Electric power and electric power measurement

Study-aid for Laboratory 2. Exercise 4.

1 Electric power

The time function of the power can be written as shown in equation (1). In case of a DC circuit the power can be calculated as the product of the voltage and current $P = UI$, where the sign of the power indicates whether it is a power of a generator or a load.

There are two types of sign conventions. Mostly the *passive sign convention* is used. In this case both the voltage and current have positive sign on the load, that means in the circuit drawing the arrows point to the same direction. The positive power flow is power dissipation, the negative power flow is power generation. The other convention is the *active sign convention*, where the positive power flow is power generation, and negative power flow is power dissipation. This convention is rarely used.

$$p_{(t)} = u_{(t)} i_{(t)} \tag{1}$$

In case of AC circuits (using sinusoidal voltage) three types of power can be defined. The apparent power can be calculated according equation (2), the effective power according (3) and the reactive power according (4). In the passive sign convention on the inductive load the reactive power has positive sign, so the inductive load uses reactive power, whereas capacitive load and generators are producing reactive power. In the active sign convention the negative reactive power is on the inductive load.

$$S = UI = \sqrt{P^2 + Q^2} \tag{2}$$

$$P = UI \cos \varphi \tag{3}$$

$$Q = UI \sin \varphi \tag{4}$$

If the used voltage and current are periodic but not sinusoidal, this means these are composed of harmonic components, then the effective power can be calculated according equation (5). In such a case when the voltage is sinusoidal and the current is periodic but not sinusoidal, the calculation of the effective power from the equation (5) becomes much simpler. In the sinusoidal voltage there are no harmonic components, therefore the effective power is the product of the voltage, the first harmonic (fundamental frequency) of the current and the $\cos\varphi$ between the two ($P = U_1 I_1 \cos\varphi_1$).

If the the voltage and current are periodic and not sinusoidal, and the reactive power is in question, then the situation is a bit more complicated. Using the equation (6) the reactive power of the harmonics with the same frequencies can be calculated. The average of the reactive power for a cycle is zero, it means that the power is returning to the source in each cycle. The average of the product of different harmonic components also fulfils this criteria, and so can be treated as reactive power. This power is called deformed power (denoted D in (7)).

The reactive power generated by the same harmonic components of the voltage and current can be calculated according equation (6) rather easily, however calculating the remaining part — the deformed power — would be rather difficult. The deformed power of such a system can be calculated from the apparent, reactive and effective power according equation (8). The equations (8) and (7) are essentially the same, just a different arrangement of the terms.

$$P = U_0 I_0 + \sum_{i=1}^{\infty} U_i I_i \cos\varphi_i \tag{5}$$

$$Q = \sum_{i=1}^{\infty} U_i I_i \sin\varphi_i \tag{6}$$

$$S = \sqrt{P^2 + Q^2 + D^2} \tag{7}$$

$$D = \sqrt{S^2 - P^2 - Q^2} \tag{8}$$

In the equations (5) and (6) U_0 is the DC voltage component of the periodic voltage, I_0 is the DC current component of the periodic current, where U_i and I_i are the harmonic components with i times the frequency according to the periodicity of the voltage and current respectively.

In electric power transmission commonly three phase AC systems are used. The effective power in such a system is the sum of the effective power in each phase. In an n phase system the power can be calculated according to equation (9).

$$P = \sum_{k=1}^{n} P_k \tag{9}$$

The electric energy is closely related to the electric power. The electric energy can be calculated according the equation (10).

$$W = \int_0^{t_m} u_{(t)} i_{(t)} \mathrm{d}t \tag{10}$$

(a) (b)

Figure 1: Measuring power with voltage and current meter

2 Measuring electric power

2.1 Measuring voltage and current

There are more possible ways to measure electric power in a circuit. In a DC circuit one of the easiest way to determine the electric power consumption on a resistor is measuring voltage and current of the resistor. In a basic circuit the voltage and current meters can be connected in two different arrangement. In the arrangement seen on figure 1a the voltage drop on the current meter causes a systematic error in the measurement. In the other arrangement on figure 1b the current of the voltage meter creates the systematic error. Measuring these two quantities the power can be calculated as the product of voltage and current $P = UI$.

The systematic error can be calculated from the resistance of the voltage and current meter. In figure 1a the voltage on the resistance of the current meter (R_{Ain}) is measured together with the voltage of the load. To calculate the power consumption of the load, the calculated voltage drop of the current meter should be subtracted from the measured voltage $P = (U - IR_{Ain})I$. If $R_{Ain} \ll R$ then the error voltage IR_{Ain} is small, and could be neglected.

To calculate the error for the arrangement on figure 1b the current of the voltage meter should be calculated. This can be done from the internal resistance of the voltage meter (R_{Vin}) and these should be subtracted from the measured current $P = U(I - U/R_{Vin})$. In this case the error could be neglected if $R_{Vin} \gg R$ because the error current is small.

2.2 Power meters basics

There are also devices for measuring the electric power directly. The most important function in the instruments measuring electric power (and also in measuring electric energy) is the calculation of the product of voltage and current. There are more possible solutions for this. One example is the electrodynamic power meter (figure 2). The multiplication is done inside the measuring device, the moment on the pointer can be calculated according the equation (11) where the product of the two currents influences the moment moving the pointer on the scale.

$$M = k_{(\alpha)} I_i I_u \cos\varphi \tag{11}$$

In the equation (11) the coefficient $k_{(\alpha)}$ is a non linear scale coefficient, I_i is the current on

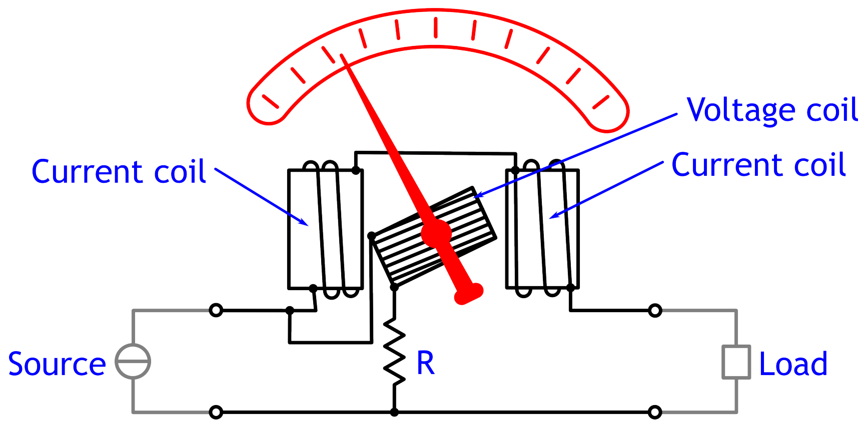


Figure 2: Schematic of the electrodynamic power meter

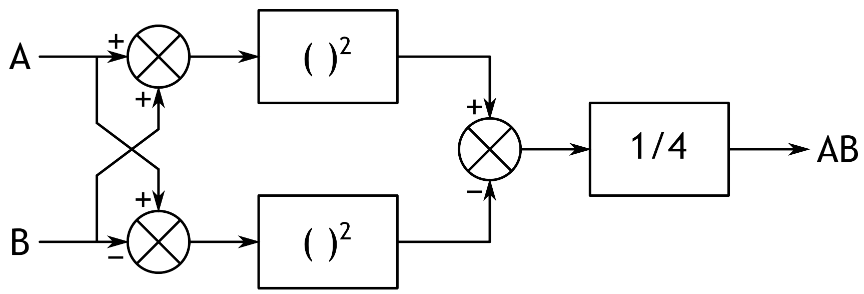


Figure 3: Block diagram of a quarter-square multiplier

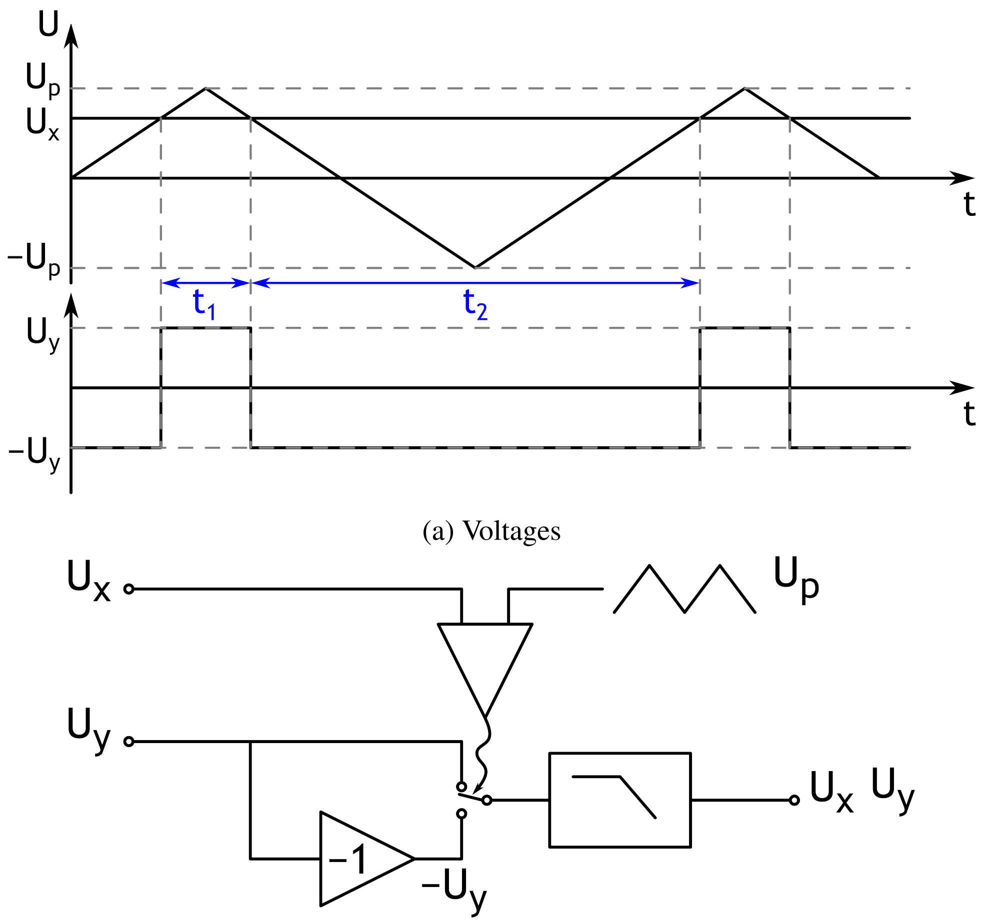
the load, I_u is a current proportional to the voltage on the load and $\cos\varphi$ is the phase angle between the voltage and current. This type of power meter is capable of measuring DC power and also AC effective power (as it can be seen from the equation (11)).

In modern electronic measuring devices the basic is also the product of the voltage and current, but this is done by electronic circuits. There are two main types of multipliers: the quarter-square multiplier and the time-division multiplier. The quarter-square multiplier uses the equation (12) as basic method to calculate the product of the two input signals. On figure 3 the block diagram of the quarter-square multiplier can be seen.

$$AB = \frac{1}{4}\left[(A+B)^2 - (A-B)^2\right] \tag{12}$$

The time-division multipliers input U_x is connected to a comparator. To the comparators other input a symmetrical triangle wave is connected. The comparator according to the voltages on the input controls an electronic switch. This switch connects U_y or $-U_y$ to the input of a low pass filter. The output value of the low pass filter is the average of the input signal (which is the output of the multiplier) so the voltage is to calculated according to equation (13). This uses the similarity of the two triangles on the figure 4a, according to equation (14). The block diagram of the time-division multiplier can be seen on figure 4b.

$$U = \frac{U_y(t_1 - t_2)}{t_1 + t_2} = -\frac{U_x U_y}{U_p} \tag{13}$$



(b) Block diagram

Figure 4: Time-division multiplier

$$\frac{U_p - U_x}{t_1} = \frac{U_p + U_x}{t_2} \tag{14}$$

2.3 Measuring DC and AC power with power meters

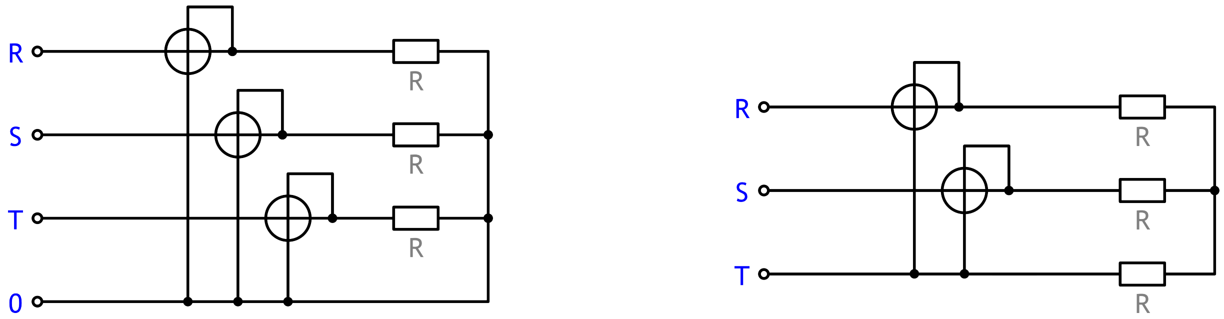
In a circuit to measure power the four pole of the instrument should be connected. In case of the electrodynamic power meter two of these are the voltage coils connections and the other two are the current coils connections. The case is similar using an electronic device, but in most cases the correct connections are made inside of the measuring device and the connections are denoted as input and output connectors.

If an electrodynamic power meter is connected to a circuit, the same two possibilities for the connection and for the systematic error applies as by the voltage and current power measurement (as described in section 2.1). In one case the voltage coil (see figure 5a) in the other case the current coil (see figure 5b) causes the systematic error, which can be calculated in view of the measuring devices parameters.

The effective power in three phase systems can also be measured by power meters. The connection of the power meters is depending on the number of wires in the system.

(a) (b)

Figure 5: Measuring power with power meter



(a) (b)

Figure 6: Measuring power in 3 phase circuit

If the system is a four wire system, that means 3 phase and a null wire is in the configuration, then – as shown in figure 6a – 3 power meters are used to measure the effective power used by the load. The total effective power of the load is the sum of the values measured by each power meter.

If the system is a three wire system, that means that only the three phase lines and no null line is used, it is enough to use two power meters to measure the effective power (Aronschaltung). The effective power in the circuit is the sum of the measured powers on each power meter. The configuration of this measurement can be seen on figure 6b.

2.4 Three voltmeters method for measuring AC power

In one phase AC circuits it is also possible to measure the power with the three voltmeters method. The circuit can be seen on the figure 7b. The power consumed by the impedance (Z) can be determined as the product of the voltage (U_Z) and current ($I = \frac{U_R}{R}$) of the impedance and the cosine of the phase angle between these two (power factor). The resistor R has a known resistance. The vector diagram on figure 7b shows the three measured voltages as vectors, and the phase angle (φ) of the voltage on the Z impedance. Using the law of cosines (in equation (15)) the effective power (P) of the impedance (Z) can be calculated according the equation (16) and $\cos\varphi$ can also be calculated according the equation (17).

$$U^2 = U_R^2 + U_Z^2 + 2U_R U_Z \cos\varphi_Z \tag{15}$$

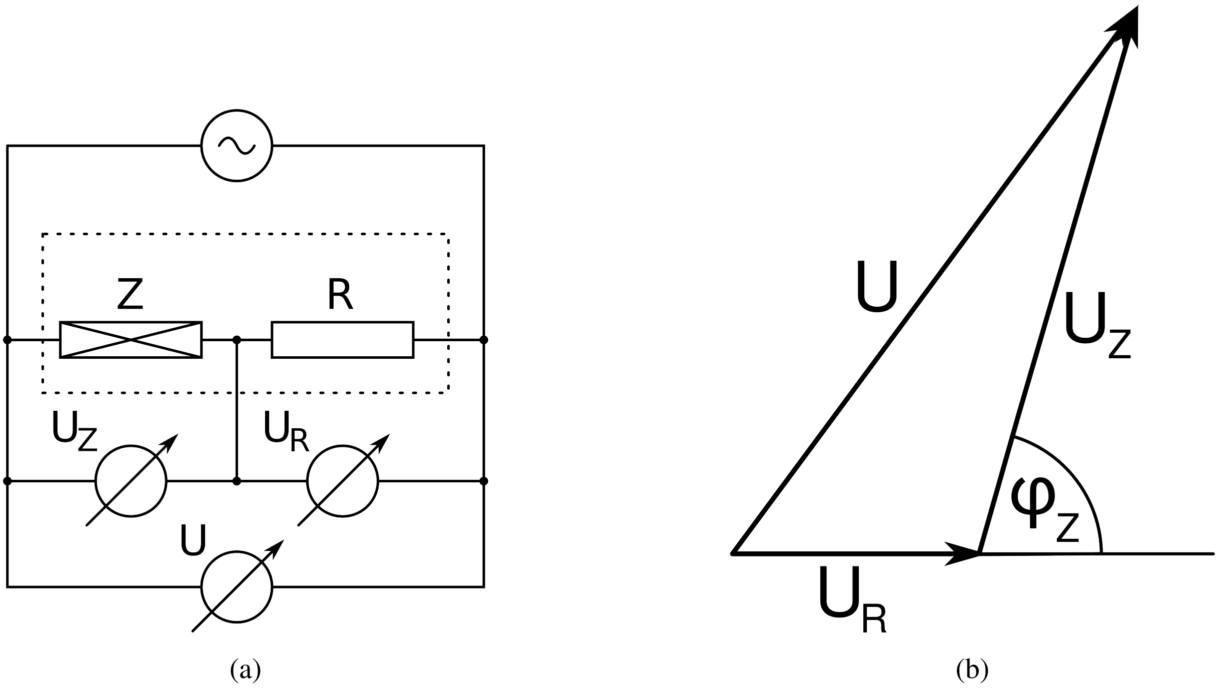


Figure 7: Measuring power wit the three voltmeters method

$$P_Z = U_Z \frac{U_R}{R} \cos\varphi_Z = \frac{U^2 - U_R^2 - U_Z^2}{2R} \tag{16}$$

$$\cos\varphi_Z = \frac{U^2 - U_R^2 - U_Z^2}{2U_R U_Z} \tag{17}$$

It is to note that the law of cosines is valid for the supplementary angle of angle φ_Z and so $U^2 = U_R^2 + U_Z^2 - 2U_R U_Z \cos(180 - \varphi_Z)$ is the form of the law. We are interested in the angle of the voltage of the measured impedance, which is measured to to direction of U_R, so the angle needed is φ_Z. The use of $\cos(180 - \varphi_Z) = -\cos\varphi_Z$ leads to the equation (15).

The measured loads capacitive or inductive property can not be decided using the the three voltmeters method for power measurement. The measured voltages have the same sign and $\cos\varphi_Z$ has positive values for both positive and negative angles.

2.4.1 Relative error of the three voltmeter measurement

To calculate the relative error of the three voltmeter measurement the relative error of the measured voltages (h) are needed. This can be calculated according equation (18) for each voltmeters measured values. (U_{max} is the maximal voltage in the measuring range, aor is the accuracy of range).

$$h = \frac{U_{max}}{U} \cdot aor; \qquad h_R = \frac{U_{max}}{U_R} \cdot aor; \qquad h_Z = \frac{U_{max}}{U_Z} \cdot aor \tag{18}$$

The relative sensitivity (c) should be also calculated according the equation (19) for all the voltmeters.

$$c = \frac{\partial P}{\partial U_g}\frac{U}{P} = \frac{2U^2}{U^2 - U_R^2 - U_Z^2}; \qquad c_Z = \frac{\partial P}{\partial U_Z}\frac{U_Z}{P} = \frac{-2U_Z^2}{U^2 - U_R^2 - U_Z^2};$$
$$c_R = \frac{\partial P}{\partial U_R}\frac{U_R}{P} = \frac{-2U_R^2}{U^2 - U_R^2 - U_Z^2}; \tag{19}$$

The relative error of the power measurement (or expanded uncertainty of the measurement) with three voltmeters can be calculated from the result of the equations (18) and (19) using the equation (20).

$$\frac{\Delta P}{P} = \frac{k}{\sqrt{3}} \cdot \sqrt{c^2h^2 + c_Z^2h_Z^2 + c_R^2h_R^2} \tag{20}$$

In the equation (20) the factor k is the so called coverage factor. If the measured values have normal distribution, and the coverage factor is chosen for 2 then the expanded uncertainty of the measurement defines a range with 95 % confidence level. That means that the measured values are in the given range with 95 % probability. Normally the coverage factor chosen is between 2 and 3 (choosing 3 as coverage factor means 99 % confidence level). These confidence levels are valid only if the measured values have normal distribution. With other distributions the coverage factors define different confidence levels. Thus in the equation (20) the factor $\frac{1}{\sqrt{3}}$ is necessary because the terms in the equation have uniform distribution.

2.5 Power analysers

Power analysers are complex devices to determine the power quality on the electric network. On the network also the voltage and the current should be sinusoidal. The distortion of the voltage and current can be described by the amount of harmonics. Most of these devices collect information about the harmonics of voltages and currents on the network, so analysing the spectrum of the power.

The inputs are the voltage and current. These signals are sampled and converted to digital values. These values are connected trough galvanic isolation to the input of a DSP. The DSP and a CPU processes the incoming signals, and creates the displayed results for the user.

Modern power analysers measure the spectrum up to 1 MHz. The amplitude of the measured harmonics power is displayed in a logarithmic scale, so the smaller harmonics can be also displayed on the screen, or the measured values are displayed in numerical form in a list. Also the measured voltage and current waveform can be displayed in such devices.