MIXTURE OF GAUSSIANS

$$p(\underline{x}) = \sum_{k=1}^{K} \pi_k \, \mathcal{N}(\underline{x} | \underline{\mu}_k, \underline{\underline{\Sigma}}_k)$$

$\underline{z}$ K-dim 1-of-K coded vector

$\begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} - z_k$ $\quad z_k \in \{0, 1\}$ $\quad \sum z_k = 1$

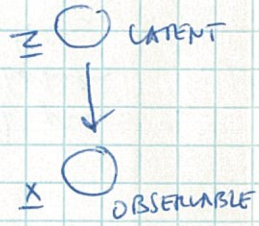
$p(z_k = 1) = \pi_k$ MIXING COEFFICIENTS

$0 \leq \pi_k \leq 1 \qquad \sum_{k=1}^{K} \pi_k = 1$

$$p(\underline{z}) = \prod_{k=1}^{K} \pi_k^{z_k}$$

$$p(\underline{x} | z_k = 1) = \mathcal{N}(\underline{x} | \underline{\mu}_k, \underline{\underline{\Sigma}}_k)$$

$$p(\underline{x} | \underline{z}) = \prod_{k=1}^{K} \mathcal{N}(\underline{x} | \underline{\mu}_k, \underline{\underline{\Sigma}}_k)^{z_k}$$



$$p(\underline{x}, \underline{z}) = p(\underline{x} | \underline{z}) \, p(\underline{z})$$

$$\downarrow$$

$$p(\underline{x}) = \sum_{\underline{z}} p(\underline{z}) \, p(\underline{x} | \underline{z}) = \sum_{k=1}^{K} \pi_k \, \mathcal{N}(\underline{x} | \underline{\mu}_k, \underline{\underline{\Sigma}}_k)$$

$$= \sum_{\underline{z}} p(\underline{x}, \underline{z})$$

$\leftarrow$ FOR EVERY DATA POINT $\underline{x}$ THERE IS A LATENT VARIABLE $\underline{z}$

CONDITIONAL PROB OF $\underline{z}$ GIVEN $\underline{x}$

$$\gamma(z_k) = p(z_k = 1 | \underline{x}) = \frac{p(z_k = 1) \, p(\underline{x} | z_k = 1)}{\sum_{j=1}^{K} p(z_j = 1) \, p(\underline{x} | z_j = 1)}$$

$\leftarrow$ RESPONSIBILITY COMPONENT k TAKES IN EXPLAINING $\underline{x}$

$\pi_k \rightarrow \gamma(z_k)$

PRIOR $\rightarrow$ POSTERIOR

$(z_k = 1)$

$$= \frac{\pi_k \, \mathcal{N}(\underline{x} | \underline{\mu}_k, \underline{\underline{\Sigma}}_k)}{\sum_{j=1}^{K} \pi_j \, \mathcal{N}(\underline{x} | \underline{\mu}_j, \underline{\underline{\Sigma}}_j)}$$

MAXIMUM LIKELIHOOD

$\{\underline{x}_1, \ldots, \underline{x}_N\}$ OBSERVATION $\rightarrow$ MODEL AS MIXTURE OF GAUSSIANS

$$\underline{\underline{X}} = \begin{array}{c|c} & 1 \quad \cdots \quad D \\ \hline 1 & \\ \vdots & \left[\; \underline{x}_n^T \;\right] \\ N & \end{array} \qquad \underline{\underline{Z}} = \begin{array}{c|c} & 1 \quad \cdots \quad K \\ \hline 1 & \\ \vdots & \left[\; \underline{z}_n^T \;\right] \\ N & \end{array}$$

OBSERVATIONS INDEPENDENT

$$\ln p(\underline{\underline{X}} | \underline{\pi}, \underline{\mu}, \underline{\underline{\Sigma}}) = \sum_{n=1}^{N} \ln \left\{ \sum_{k=1}^{K} \pi_k \, \mathcal{N}(\underline{x}_n | \underline{\mu}_k, \underline{\underline{\Sigma}}_k) \right\} \qquad (*)$$