MIXTURE MODELS & EM ALGORITHM

└ GAUSSIAN MIXTURE MODELS

K-MEANS CLUSTERING

$\{\underline{X}_1, \dots, \underline{X}_N\}$ OBSERVATIONS D-DIM, RANDOM VARIABLE $\underline{X}$

└ K CLUSTERS

CLUSTERS PROTOTYPES $\{\underline{\mu}_k\}_1^K$ D-DIM (CLUSTER CENTERS)

TASK: ASSIGN $\underline{X}_i$ TO CLUSTER, THAT

$$\sum \|\underline{X}_i - \underline{\mu}_j\|^2 = \text{MIN}$$

THE CLOSEST

$\underline{X}_k \rightarrow r_{nk} \in \{0,1\}$ $k = 1 \dots K$

$(0 \dots 0, 1\, 0 \dots 0)$ 1-OF-K CODING

DISTORTION MEASURE

$$J = \sum_{n=1}^{N} \sum_{k=1}^{K} r_{nk} \|\underline{X}_n - \underline{\mu}_k\|^2 = \text{MIN}$$

└ $r_{nk} = ?$

$\underline{\mu}_k = ?$

SEQUENTIAL ALGORITHM: – INIT $\underline{\mu}_k$

↻ – $\{\underline{\mu}_k\}$ FIXED, J MIN FOR r_{nk} (E)

– $\{r_{nk}\}$ FIXED, J MIN FOR $\underline{\mu}_k$ (M)

(E): J LINEAR IN $\{r_{nk}\}$

J MIN → ASSIGN $\underline{X}_n$ TO THE CLOSEST $\underline{\mu}_k$

$$r_{nk} = \begin{cases} 1 & \text{IF } k = \arg\min \leftarrow \|\underline{X}_n - \underline{\mu}_k\|^2 = \text{MIN} \\ 0 & \text{OTHERWISE} \end{cases}$$

(M) J QUADRATIC IN $\{\underline{\mu}_k\}$

$$\frac{\partial}{\partial \underline{\mu}_k} \rightarrow 2 \sum_{n=1}^{N} r_{nk} (\underline{X}_n - \underline{\mu}_k) = 0 \Rightarrow \underline{\mu}_k = \frac{\sum_n r_{nk} \underline{X}_n}{\sum_n r_{nk}}$$