$$\boxed{\begin{aligned} \underline{m}_N &= \Sigma_N \left(\Sigma_{w_0}^{-1} \underline{m}_0 + \beta \Phi^T \underline{t} \right) \\ \Sigma_N^{-1} &= \Sigma_{w_0}^{-1} + \beta \Phi^T \Phi \end{aligned}}$$

ISOTROPIC NOISE (EVERY DATA EQUALLY NOISY)

$$p(\underline{w}) = \mathcal{N}(\underline{w} \mid \underline{m}_0, \alpha^{-1} I)$$
$$\downarrow$$
$$p(\underline{w}) = \mathcal{N}(\underline{w} \mid \underline{0}, \alpha^{-1} I)$$

(GAUSSIAN) POSTERIOR:

$$p(\underline{w} \mid \underline{t}, \beta, \alpha) = \mathcal{N}(\underline{m}_N, \Sigma_N)$$

$$\boxed{\underline{m}_N = \Sigma_N \left(\Sigma_{w_0}^{-1} \underbrace{\underline{m}_0}_{\underline{0}} + \beta \Phi^T \underline{t} \right)} = \boxed{\Sigma_N \left(\alpha \underline{m}_0 + \beta \Phi^T \underline{t} \right)} \rightarrow \boxed{\Sigma_N \beta \Phi^T \underline{t}}$$

$$\boxed{\Sigma_N^{-1} = \alpha I + \beta \Phi^T \Phi} \rightarrow \boxed{\Sigma_N^{-1} = \alpha I + \beta \Phi^T \Phi}$$

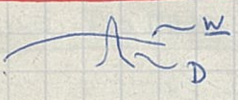
$$\ln p(\underline{w} \mid \underline{t}) = -\frac{\beta}{2} \sum_{n=1}^{N} \left(t_n - \underline{w}^T \phi(x_n) \right)^2 - \frac{\alpha}{2} \underline{w}^T \underline{w} + \text{const}$$

$\uparrow$ $\arg\min_{\underline{w}}$

like REGULARIZATION FOR $\boxed{\lambda \equiv \frac{\alpha}{\beta}}$

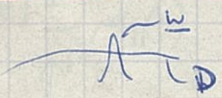
DATA MORE INFORMATIVE THAN PRIOR: $\beta \gg \alpha$ $(\alpha \to 0)$

$$\underline{w}^*_{MAP} = \underline{w}^*_{ML} = (\Phi^T \Phi)^{-1} \Phi^T \underline{t}$$



PRIOR MORE INFORMATIVE THAN DATA $\beta \ll \alpha$ $(\beta \to 0)$

$$\underline{w}^*_{MAP} \simeq \underline{m}_0$$



(example)