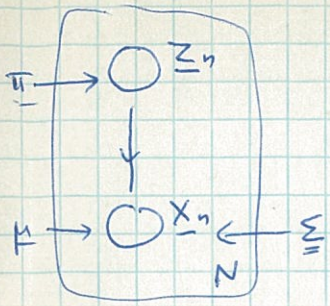

PROBLEM WITH SINGULARITIES IF:

$\underline{x}_n = \underline{\mu}_j$

$\sigma_k^2 \underline{\underline{I}} = \underline{\underline{\Sigma}}_k$

$$\mathcal{N}(\underline{x}_n | \underline{\mu}_k, \underline{\underline{\Sigma}}_k) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_k}$$

IF $\sigma_k \to \emptyset$ LIKELIHOOD $\to \infty$

MAXIMIZING LIKELIHOOD NOT WELL POSED
↳ DETECTION, HEURISTICS
(NOT A PROBLEM WITH SINGLE GAUSSIAN)
(OTHER MULTIPLICATIVE FACTORS)

MAXIMIZING LOG LIKELIHOOD → SUM IN LOGARITHM!
↳ <u>EM (EXPECTATION MAXIMIZATION) FOR GAUSSIAN MIXTURES</u>

$$\frac{\partial(*)}{\partial \underline{\mu}_k} = \emptyset \to \quad 0 = -\sum_{n=1}^{N} \underbrace{\frac{\pi_k \mathcal{N}(\underline{x}_n | \underline{\mu}_k, \underline{\underline{\Sigma}}_k)}{\sum_j \pi_j \mathcal{N}(\underline{x}_n | \underline{\mu}_j, \underline{\underline{\Sigma}}_j)}}_{\gamma(z_{nk})} \underline{\underline{\Sigma}}_k^{-1} (\underline{x}_n - \underline{\mu}_k) \quad \times \Sigma_k \to$$

$$\sum_n \gamma(z_{nk}) (\underline{x}_n - \underline{\mu}_k)$$

$$\mu_k = \frac{1}{N_k} \sum_{n=1}^{N} \gamma(z_{nk}) \underline{x}_n \quad (*) \qquad N_k = \sum_{n=1}^{N} \gamma(z_{nk})$$

EFFECTIVE # OF POINTS ASSIGNED TO k CLUSTER

$$\frac{\partial(*)}{\partial \underline{\underline{\Sigma}}_k} = \emptyset \to \quad \underline{\underline{\Sigma}}_k = \frac{1}{N_k} \sum_{n=1}^{N} \gamma(z_{nk}) (\underline{x}_n - \mu_k)(\underline{x}_n - \underline{\mu}_k)^T \quad (*)$$

MAXIMIZING $\ln p(\underline{X} | \underline{\pi}, \underline{\mu}, \underline{\Sigma})$ WITH RESPECT TO MIXING COEFFICIENTS π_k

$$\ln p(\underline{X} | \underline{\pi}, \underline{\mu}, \underline{\Sigma}) + \lambda \left(\sum_{k=1}^{K} \pi_k - 1 \right)$$

$$\frac{\partial}{\partial \pi_k} \quad 0 = \sum_{n=1}^{N} \frac{\mathcal{N}(\underline{x}_n | \underline{\mu}_k, \underline{\Sigma}_k)}{\sum_j \pi_j \mathcal{N}(\underline{x}_n | \underline{\mu}_j, \underline{\Sigma}_j)} + \lambda \qquad \times \pi_k, \sum_{k=1}^{K}$$

$$\frac{\partial}{\partial \lambda} \quad \lambda = -N \qquad \Big\} \quad \boxed{\pi_k = \frac{N_k}{N}} \quad (*)$$

$\xrightarrow{(*)(*)(*)}$ ITERATIVE SCHEME