PROBABILISTIC GENERATIVE MODELS (CH. 4.2)

$$p(C_k),\ p(x|C_k) \longrightarrow p(C_k|x)$$

$K=2$

$$p(C_1|x) = \frac{p(x|C_1)p(C_1)}{p(x|C_1)p(C_1)+p(x|C_2)p(C_2)} = \frac{1}{1+\exp(-a)} = \sigma(a) \quad (*)$$

$$a = \ln\frac{p(x|C_1)p(C_1)}{p(x|C_2)p(C_2)} \quad (*)$$

$\sigma(-a) = 1-\sigma(a)$ LOGISTIC SIGMOID (FIG. 4.9)

$a = \ln\left(\frac{\sigma}{1-\sigma}\right)$ LOGIT FUNCTION

$\ln\frac{p(C_1|x)}{p(C_2|x)}$

$K>2$

$$p(C_k|x) = \frac{p(x|C_k)\,p(C_k)}{\sum_j p(x|C_j)\,p(C_j)} = \frac{\exp(a_k)}{\sum_j \exp(a_j)}$$

NORMALIZED EXPONENTIAL, SOFTMAX FUNCTION

$$a_k = \ln p(x|C_k)\,p(C_k)$$

IF $a_k \gg a_j \quad \forall j \neq k \qquad p(C_k|x) \simeq 1 \qquad p(C_j|x) \simeq \phi$

CONTINUOUS INPUTS (CH. 4.2.1)

$$p(\underline{x}|C_k) = \frac{1}{(2\pi)^{D/2}}\frac{1}{|\underline{\underline{\Sigma}}|^{1/2}}\exp\left\{-\frac{1}{2}(\underline{x}-\underline{\mu}_k)^T\underline{\underline{\Sigma}}^{-1}(\underline{x}-\underline{\mu}_k)\right\} \quad (*)$$

$K=2$

$$p(C_1|\underline{x}) = \sigma(\underline{w}^T\underline{x}+w_0) \quad (*)$$

$$w_0 = -\frac{1}{2}\underline{\mu}_1^T\underline{\underline{\Sigma}}^{-1}\underline{\mu}_1 + \frac{1}{2}\underline{\mu}_2^T\underline{\underline{\Sigma}}^{-1}\underline{\mu}_2 + \ln\frac{p(C_1)}{p(C_2)}$$

$$\underline{w} = \underline{\underline{\Sigma}}^{-1}(\underline{\mu}_1-\underline{\mu}_2) \quad (**)$$

IF COMMON COVARIANCE & $\underline{x}$ LINEAR UNDER $\sigma(\cdot)$

$p(C_k|\underline{x}) = \text{const}$ → DECISION BOUNDARY LINEAR, ALSO IN INPUT SPACE

$K>2$

$$a_k(\underline{x}) = \underline{w}_k^T\underline{x} + w_{k0}$$

$$w_{k0} = -\frac{1}{2}\underline{\mu}_k^T\underline{\underline{\Sigma}}^{-1}\underline{\mu}_k + \ln p(C_k)$$

$$\underline{w}_k = \underline{\underline{\Sigma}}^{-1}\underline{\mu}_k$$

ALSO LINEAR BOUNDARIES