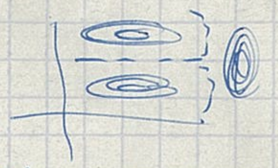
IF FOR EVERY CLASS $p(\underline{x}|C_k)$ HAS OWN COVARIANCE $\underline{\underline{\Sigma}}_k$

└ DECISION BOUNDARIES QUADRATIC (FIG. 4.11)



MAXIMUM LIKELIHOOD (CH. 4.2.2)

$K=2$, COMMON $\underline{\underline{\Sigma}}$, $t_n \begin{cases} 1 & C_1 \\ 0 & C_2 \end{cases}$ $\{x_n, t_n\}_{n=1}^{N}$

LET: $p(C_1) = \pi$, $p(C_2) = 1-\pi$

$$p(x_n, C_1) = p(C_1)\, p(x_n|C_1) = \pi\, N(x_n|\mu_1, \underline{\underline{\Sigma}}) \quad t_n = 1$$

$$p(x_n, C_2) = p(2)\, p(x_n|C_2) = (1-\pi)\, N(x_n|\mu_2, \underline{\underline{\Sigma}}) \quad t_n = \phi$$

$$p(\underline{t}|\pi, \mu_1, \mu_2, \underline{\underline{\Sigma}}) = \prod_{n=1}^{N} \left[\pi N(x_n|\mu_1, \underline{\underline{\Sigma}})\right]^{t_n} \left[(1-\pi) N(x_n|\mu_2, \Sigma)\right]^{1-t_n}$$

$\ell \searrow \pi$

$$\frac{\partial}{\partial \pi} \rightarrow \quad \sum \{t_n \ln \pi + (1-t_n)\ln(1-\pi)\} + \text{CONST}$$

$$\pi = \frac{1}{N}\sum t_n = \frac{N_1}{N} = \frac{N_1}{N_1+N_2}$$

$\ell \searrow \mu_1$

$$\frac{\partial}{\partial \mu_1} \quad \sum t_n \ln N(x_n|\mu_1, \Sigma) = -\frac{1}{2}\sum t_n (x_n-\mu_1)^T \underline{\underline{\Sigma}}^{-1}(x_n-\mu_1) + \text{CONST}$$

$$\mu_1 = \frac{1}{N_1}\sum t_n x_n$$

$$\frac{\partial}{\partial \mu_2} \quad \mu_2 = \frac{1}{N_2}\sum (1-t_n) x_n$$

$$\frac{\partial}{\partial \Sigma} \quad -\frac{1}{2}\sum t_n \ln|\Sigma| - \frac{1}{2}\sum t_n (x_n-\mu_1)^T \Sigma^{-1}(x_n-\mu_1) - \ldots$$

$$- \frac{1}{2}\sum (1-t_n)\ln|\Sigma| - \frac{1}{2}\sum (1-t_n)(x_n-\mu_2)^T \Sigma^{-1}(x_n-\mu_2)$$

$$= -\frac{N}{2}\ln|\Sigma| - \frac{N}{2}\,\text{tr}\{\underline{\underline{\Sigma}}^{-1}\underline{\underline{S}}\}$$

$$\left.\begin{aligned} \underline{\underline{S}} &= \frac{N_1}{N}\underline{\underline{S}}_1 + \frac{N_2}{N}\underline{\underline{S}}_2 \\ \underline{\underline{S}}_1 &= \frac{1}{N_1}\sum_{C_1}(x_n-\mu_1)(x_n-\mu_1)^T \\ \underline{\underline{S}}_2 &= \frac{1}{N_2}\sum_{C_1}(x_n-\mu_2)(x_n-\mu_2)^T \end{aligned}\right\} \quad \underline{\underline{\Sigma}} = \underline{\underline{S}} \quad \Rightarrow (**)$$

EXTENSION TO $K > 2$ CASE