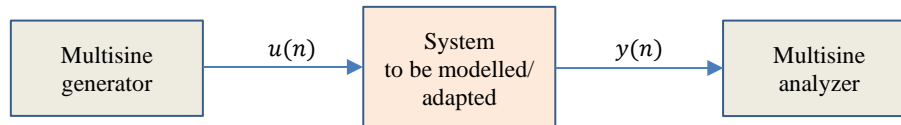# Measurement Theory Homework #2

This homework covers multisine measurement technology, model-fitting, adaptive systems, and Kalman filtering. The homeworks are parametrized separately for every individual in the table below. To solve the problems, the use of MATLAB is recommended, but the use of any similar program is allowed.

1.1. „Using" a multisine signal generator generate a discrete sequence $u(n), n = 0, 1, ...$, consisting of $M = 50$ subsequent harmonic components, where the magnitude of the components is one, the initial phase is random, and the mean of the sequence is one! The multisine signal generator should correspond to the structure of Figure 46 in the lecture notes. Please take care to provide excitation within the whole frequency range of the possible operation (max. 3 points)! Determine the maximum signal level, and compare it to that case, where all the initial phase values of the harmonics are zero (max. 2 points)!

1.2. Based on Figure 46 create a multisine analyser, which is capable to measure the magnitude and the phase[1] of the harmonics of its input! Using the analyser check whether the generated sequence meets the specification! Give a graphical illustration by plotting the difference of the input signal (see 1.1.) and the reconstructed signal (max. 5 points)!

1.3. This sequence $u(n)$ serves as input of the system to be modelled/adapted, which can be described by the following transfer functions (the assignment is in the table below):

A: $\frac{(1-r)z^{-1}}{1+rz^{-4}}$, B: $\frac{(1-r)z^{-1}}{1-rz^{-4}}$, C: $\frac{(1-r)z^{-1}(1+z^{-1})}{2(1-rz^{-4})}$, D: $\frac{(1-r)z^{-1}(1+z^{-2})}{2(1-rz^{-4})}$, E: $\frac{(1-r)z^{-1}(1-z^{-1})}{2(1-rz^{-4})}$, F: $\frac{(1-r)z^{-1}(1-z^{-2})}{2(1-rz^{-4})}$ (1)
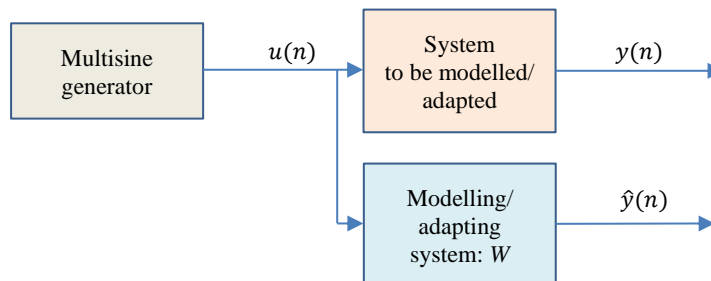
Based on the known magnitude and phase of the input signal „measure" the output signal of the system to be modelled/adapted and compute the magnitude and phase of the transfer function at the multisine frequencies (max. 4 points)!

```
Multisine     u(n)     System          y(n)     Multisine
generator          to be modelled/             analyzer
                      adapted
```

1.4. We try to model the system to be adapted by a linear combinator. This produces the following output:

$$\hat{y}(n) = w_1 u(n-1) + w_2 u(n-2) + ... + w_P u(n-P). \tag{2}$$

Let's operate the system to be modelled/adapted until it reaches its stationary state, i.e. the transients are over, and after it compute the weighting factors of (2) using the recursive LS method! How many steps were needed to reach the stationary state? After how many steps could you stop the LS method? (Let's denote column vector consisting of the weights of the linear combinator by W, and the regression vector by X!) (max. 6 points).

```
Multisine     u(n)     System          y(n)
generator          to be modelled/
                      adapted

                   Modelling/           ŷ(n)
                   adapting
                   system: W
```

1.5. Expand into geometric series the transfer function (1)! Compare the expanded form and the coefficients of the linear combinator with the weighting function of the system (1) (max. 2 points)!

2. Apply the LMS method for task 1.4:

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[1] Be careful with the phase values of MATLAB, since on one hand the phase is a multivalued function while on the other system responses have typically continuous phase characteristics.

$$W(n + 1) = W(n) + 2\mu e(n)X(n).$$

The parameters should be zero at the beginning. Let's run the algorithm! After finding the (approximate) solution, stop running, and reduce the value of *r* by *q*, and continue the algorithm until finding the new solution. The learning factor *µ* should be chosen by you. Draw the coefficient values as the function of the iteration steps (convergence diagram)! The commented program list and the convergence diagram are to be submitted[2] (max. 5 points)!

3. Perform the model fitting for both transfer functions of task 1 (*r* and *r-q* cases) by using the model of the form

$$\hat{y}(n) = a_1 u(n-1) + a_2 u(n-2) + a_3 u(n-3) - b_1 \hat{y}(n-1) - b_2 \hat{y}(n-2) - b_3 \hat{y}(n-3) - b_4 \hat{y}(n-4),$$

and applying the *equation-error formulation* method! The commented program list, and the convergence diagram are to be submitted (max. 5 points)!

4. Derive the state variable formulation of the system having transfer function (1)! Develop the program of the Kalman predictor to estimate the states of this system! The observation noise and the system noise should be generated in such a way that they are WGN processes having covariance matrices $R(n) = \sigma_n^2 I$ and $Q(n) = \sigma_w^2 I$! The standard deviation σ_n should be set to 2% of the peak value of the input signal! The standard deviation σ_w should be set to be 5% of the magnitude of each signal component! After reaching the stationary state of the system to be investigated, operate the predictor using the input signal of 1.1 both for *r* and *r-q*, and plot the state variables of the error system and the $trace P(n)$ as a function of the iteration steps until reaching the stationary state! Do not forget the proper setting $P(0)$ (max. 8 points)!

It is to be submitted: the solutions of the problems according to the given requirements concerning form and content, the commented lists of the programs, and the conclusions of the experiences.

Submission is requested to the portal: https://hf.mit.bme.hu as a single file in *pdf* format.

Date of publishing: **April 19, 2023.**
Deadline of submission**: May 23, 2023.**
The condition of acceptance: min. **16 points** (40%). **Good luck!**

Neptun code	system	r	q	P
CI6GN0	A	0.80	0.10	15
CJJFTJ	B	0.80	0.11	17
EF1IQD	C	0.81	0.12	13
F8S52E	D	0.81	0.13	11
FGB002	E	0.82	0.14	15
H3I99B	F	0.82	0.15	17
H8B414	A	0.83	0.16	19
HY0C3E	B	0.83	0.17	17
ITGBNO	C	0.84	0.18	19
J1QWH1	D	0.84	0.19	13
LUC34P	E	0.85	0.20	11
M2JXJY	F	0.85	0.21	17
NAD7Y8	A	0.86	0.22	15
PLEUO1	B	0.86	0.23	17
Q5ZGIR	C	0.87	0.24	19
RXKUBJ	D	0.87	0.25	11
SUT0TD	E	0.88	0.10	15
TRKRXD	F	0.88	0.11	13
U16MCS	A	0.89	0.12	17

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[2]The convergence drawings are enough for the five highest magnitude coefficients, preferably in a single diagram with appropriate identification.