

M=0 MODEL

$$\underline{\phi}(x_n) = [1] \quad \underline{w} = [w_0] \quad \underline{\phi} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_N$$

$$\underline{\phi}^T \underline{\phi} = N$$

$$\underline{w}_{LS}^{(0)} = \frac{1}{N} \sum_{n=1}^N t_n$$

M=1 MODEL

$$\underline{\phi}(x_n) = [1 \ x_n] \quad \underline{\phi} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}_N$$

$$\underline{\phi}^T \underline{\phi} = \begin{bmatrix} N & \sum x_n \\ \sum x_n & \sum x_n^2 \end{bmatrix}$$

$$\text{LET } \mu = \frac{1}{N} \sum x_n, \quad m = \frac{1}{N} \sum x_n^2, \quad r = \frac{1}{N} \sum t_n x_n$$

$$\bar{t} = \frac{1}{N} \sum t_n$$

$$\underline{\phi}^T \underline{t} = \begin{bmatrix} \sum t_n \\ \sum t_n x_n \end{bmatrix}$$

$$\underline{w}_{LS}^{(1)} = \frac{1}{m - \mu^2} \begin{bmatrix} m \bar{t} - \mu r \\ -\mu \bar{t} + r \end{bmatrix}$$

IF DATA SOURCE: $t_n = a + e_n$ ($= a \cdot x_n^0 + e_n = w_0 \cdot x_n^0 + e_n$)

$$\underline{w}_{LS}^{(0)} = \frac{1}{N} \sum t_n = a + \frac{1}{N} \sum e_n \approx a$$

$$\bar{t} \rightarrow a$$

$$r \rightarrow a\mu$$

$$\underline{w}_{LS}^{(1)} \approx \frac{1}{m - \mu^2} \begin{bmatrix} a(m - \mu^2) \\ -a\mu + a\mu \end{bmatrix} = \begin{bmatrix} a \\ 0 \end{bmatrix}$$

IF DATA SOURCE: $t_n = a \cdot x_n + e_n$ ($= \cancel{w_0} \cdot x_n^0 + a \cdot x_n^1 + \dots$)

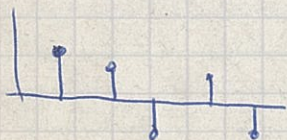
$$w_0 = 0, \quad w_1 = a$$

$$\left. \begin{array}{l} \bar{t} \rightarrow a\mu \\ r \rightarrow a m \end{array} \right\}$$

$$\underline{w}_{LS}^{(0)} \approx a \cdot \frac{1}{N} \sum x_n \quad (\text{MODEL ERROR})$$

$$\underline{w}_{LS}^{(1)} = \frac{1}{m - \mu^2} \begin{bmatrix} m a \mu - \mu a m \\ -\mu^2 a + a m \end{bmatrix} = \begin{bmatrix} 0 \\ a \end{bmatrix}$$

(BISITOR)



M=0

M=1

M=2

M=3

⋮

M=9



MODEL ERROR

$$E(\underline{w}^*) = \min E(\underline{w})$$

