PHASE-LOCKED LOOP

- Basic loop configuration
- Operation principle of phase-locked loop
- Loop equations and nonlinear baseband model
- Linear operation of the PLL
  - Linear baseband model
  - Transfer functions
  - PLL with active loop filter (Most commonly used PLL configuration)
  - Stability considerations
- An example for PLL application: Coherent FM demodulator

References for phase-locked loop:

[1] G. Kolumbán, "Phase-Locked Loops"
Article in *The Encyclopedia of RF and Microwave Engineering*, K. Chang, (Ed.), vol. 4, pp. 3735–3767, Wiley, New York, 2005.

[2] G. Kolumbán, "Phase-Locked Loops"
Article in *The Encyclopedia of Electrical and Electronics Engineering*, J. G. Webster, Ed., vol. 16, pp. 158–188, Wiley, New York, 1999.

Phase-locked loop is one of the most commonly used circuit in both telecommunication and measurement engineering. Depending on the operation principle of loop components we distinguish

- Analog
- Digital
- Hybrid

phase-locked loops. Only the analog phase-locked loop (APLL) is discussed in this course. For the sake of simplicity, we will call this circuit PLL

ANALOG PHASE-LOCKED LOOP

Circuit configuration:

- Phase detector (PD) is an analog multiplier
- All loop components are analog circuits

Mathematical model:

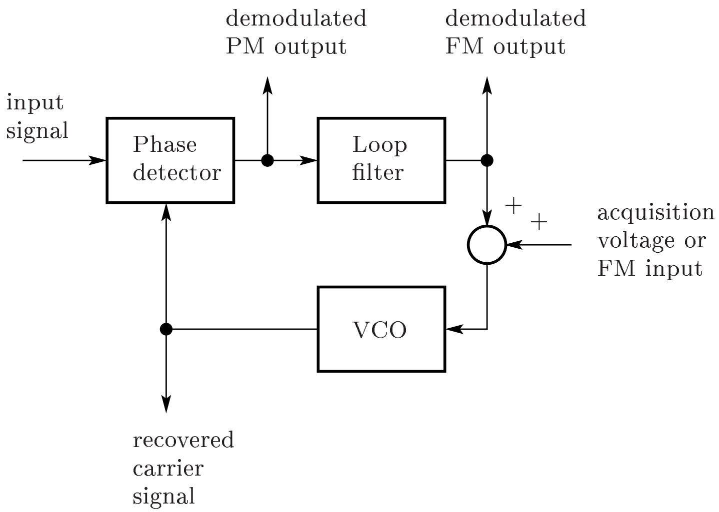
- Operation of analog phase-locked loop is modeled by an ordinary differential equation

Conditions:

- For the sake of simplicity, only the noise-free case is studied here
- We assume that the only source of nonlinearity is the phase detector, the other loop components are assumed to be linear

BASIC LOOP CONFIGURATION

PLL block diagram showing inputs and outputs for various applications



A few important applications:

- Demodulation of FM and PM signals
- FM modulator
- Carrier recovery

The PLL is a *nonlinear feedback* system that tracks the phase of input signal

The basic PLL configuration contains a

- *Phase detector* (PD)
- Time-invariant linear *loop filter* and
- *Voltage-controlled oscillator* (VCO); the oscillator to be synchronized

QUALITATIVE CHARACTERIZATION OF LOOP COMPONENTS

Phase detector (PD):

- Analog multiplier
- PD produces an error signal that is proportional to the *phase error*, i.e., to the difference between the phases of input and output signals of the phase-locked loop

Loop filter:

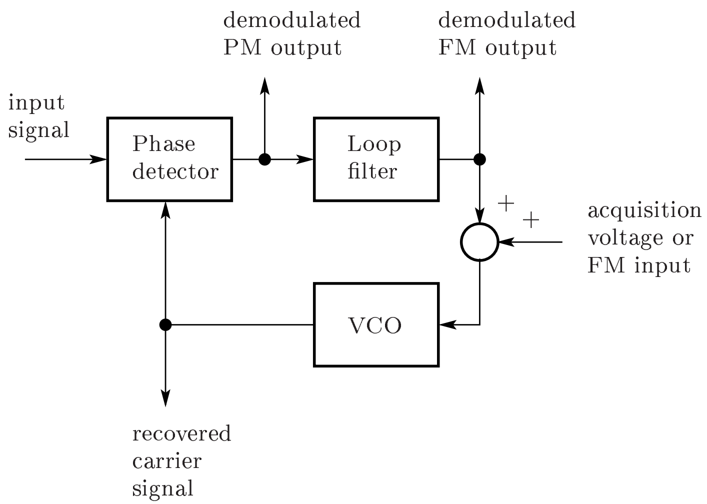
- Low-pass filter
- It is characterized by its transfer function $F(s)$
- Low-pass filter suppresses the noise and unwanted PD outputs. It determines the *dynamics* of phase-locked loop

Voltage-controlled oscillator (VCO):

- VCO generates a sinusoidal signal
- The instantaneous VCO frequency is controlled by its input voltage

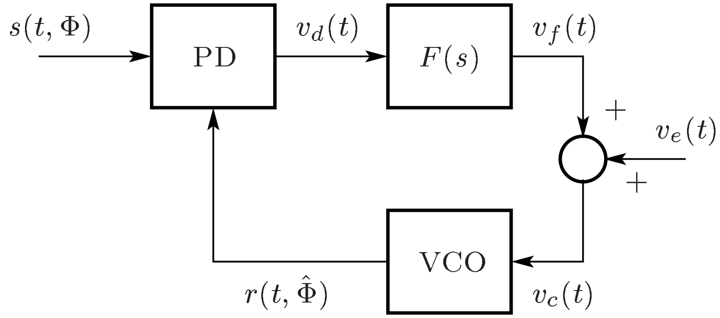
OPERATION PRINCIPLE OF PHASE-LOCKED LOOP – Part I

Basic loop configuration



PLL block diagram

Voltages appearing in the loop are also shown



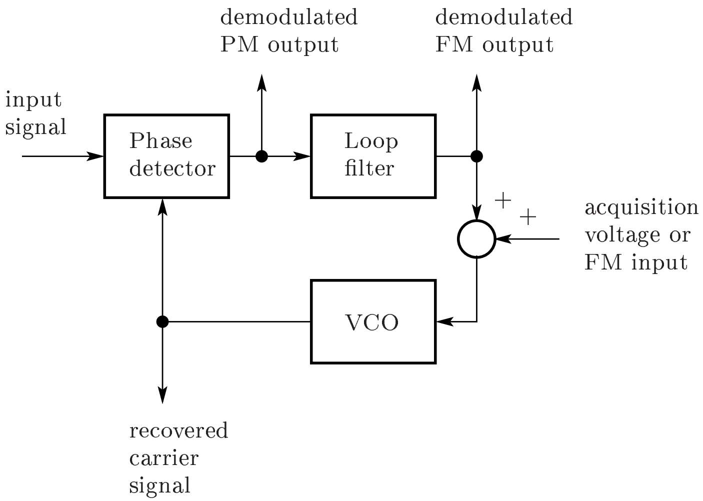
Phase detector (PD) compares the phase of the input signal $s(t, \Phi)$ against the phase of the VCO output $r(t, \hat{\Phi})$ and produces an error signal $v_d(t)$

This error signal is then filtered, in order to remove noise and other unwanted components of the input spectrum

The sum of filter output $v_f(t)$ and an additive external control voltage $v_e(t)$ controls the instantaneous VCO frequency

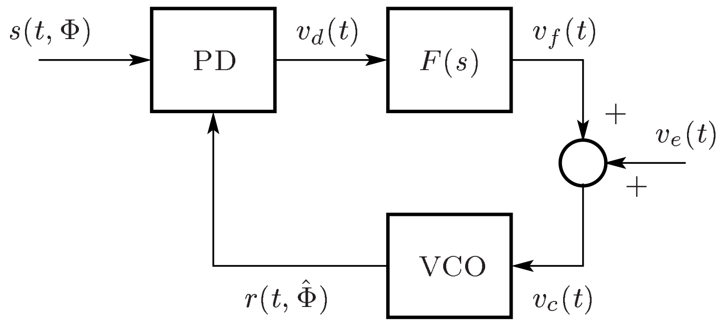
OPERATION PRINCIPLE OF PHASE-LOCKED LOOP – Part II

Basic loop configuration



PLL block diagram

Voltages appearing in the loop are also shown



A nonzero output voltage must be provided by the PD, in order to tune the VCO frequency to the input one if the input frequency differs from the VCO center frequency

Consequently, the PLL tracks the phase of input signal with some phase error. However, this phase error can be kept very small in a well-designed PLL

IMPORTANT PLL CHARACTERISTICS – Part I

Acquisition and Tracking

In every application, the PLL tracks the phase of the input signal. However, before a PLL can track, it must first reach the phase-locked condition

In general, the VCO center frequency ω_0 differs from the frequency ω_i of the input signal

Therefore, first the VCO frequency has to be tuned to the input frequency by the loop. This process is called *frequency pull-in*

Then the VCO phase has to be adjusted according to the input phase. This process is known as *phase lock-in*

Both the frequency pull-in and phase lock-in processes are parts of acquisition which is a highly nonlinear process and is very hard to analyze

After acquisition the PLL achieves the *phase-locked* condition, where the PLL tracks the input phase. Under this *phase-locked condition*, the VCO frequency is equal to the input frequency

Pull-in Range

$\Delta\omega_P = |\omega_i - \omega_0|$ is the maximum initial frequency difference between the input and VCO center frequencies both in positive and negative directions, for which the PLL eventually achieves the phase-locked condition. The pull-in range is related to the dynamics of the PLL

Lock-in Range

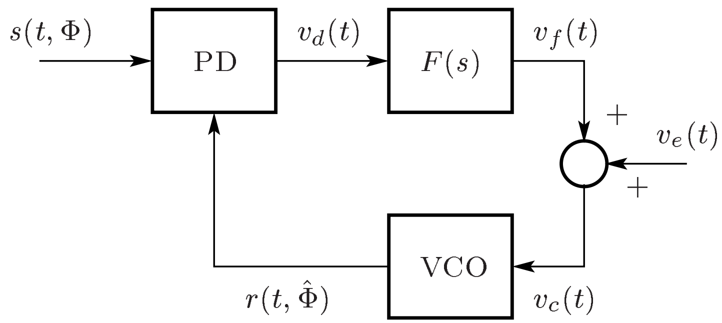
$\Delta\omega_L = |\omega_i - \omega_0|$ is the frequency range over which the PLL achieves the phase-locked condition without cycle slips, i.e., $-\pi < \theta_e(t) < \pi$ during the entire lock-in process

Hold-in Range

Suppose the phase-locked condition has been achieved in the PLL. Now vary the input frequency ω_i slowly and the VCO frequency will follow it. The hold-in range $\Delta\omega_H = |\omega_i - \omega_0|$ is determined by the lower and upper values of ω_i, for which the phase-locked condition is lost. The hold-in range represents the maximum static tracking range and is determined by the saturation characteristics of the nonlinear loop elements of the PLL

LOOP EQUATIONS AND NONLINEAR BASEBAND MODEL

PLL block diagram



$F(s)$ denotes the transfer function of the loop filter

In order to write the differential equations in compact form, the operation of differentiation d/dt in the time domain will be denoted by the multiplication of the *Heaviside operator* p

Note, the Heaviside operator is valid in the time domain, while s denotes the *complex frequency*. If the transfer function $F(s)$ of a linear network is given in the complex frequency domain s then the transfer function in operator form may be expressed as $F(p) = F(s)|_{s=p}$

DEVELOPMENT OF LOOP EQUATIONS

In the equations to be developed, the time variable t is suppressed for conciseness where it does not cause misunderstanding

Input signal

Let the phase $\Phi(t)$ of input signal $s(t, \Phi)$ be expressed with respect to the VCO center frequency ω_0 as

$$\Phi = \omega_0 t + \theta_i$$

Then the input signals becomes

$$s(t, \Phi) = \sqrt{2} A \sin \Phi = \sqrt{2} A \sin(\omega_0 t + \theta_i)$$

where $A(t)$ describes the amplitude modulation of input signal and $\theta_i(t)$ is the *input phase modulation*, i.e., the PM of the input signal. Note that $\theta_i(t)$ also incorporates the input frequency error $\Delta\omega_i = \omega_i - \omega_0$

Output signal of voltage controlled oscillator (VCO)

VCO output is the output signal of phase-locked loop. Since the VCO phase $\hat{\Phi}(t)$ tracks the phase $\Phi(t)$ of input signal we call it loop estimate of $\Phi(t)$. It is expressed with respect to the VCO center frequency as

$$\hat{\Phi} = \omega_0 t + \theta_o$$

Then the VCO output is obtained as

$$r(t, \hat{\Phi}) = \sqrt{2} V_o \cos \hat{\Phi} = \sqrt{2} V_o \cos(\omega_0 t + \theta_o)$$

In the above equations, $\theta_o(t)$ and V_o denote the phase and rms amplitude of VCO output, respectively

Transfer function of voltage controlled oscillator (VCO)

The frequency of a voltage controlled oscillator is determined by the VCO control voltage $v_c(t)$. The instantaneous VCO frequency referenced to ω_0 varies linearly with the control voltage $v_c(t)$

$$\frac{d\hat{\Phi}}{dt} - \omega_0 = \frac{d}{dt}[\omega_0 t + \theta_o(t)] - \omega_0 = \frac{d\theta_o}{dt} \equiv K_v v_c$$

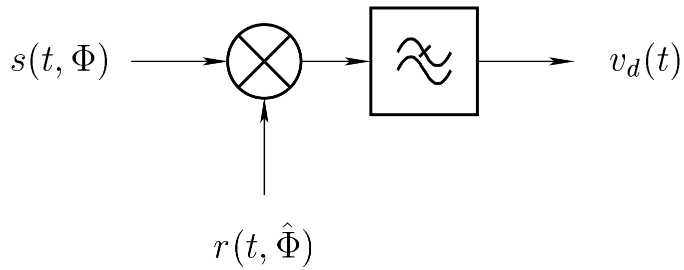
where K_v is the VCO gain in $\frac{\text{rad}}{\text{Vs}}$. Note if $v_c(t) = 0$ then the VCO frequency is equal to the center frequency ω_0

Transfer function of phase detector (PD)

Block diagram of a phase detector

Note a PD consists of

- An analog multiplier
- A low-pass filter



The analog multiplier in the PD multiplies the input signal $s(t,\Phi) = \sqrt{2}A\sin(\omega_0 t + \theta_i)$ and VCO output $r(t,\hat{\Phi}) = \sqrt{2}V_o\cos(\omega_0 t + \theta_o)$ and produces both the difference- and sum-frequency terms. The low-pass filter eliminates the sum-frequency component. The PD output is obtained as

$$v_d = \mathcal{FILT}\{s(t,\Phi)r(t,\hat{\Phi})\} = AV_o\sin(\theta_i - \theta_o) = AV_o\sin\theta_e = K_d A\sin\theta_e$$

where the *phase error* is defined by

$$\theta_e(t) = \theta_i(t) - \theta_o(t)$$

and $K_d = V_o$, a dimensionless quantity, is the gain of PD

Properties of phase detector

$$v_d = K_d A \sin \theta_e$$

- Phase detector is a nonlinear device
- Its output depends on the difference of input and VCO phases
- Its output also depends on $A(t)$, i.e., on the AM of input signal

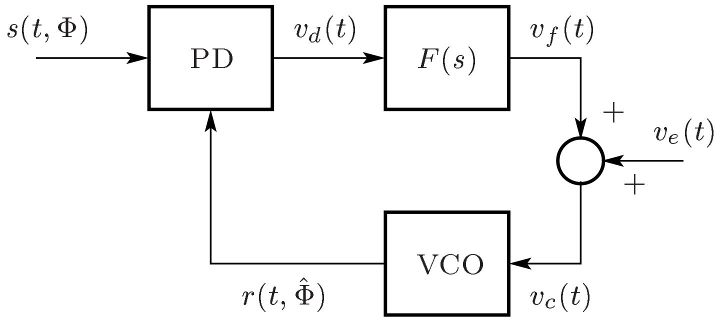
Loop filter and adder

The VCO control voltage $v_c(t)$ is the sum of the loop filter output $v_f(t)$ and external control voltage $v_e(t)$

$$v_c(t) = v_f(t) + v_e(t) = F(p)v_d(t) + v_e(t)$$

where $F(p) = F(s)|_{s=p}$ and $p = \frac{d}{dt}$ is the Heaviside operator

Equations we obtained up to this point



$$\frac{d\theta_o}{dt} = K_v v_c \quad \Longrightarrow \quad p\theta_o = K_v v_c$$

$$v_c = F(p)v_d + v_e$$

$$v_d = K_d A \sin\theta_e$$

$$\theta_e = \theta_i - \theta_o$$

LOOP EQUATIONS

$$\begin{aligned}\theta_o =& \frac{K_v}{p} v_c = \frac{K_v F(p)}{p} v_d + \frac{K_v}{p} v_e = \frac{K_v K_d F(p)}{p} A\sin\theta_e + \frac{K_v}{p} v_e \\ =& \frac{KF(p)}{p} A\sin\theta_e + \frac{K_v}{p} v_e\end{aligned}$$

where $K = K_d K_v$ defines the *loop gain* in rad/Vs

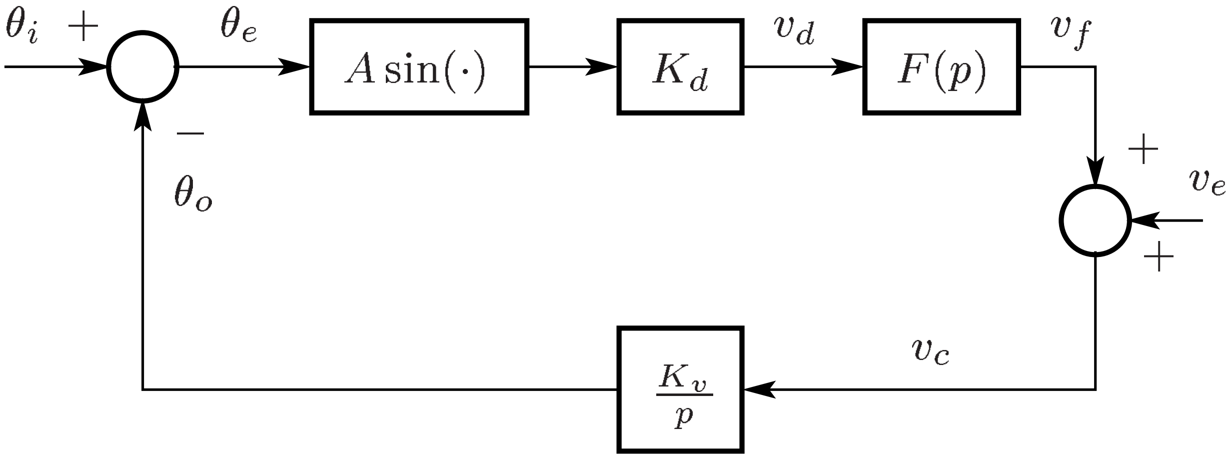
$$\theta_e = \theta_i - \theta_o = \theta_i - \frac{KF(p)}{p} A\sin\theta_e - \frac{K_v}{p} v_e$$

Loop equations

$$\theta_e = \theta_i - \theta_o$$

$$\theta_o = \frac{KF(p)}{p} A \sin\theta_e + \frac{K_v}{p} v_e = \frac{K_v}{p}[F(p) K_d A \sin\theta_e + v_e]$$

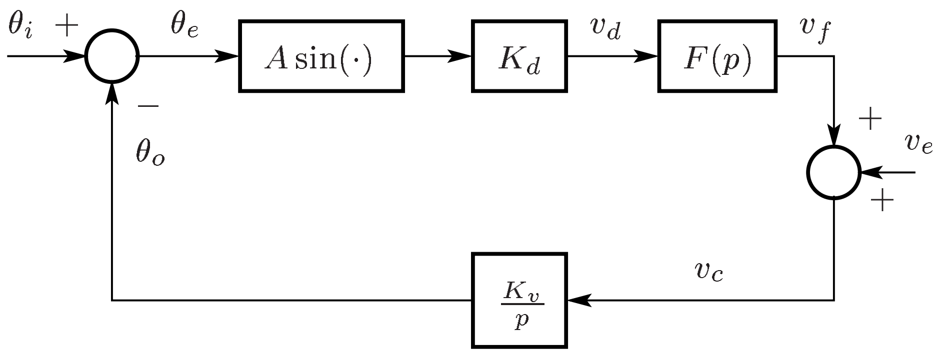
NONLINEAR BASEBAND MODEL



Recall:

$$\begin{aligned} v_d &= K_d A \sin\theta_e \\ v_f &= F(p) v_d \\ v_c &= v_f + v_e \end{aligned}$$

Properties of nonlinear baseband model



Real input and output signals:

$$\begin{aligned} s(t,\Phi) &= \sqrt{2}A\sin(\omega_0 t + \theta_i) \\ r(t,\hat{\Phi}) &= \sqrt{2}V_o\cos(\omega_0 t + \theta_o) \end{aligned}$$

Note:

- *Baseband* model contains only low-pass signals because the carrier has been removed
- Input and output signals of baseband model are the input θ_i and output θ_o phase modulations
- Real input and output signals do *not appear* in the baseband model they have to be calculated from θ_i and θ_o
- Since the VCO can generate only angle modulated signals, only angle modulated signals can be produced by the PLL
- Because of the nonlinear PD characteristic, this model is nonlinear, consequently, its analysis must be performed in time domain. Transfer function concept may not be used

LINEAR OPERATION OF PLL

The linear operation of PLL assumes that

- Phase-locked condition has been achieved and is maintained
- Phase error remains in the neighborhood of its quiescent value, i.e. we may linearize the PLL using the small-signal approximation

Mathematical background of linearization: Taylor series representation

Steps of linearization

1. Determination of the quiescent point
2. Approximation of nonlinear characteristic by its tangent (Linear term in the Taylor series approximation)

Determination of quiescent point

If a PLL operates in steady-state and all its input signals are constant then the PLL is operating in the quiescent point

Let the PLL loop equation rearranged as

$$\theta_e = \theta_i - \theta_o = \theta_i - \frac{KF(p)}{p} A \sin\theta_e - \frac{K_v}{p} v_e \quad \Rightarrow \quad p\theta_e = p\theta_i - KF(p)A\sin\theta_e - K_v v_e$$

Under steady-state conditions, all signals are constant, but a constant input frequency error may be present

$$\begin{aligned} \theta_e(t) &= \theta_{ss} \\ v_e(t) &= v_{e0} \\ \theta_i(t) &= (\omega_i - \omega_0)t + \theta_{i0} = \Delta\omega_i t + \theta_{i0} \end{aligned}$$

Since the Heaviside operator means derivation d/dt in the time domain we get

$$0 = \Delta\omega_i - KF(0)A\sin\theta_{ss} - K_v v_{e0}$$

$$0 = \Delta\omega_i - KF(0)A\sin\theta_{ss} - K_v v_{e0}$$

From which the **quiescent point** θ_{ss} of PLL is obtained as

$$\theta_{ss} = \sin^{-1}\left(\frac{\Delta\omega_i - K_v v_{e0}}{KF(0)A}\right)$$

where $F(0)$ is the dc gain of loop filter

Note: To get the quiescent point, a nonlinear dc analysis had to be performed

To get the best system performance, the quiescent value of phase error has to be set to zero, i.e.,

$$\theta_{ss} = 0$$

It can be achieved if the dc gain of loop filter goes infinite $F(0) \to \infty$. This conditions may be satisfied by the most commonly used active loop filter. In the remaining part of discussion we assume that an active loop filter is used

Mathematical background of linearization: **Taylor series approximation**

$$y = g(\theta) = g(\theta_{ss}) + \frac{1}{1!}\left.\frac{dg(\theta)}{d\theta}\right|_{\theta_{ss}} (\theta - \theta_{ss}) + \cdots + \frac{1}{n!}\left.\frac{dg^n(\theta)}{d\theta^n}\right|_{\theta_{ss}} (\theta - \theta_{ss})^n + \ldots$$

Only the linear term is considered in the **small-signal model**

$$\Delta y = y - g(\theta_{ss}) = \frac{1}{1!}\left.\frac{dg(\theta)}{d\theta}\right|_{\theta_{ss}} (\theta - \theta_{ss}) = \left.\frac{dg(\theta)}{d\theta}\right|_{\theta_{ss}} \Delta\theta$$

where $\left.\frac{dg(\theta)}{d\theta}\right|_{\theta_{ss}}$ is the tangent of the nonlinear function $f(\theta)$ at the quiescent point θ_{ss}, Δy and $\Delta\theta$ are called perturbations

If $\theta_{ss} = 0$ and $g(0) = 0$ then the variables θ and y, and their perturbations $\Delta\theta$ and Δy, respectively, become identical. Consequently, we obtain

$$y = \left.\frac{dg(\theta)}{d\theta}\right|_{\theta_{ss}} \theta$$

Linearization of nonlinear baseband model

(Determination of the small-signal model)

The only nonlinear loop component is the phase detector

$$v_d = K_d A \sin\theta_e$$

Since $\theta_{ss} = 0$ and $v_d(0) = 0$, $\Delta v_d = v_d$ and $\Delta\theta_e = \theta_e$. If during the operation the phase error always remains in the neighborhood of θ_{ss} then we may linearize the phase detector

$$v_d = K_d A \sin\theta_e \approx K_d A \theta_e$$

Substituting $\sin\theta_e \approx \theta_e$ in the nonlinear loop equation, the linear loop equations are obtained as

$$\theta_e = \theta_i - \theta_o$$

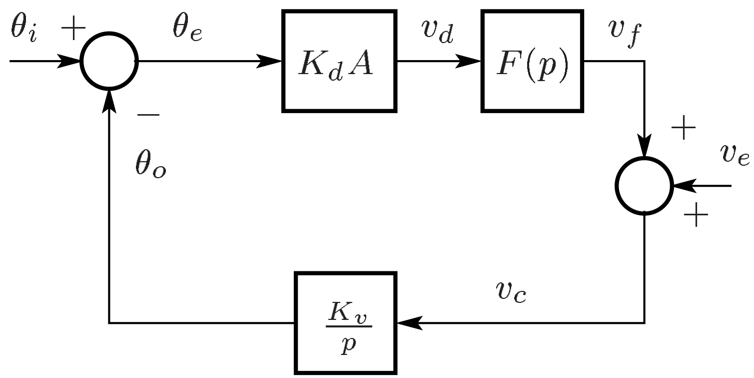
$$\theta_o = \frac{KF(p)}{p} A\theta_e + \frac{K_v}{p} v_e = \frac{K_v}{p}[F(p) K_d A \theta_e + v_e]$$

Linearized loop equations

$$\theta_e = \theta_i - \theta_o$$

$$\theta_o = \frac{KF(p)}{p} A\theta_e + \frac{K_v}{p} v_e = \frac{K_v}{p}[F(p)K_d A\theta_e + v_e]$$

LINEAR BASEBAND MODEL



Recall:

$$\begin{aligned} v_d &= K_d A\theta_e \\ v_f &= F(p)v_d \\ v_c &= v_f + v_e \end{aligned}$$

Based on the linear baseband model, the *transfer functions* may be developed

To show explicitly the dependence of PLL parameters on the amplitude of input signal, A is not lumped with K_d

TRANSFER FUNCTIONS

A linear (and only a linear) system may be characterized by its transfer functions. The *transfer function* expresses the output signal of the linear system as a function of an input signal

Transfer function gives the response of a linear system to an arbitrary input in *closed form*

A linear system may have many inputs and outputs, transfer functions may be developed between each pair of output and input

The transfer functions may be expressed starting from

- Loop equations
- Linear baseband model applying the rules of block diagram algebra

An example: Express $\Theta_o(s)$ as a function of $\Theta_i(s)$ in the complex frequency domain s

Step 1: Linearized PLL loop equations in the time domain

$$\theta_o = \frac{KF(p)}{p} A\theta_e + \frac{K_v}{p} v_e = \frac{K_v}{p} [F(p) K_d A\theta_e + v_e]$$

$$\theta_e = \theta_i - \theta_o$$

Step 2: Transformation of the signals and system into the complex frequency domain s by means of Laplace transform

$$p = s$$

$$\Theta_i(s) = \mathcal{L}\{\theta_i(t)\} \quad \dots \quad V_c(s) = \mathcal{L}\{v_c(t)\}$$

Recall: Fourier transform can be determined from the Laplace transform by substituting $s = j2\pi f$

Step 3: Development of the transfer function $H(s)$

$$\Theta_o(s) = \frac{KF(s)}{s} A\Theta_e(s) + \frac{K_v}{s} V_e(s) \quad \text{where} \quad V_e(s) = 0$$

$$\Theta_e(s) = \Theta_i(s) - \Theta_o(s)$$

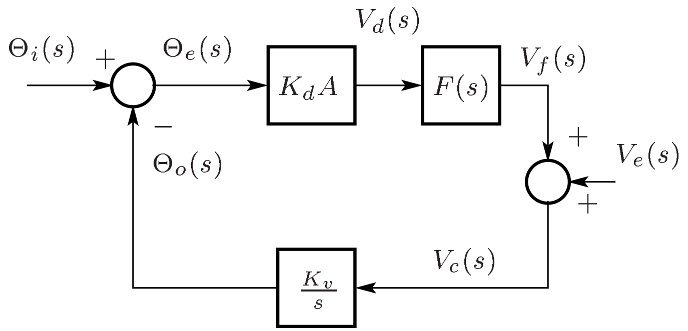
$$\Theta_o(s) = \frac{AKF(s)}{s} [\Theta_i(s) - \Theta_o(s)]$$

$$\left[1 + \frac{AKF(s)}{s}\right] \Theta_o(s) = \frac{AKF(s)}{s} \Theta_i(s)$$

$$\Theta_o(s) = \frac{AKF(s)}{s + AKF(s)} \Theta_i(s) = H(s)\Theta_i(s)$$

where $H(s)$ is the *closed-loop transfer function*

PLL transfer functions



$$\Theta_e(s) = [1 - H(s)]\left[\Theta_i(s) - \frac{K_v}{s}V_e(s)\right]$$

$$\Theta_o(s) = H(s)\Theta_i(s) + [1 - H(s)]\,\frac{K_v}{s}V_e(s)$$

Note: Only two transfer functions

- *Closed-loop transfer function* $H(s)$
- *Error function* $[1 - H(s)]$

are required to characterize completely the PLL

Closed-loop transfer function (Low-pass characteristic)

$$H(s) = \frac{AKF(s)}{s + AKF(s)}$$

Error function (High-pass characteristic)

$$1 - H(s) = \frac{s}{s + AKF(s)}$$

Parameters of closed-loop transfer and error functions are determined by

- Loop gain $K = K_d K_v$
- Transfer function of loop filter $F(s)$

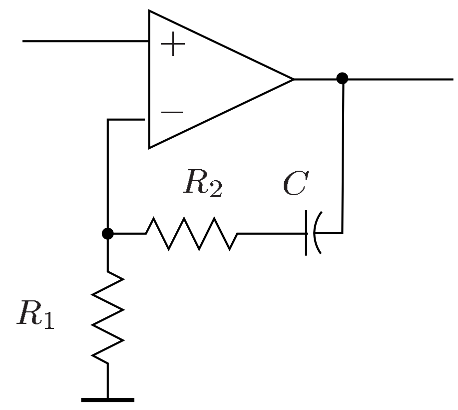
and, unfortunately, by

- Amplitude (and if there is any, the AM) $A(t)$ of input signal

In the majority of applications, this dependence on $A(t)$ is not allowed. Solution: An AGC circuit preceding the PLL is used to fix the amplitude of input signal

PLL IMPLEMENTED WITH ACTIVE LOOP FILTER

Circuit diagram of active loop filter



If an ideal operational amplifier (op amp) is used then the transfer function of loop filter is

$$F(s) = \frac{1 + s\tau_2}{s\tau_1}, \qquad \text{where} \quad \tau_1 = R_1 C \ \text{ and } \ \tau_2 = (R_1 + R_2)C$$

Due to the infinite dc gain of ideal op amp, $F(0) \to \infty$ and, consequently, the steady-state phase error θ_{ss} is equal to zero

Closed-loop transfer function (PLL implemented with active loop filter)

$$H(s) = \frac{2\xi\omega_n s + \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

where the *natural frequency* ω_n of the loop is defined by

$$\omega_n = \sqrt{\frac{AK}{\tau_1}}$$

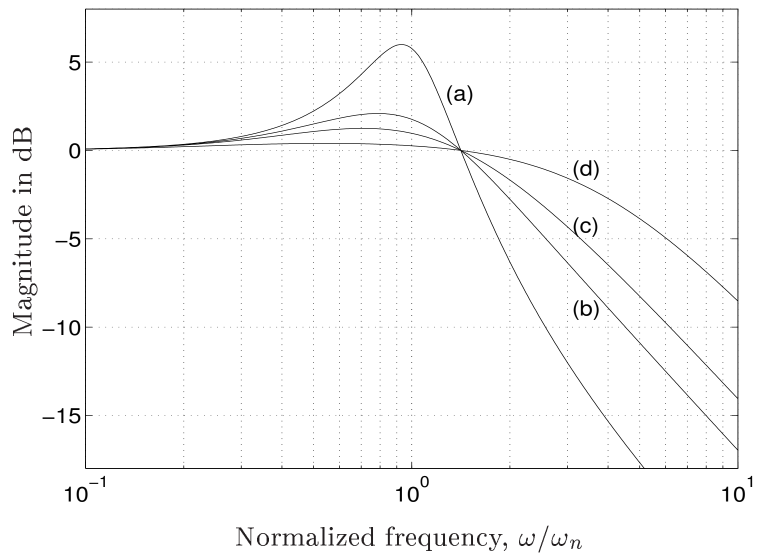
and the *damping factor* ξ of the PLL is defined by

$$\xi = \frac{\tau_2\omega_n}{2}$$

PLL implemented with an active loop filter is a second-order, type-two feedback system. Unfortunately, both the natural frequency ω_n and damping factor ξ depend on $A(t)$ which may be an AM or the effect of a time-varying channel

Frequency response of the PLL implemented with ideal loop filter

$$\Theta_o(f) = H(s)|_{s=j2\pi f}\,\Theta_i(f) = H(f)\Theta_i(f)$$



Transfer response has a low-pass characteristic to the input PM Parameter is the damping factor $(0.3 \leq \xi \leq 2)$

Recall: The real input and output signals measured in a built PLL may be calculated from θ_i and θ_o

$$s(t,\Phi) = \sqrt{2}A\sin(\omega_0 t + \theta_i)$$
$$r(t,\hat{\Phi}) = \sqrt{2}V_o\cos(\omega_0 t + \theta_o)$$

Error function (PLL implemented with active loop filter)

$$1 - H(s) = \frac{s^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

where the *natural frequency* ω_n of the loop is defined by

$$\omega_n = \sqrt{\frac{AK}{\tau_1}}$$

and the *damping factor* ξ of the PLL is defined by

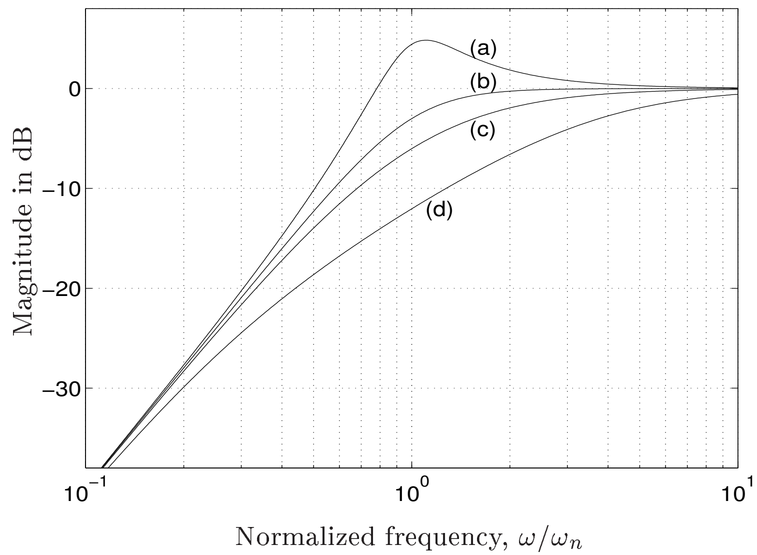
$$\xi = \frac{\tau_2 \omega_n}{2}$$

Note again, both the natural frequency ω_n and damping factor ξ depend on $A(t)$. This dependence may be prevented by an AGC circuit preceding the PD. The duty of AGC is to remove $A(t)$ caused by either AM or introduced by the time-varying channel

Error response of the PLL implemented with ideal loop filter

$$\Theta_e(f) = \left[1 - H(s)\big|_{s=j2\pi f}\right]\Theta_i(f)$$
$$= [1 - H(f)]\Theta_i(f)$$

Error response has a high-pass characteristic to the input PM
Parameter is the damping factor $(0.3 \leq \xi \leq 2)$

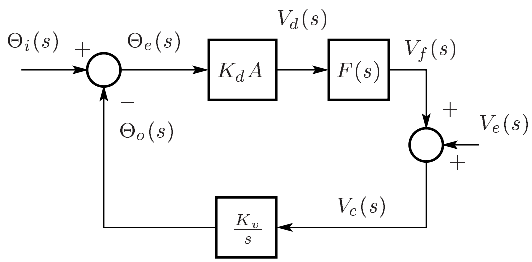


Recall: $\theta_i(t)$ and $\theta_e(t)$ cannot be measured in a built PLL. The signals that may be measured in a built PLL are calculated from θ_i and θ_e

$$s(t,\Phi) = \sqrt{2}A\sin(\omega_0 t + \theta_i)$$
$$v_d(t) = K_d A \sin\theta_e$$

STABILITY CONSIDERATIONS

PLL baseband model



where the *error signal* is

$$\Theta_e(s) = \Theta_i(s) - \Theta_o(s)$$

Conclusion: PLL is a negative feedback system which may become unstable

Mathematical background of stability analysis

Transient response is determined by the characteristic equation

A system is stable, if it does not generate an output without an input signal. Transient response is determined by the *characteristic equation*. A system is stable if all roots of characteristic equation have a negative real value

The characteristic equation is equal to the denominator of closed-loop transfer function $H(s)$

STABILITY CONDITION

The characteristic equation is equal to the denominator of closed-loop transfer function $H(s)$

Consequently, a *linear* system is *asymptotically stable* if *all poles* of its transfer function, that is, the roots of the denominator of closed-loop transfer function $H(s)$ are in the *left side* of the s-plane

A *necessary* and *sufficient* condition for the stability of a linear feedback system is that all the poles of the closed-loop transfer function lie in the left half s-plane

Stability of PLL implemented with active loop-filter

Closed-loop transfer function

$$H(s) = \frac{2\xi\omega_n s + \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Characteristic equation (denominator of closed-loop transfer function)

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

The two poles of PLL implemented with an active loop filter always lie in the left half-plane, consequently, this circuit is *unconditionally stable*

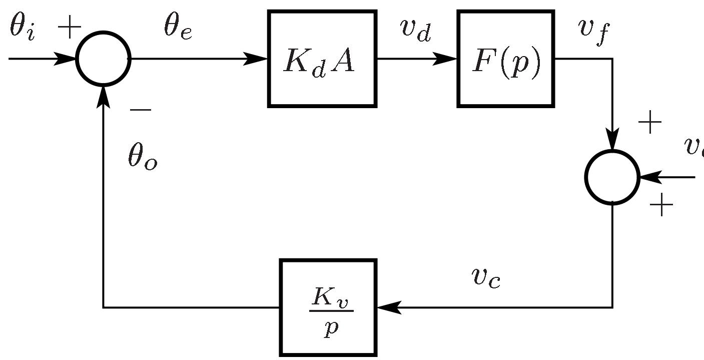
Even if the amplitude $A(t)$ of input signal varies and changes the closed-loop parameters ω_n and ξ, the PLL implemented with an active loop filter remains always stable

An example for PLL application: COHERENT FM DEMODULATOR

FM waveform: $s(t) = A_c \sin\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$

Input of the FM demodulator

$$s(t, \Phi) = \sqrt{2}A\sin(\omega_0 t + \theta_i)$$



To simplify the problem, assume $\omega_0 = 2\pi f_c$

Input FM

$$\theta_i(t) = 2\pi k_f \int_0^t m(\tau)d\tau$$

Output of the FM demodulator

$$v_c(t)$$

Basic equations in the complex frequency domain:

$$\Theta_o(s) = H(s)\Theta_i(s)$$

$$\theta_i(t) = 2\pi k_f \int_0^t m(\tau)d\tau \quad \Longrightarrow \quad \Theta_i(s) = 2\pi k_f \frac{1}{s} M(s)$$

$$\Theta_o(s) = \frac{K_v}{s} V_c(s)$$

Development of FM demodulator output in the complex frequency domain:

$$V_c(s) = \frac{s}{K_v}\Theta_o(s) = \frac{s}{K_v}H(s)\Theta_i(s) = \frac{s}{K_v}H(s)2\pi k_f \frac{1}{s}M(s) = \frac{2\pi k_f}{K_v}H(s)M(s)$$

If the maximum modulation frequency is much less than the PLL natural frequency

$$\max\{f_m\} << f_n = \frac{\omega_n}{2\pi} \quad \Longrightarrow \quad H(s) \approx 1$$

and

$$V_c(s) \approx \frac{2\pi k_f}{K_v} M(s)$$

FM demodulator output in the complex frequency domain:

$$V_c(s) = \frac{2\pi k_f}{K_v} M(s)$$

FM demodulator output in the time domain:

$$v_c(t) = \frac{2\pi k_f}{K_v} m(t)$$

Coherent FM and PM demodulation by an analog phase-locked loop