) halogen lamp or traditional filament lamp;

b) compact fluorescent lamp or LED lamp!

Measure the voltage, current, apparent, effective and reactive power of the light sources in the 0...230 V interval by

a) electronic power meter;

b) oscilloscope!

Apply the signal processing set on the oscilloscope in task 3!

Plot the *P* – *U* diagram for all the measured light sources! Choose the measurement points carefully!

What is the relation between the effective power, the voltage, and the luminous flux (light flux)? What are the qualitative experiences?

In the case of one light source and one setting selected by the instructor plot the voltage and current waveforms!

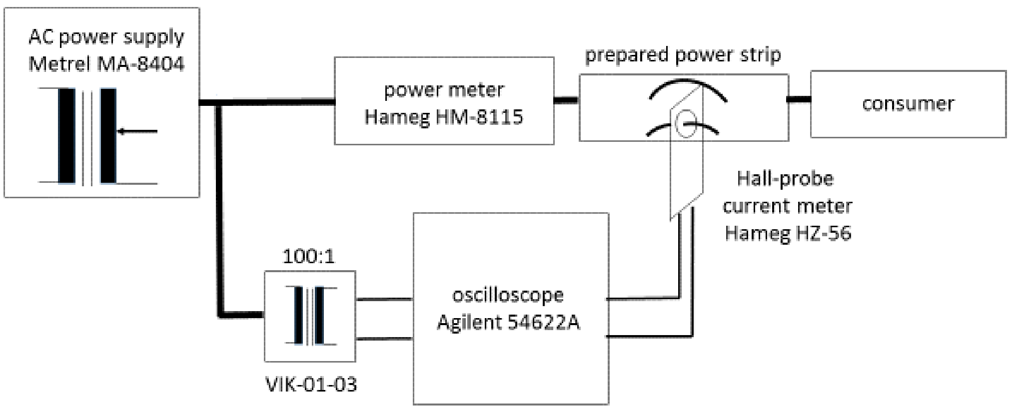

Figure 4-4. Power measurement at a voltage of the mains.

6. Measurement of the distortion factor of the mains and the crest factor of the current

Determine the distortion factor of the voltage measured on the light source chosen in the previous task!

Determine the crest factor of the current measured on the light source (at a voltage defined by the instructor) chosen in the previous task!

7. Error analysis

Calculate the error of the effective power measurement for all the results obtained in the measurement tasks 2, 3, 4, and 5! In the case of measurement task 5 use only the power data measured at the nominal voltage! The calculation should consider the data sheets of the electronic power meter, oscilloscope and the other devices used for the measurements!

Optional measurement task:

8. Measurement of different power quantities of a personal computer

Measure the voltage, current, apparent, effective and reactive power of a personal computer by

a) electronic power meter;

b) oscilloscope!

Apply the signal processing set on the oscilloscope in task 3!

Test questions

1. How to calculate the instantaneous power?
2. What is the meaning (by convention) of the positive or negative sign of a DC power?
3. In an AC circuit the effective (r.m.s.) value of the voltage and the current measured on a two-pole is U and I, respectively. The phase angle between the voltage and the current is φ (the voltage leads to the current if φ is positive). How to calculate the effective, the reactive and the apparent power of the two-pole? How do these quantities change if the sign of φ reverses?
4. How do we define the effective and the reactive power if the waveform is not sinusoidal but it is periodic? Denote the DC components of the voltage and the current U_0 and I_0, respectively, U_i and I_i the effective value of the i-th over-harmonic of the voltage, i.e. the current and φ_i the phase angle between these over-harmonics (i = 1 belongs to the fundamental harmonic and the voltage leads to the current if φ_i is positive).
5. How to calculate the effective power in a circuit where the waveform of the voltage is sinusoidal, but that of the current – due to the nonlinearity of the circuit – is not sinusoidal but the period times are identical?
6. What is the definition of the electrical energy (work, consumption)?
7. Describe the principle of the electrodynamic wattmeter!
8. Describe the method of the three voltmeter effective power measurement!
9. How the effective power can be measured if no wattmeter is available, only general laboratory instruments can be used?
10. How a digital storage oscilloscope can be used for power measurement?
11. How to calculate the measurement error if (a) the three voltmeter method, (b) an oscilloscope have been used for the measurement?
12. How to choose the measurement points if an unknown power – voltage characteristics should be measured? Too many points result in slow measurement, too few points result in the loss of the determination of the interesting points of the characteristics.

Test problems

1. Draw an AC generator of known frequency, source voltage and internal impedance. Connect a load impedance composed of an inductance and a series resistance to it. Take the positive direction of the voltage and the current as you like and write the relationships for the calculation of the current, the voltage on the terminals of the generator and the power respectively.
2. See Problem 1. Plot the time functions of the voltage, the current and the power for an interval of one period time if
   - a.) the value of the inductance is zero,
   - b.) the value of the resistance is zero,
   - c.) if the inductance is substituted by a capacitance and the resistance is zero.

3. Draw the voltage and the current on a phasor diagram in the above cases.

4. Suppose the internal impedance of a 50 Hz voltage generator is zero, the source voltage is 110 V and in the load impedance the inductance has a value of 1.2 H and the value of the series resistance is 40 ohm. What are the effective value and the phase angle of the current? How much are the real and the imaginary components of the current? How much loss is generated in the impedance? How much are the reactive and the apparent powers?

5. See Problem 4. What should be the value of an additional resistance connected series to the load impedance to get a phase angle of 45° between the terminal voltage and the current? Denote the resulted impedance by Z_E.

6. Suppose the internal impedance of a 50 Hz voltage generator is zero, the source voltage is 110 V and let the load impedance be Z_E. What are the effective value and the phase angle of the current? How much are the real and the imaginary components of the current? How much loss is generated in the impedance? How much are the reactive and the apparent powers?

7. Draw the phasor diagram of the current and the voltage in Problem 6.

8. What should be the value of the phase improving capacitance connected parallel to the Z_E impedance to get a phase angle of 0° between the terminal voltage and the current? It is known that the loss factor of the capacitance is less than 10^{-4}. In this case, how much is the absolute value and the phase angle of the resulting impedance? How much is the loss in the capacitance?

9. Study the manual of the electronic power meter of Hameg, type HM 8115. Analyse whether this instrument can be used to measure the terminal voltage, the current and the power of the devices in the measurement tasks 2, 3, 4, and 5. If yes, draw the connection arrangement. Consider the use of the instrument. Summarize the elementary steps from the drawing of the connection arrangement till the determination of the final result and the measurement uncertainty. (An elementary step is e.g. the formation of the cabling according to the connection diagram, the switch on, the setting of an operation point, taking and recording the readings etc.).

10. Study the manual of the digital storage oscilloscope Agilent 54622A. Analyse whether this instrument can be used to measure the terminal voltage, the current and the power of the devices in the measurement tasks 2, 3, 4, and 5. If yes, draw the connection arrangement. Consider the use of the instrument. Summarize the elementary steps from the drawing of the connection arrangement till the determination of the final result and the measurement uncertainty.