Artificial Intelligence
Uninformed search

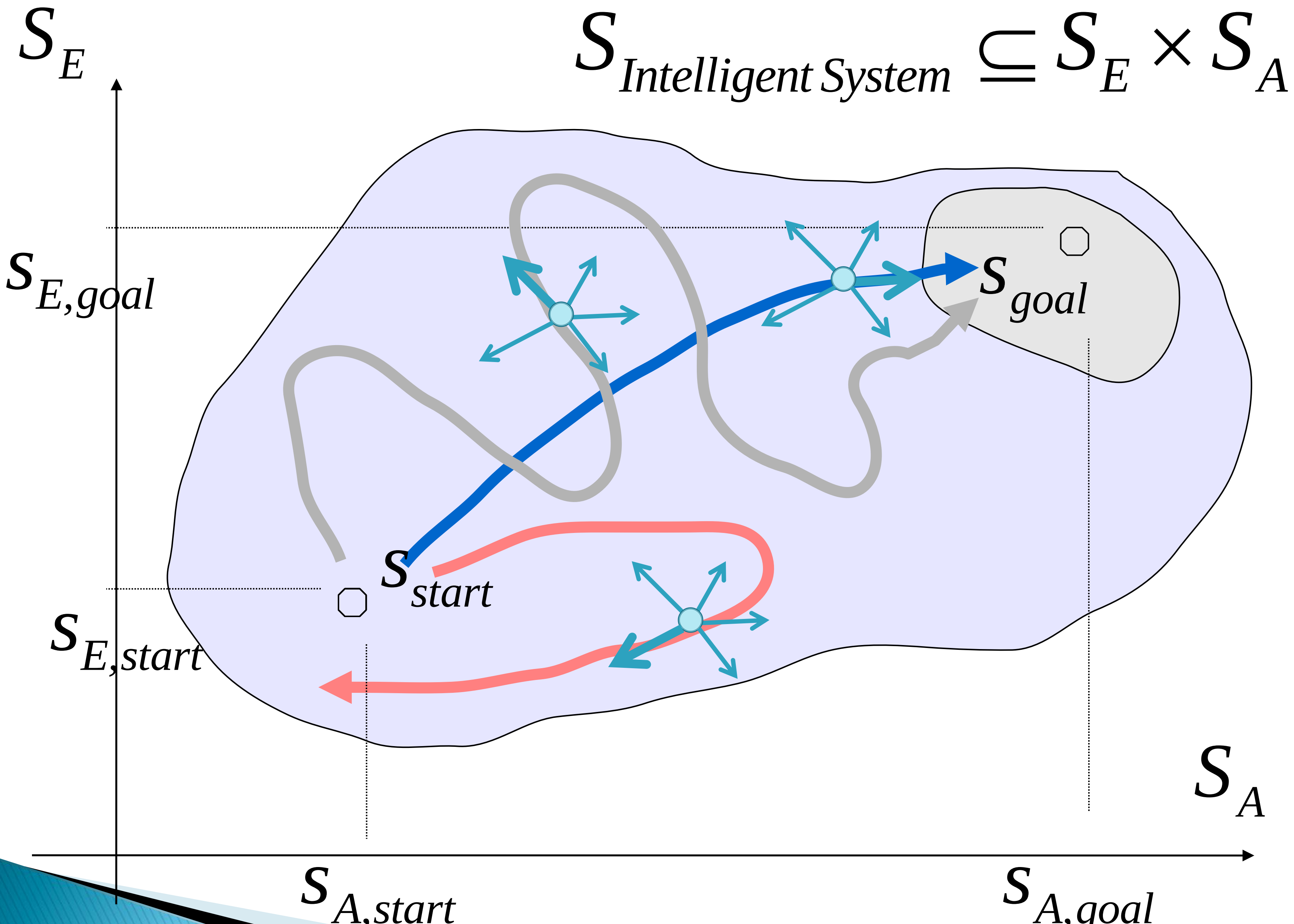
More about

Textbook, Chapter 3, Soving Problems by Searching

Outline

- Problem-solving (goal-oriented) agents
- Solving
  - Single state (fully observable)
  - Multiple state (search with partial information)

  Problem Types
- How to define a problem?
  - Example problems
- What algorithm can solve it actually?
  - Uninformed search algorithms


$S_{Intelligent\ System} \subseteq S_E \times S_A$
S_E
$S_{E,goal}$
$S_{E,start}$
S_{start}
S_{goal}
S_A
$S_{A,start}$
$S_{A,goal}$

AI as "symbol manipulation"
– expressing the goal and its direction

- The Logic Theorist, 1955 ➔ see lectures on logic
- The Dartmouth conference ("birth of AI", 1956)
- List processing (Information Processing Language, IPL)
- Means-ends analysis ("reasoning as search") ➔ see lecture on planning
- The **General Problem Solver**
- Heuristics to limit the search space ➔ see lecture on informed search
- The **Physical Symbol System Hypothesis**
  - intelligent behavior can be reduced to/emulated by symbol manipulation

    (A. Newel, H. Simon: Computer science as empirical inquiry: symbols and search, 1975)
- The unified theory of cognition (1990, cognitive architectures)

➔ expressing (human) problem solving symbolically

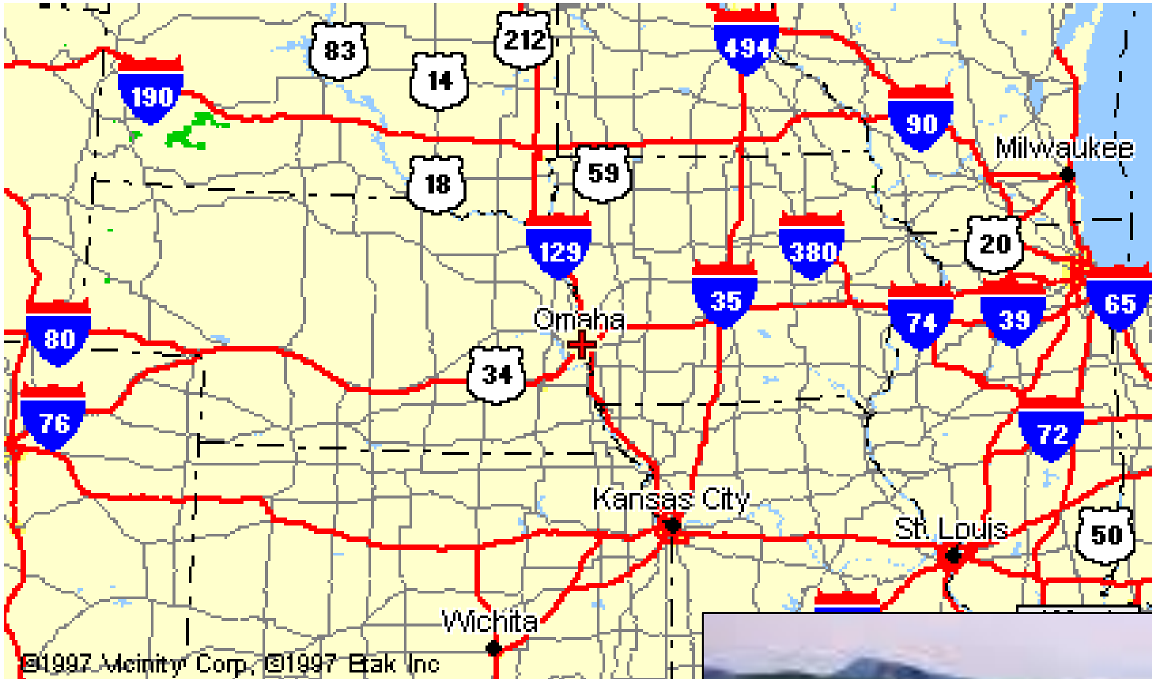
AI as “symbol manipulation” – required? enough?

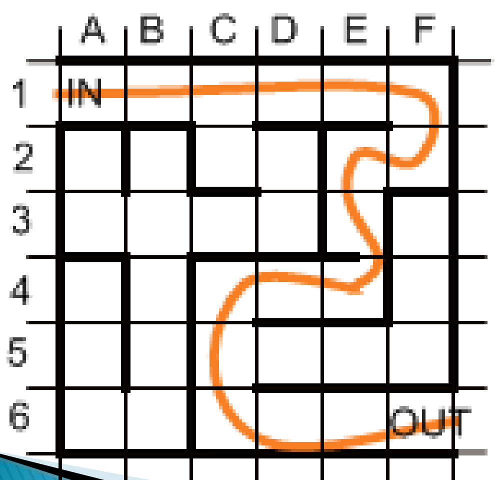
- The Box and Banana problem
  - Human, monkey, pigeon, crow, ..

Problem-solving agent

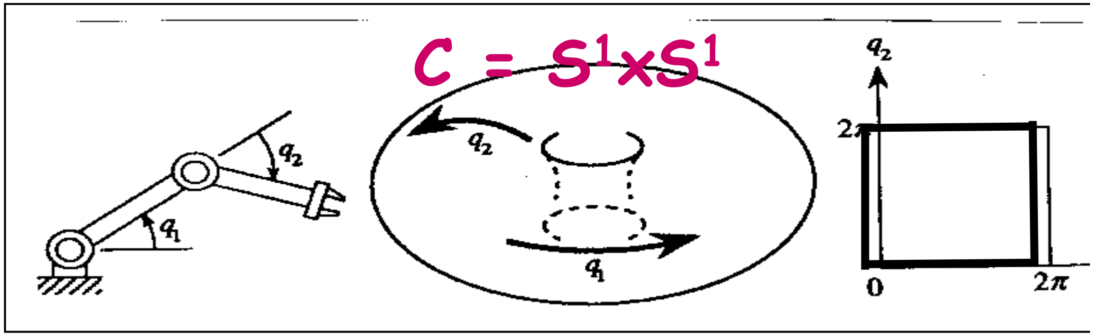
- Four general steps in problem solving:
  - (1) Goal formulation
    - What are the demanded, successful world states (**state-space of the problem**)
  - (2) Problem formulation
    - What actions and states are possible/legal to consider, given the goal
  - (3) Problem solving with search
    - Determine the possible sequence of actions that lead to the states of known values and then choose the best sequence.
  - (4) Executing the solution
    - Given the solution, perform its prescribed actions.

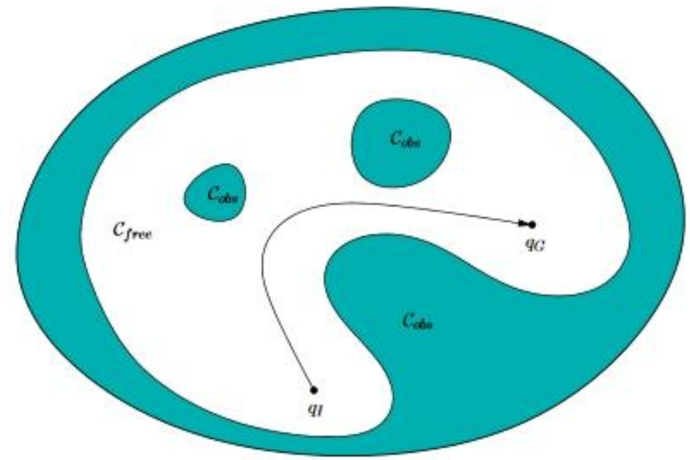
Physical State-Spaces

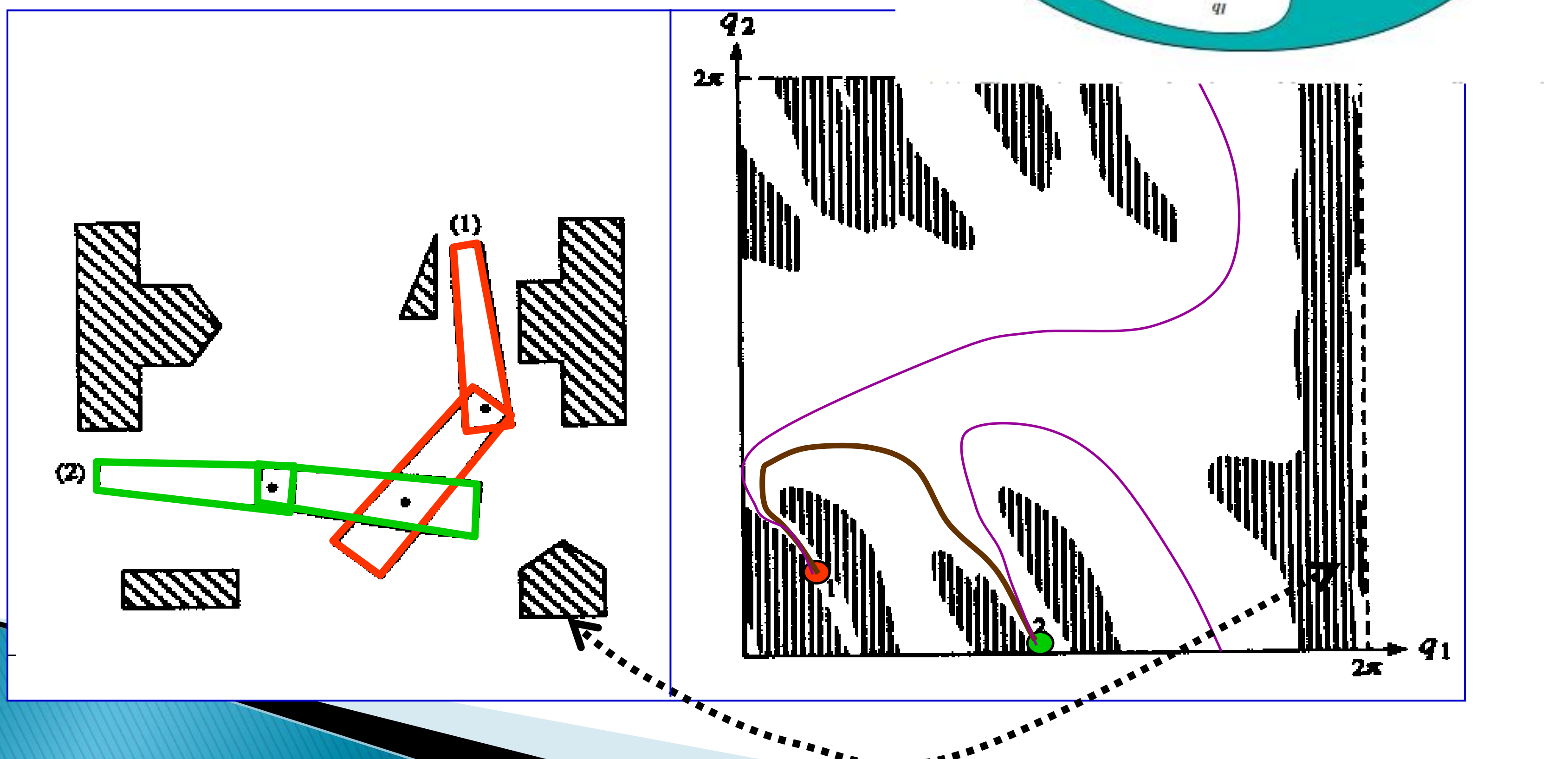




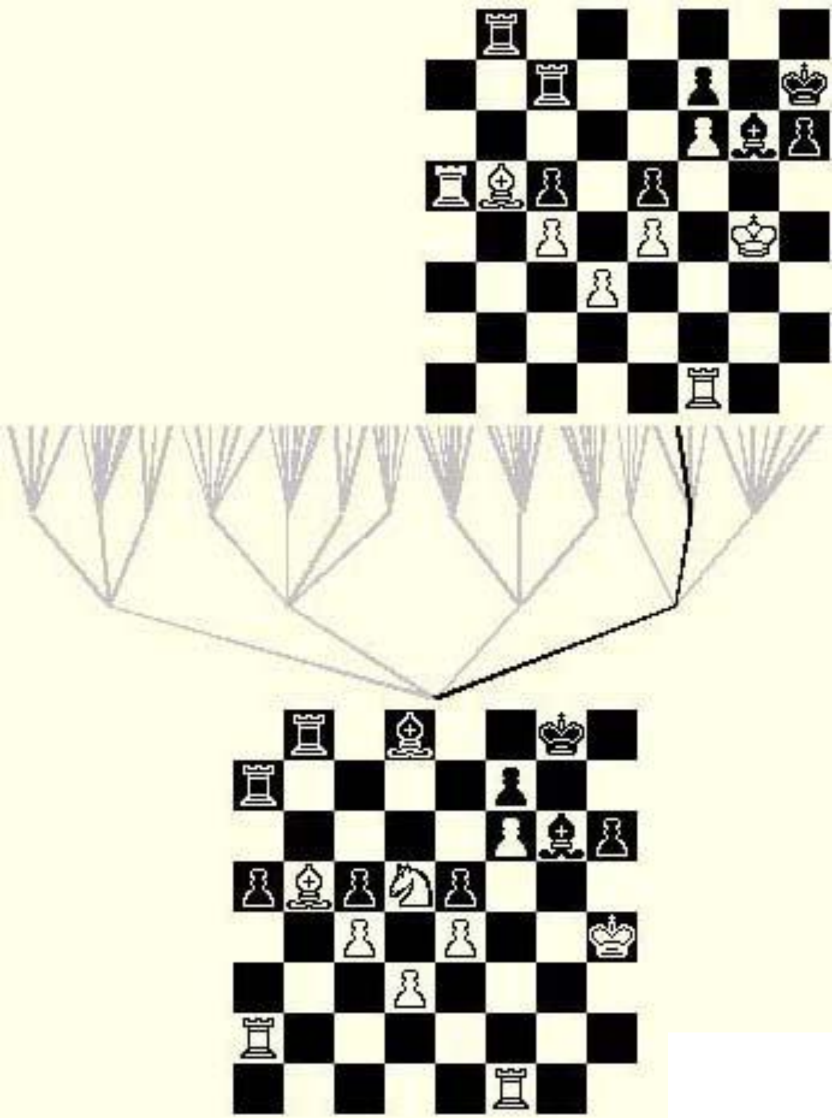
Reduce robot to a point → Configuration State-Spaces

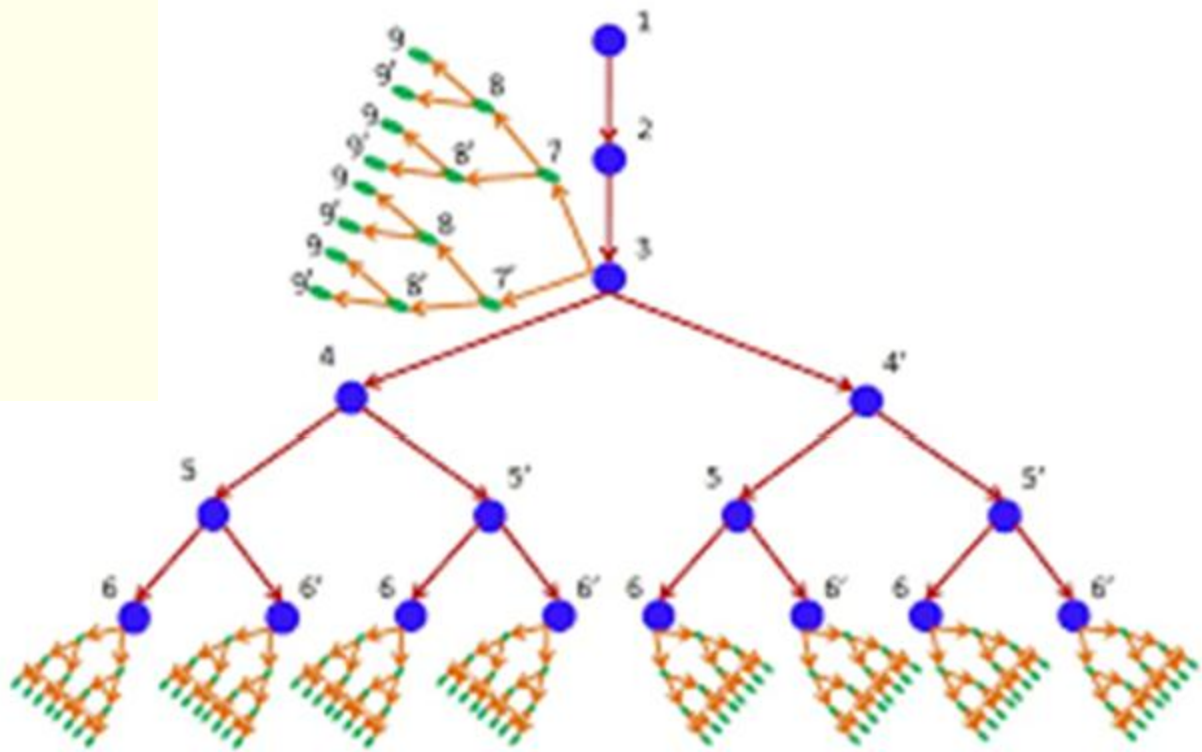






Abstract State-Spaces

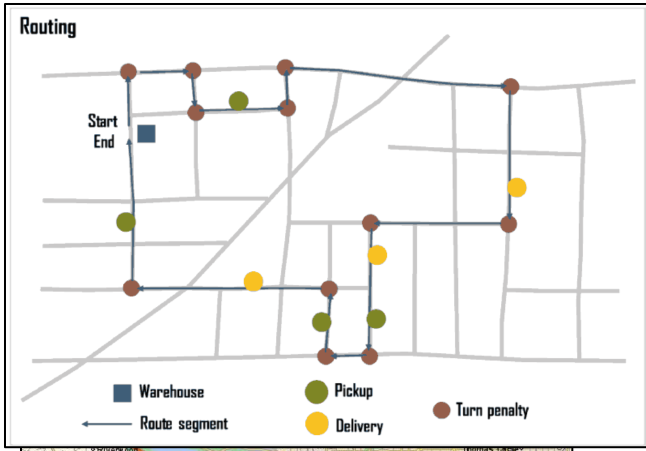




Example: Romania

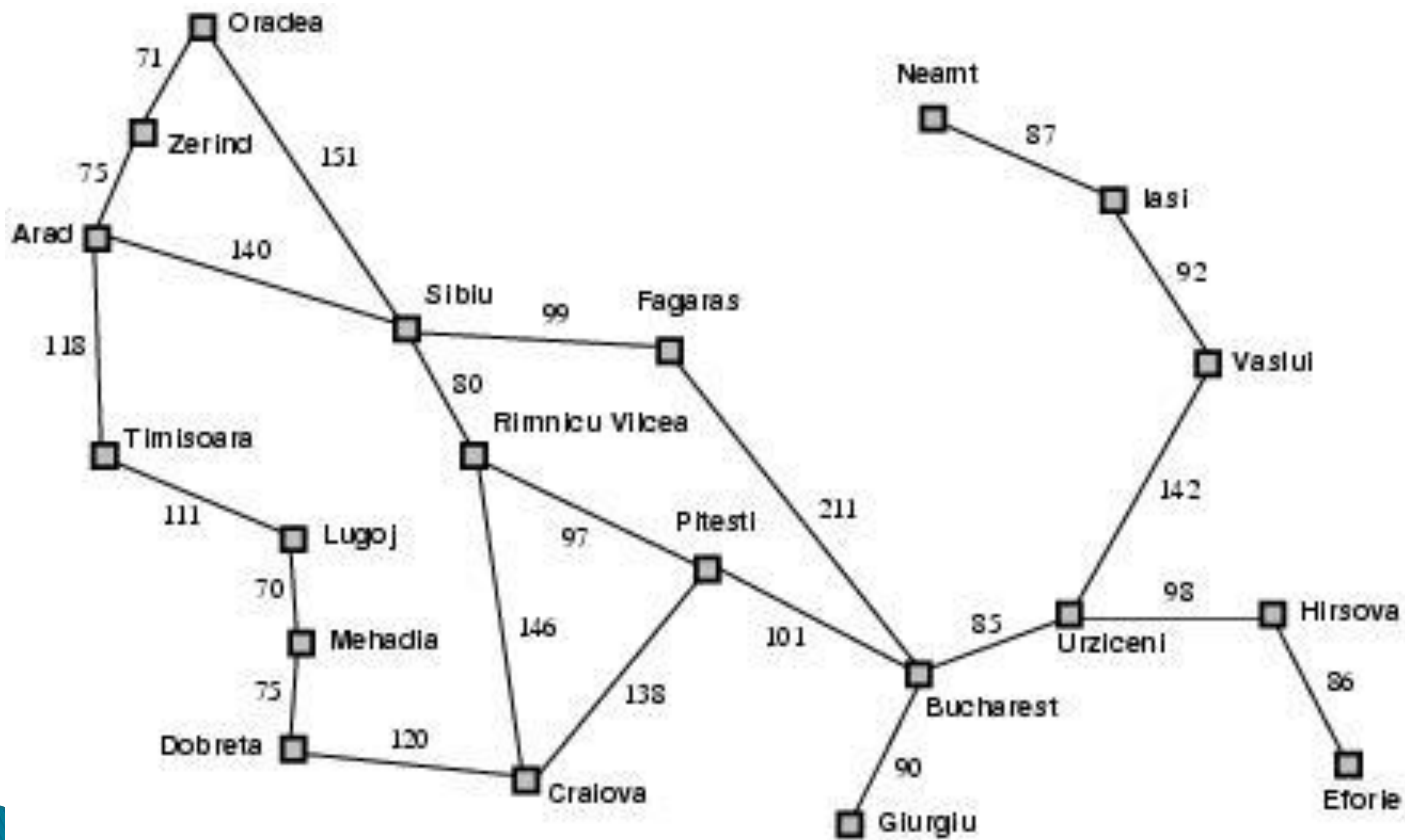
- On holidays in Romania; currently in town of Arad
  - Flight leaves home tomorrow from Bucharest
- Formulating the goal
  - Be in (time at the airport in) Bucharest
- Formulating the problem
  - States: various cities (closer or further from the goal!)
  - Actions: driving from a city to a city
- Finding solution
  - Sequence of cities; e.g. Arad, Sibiu, Fagaras, …, Bucharest (ending in the goal!)
- Executing the solution

Selecting a state space



- Real world is complex. State space must be *abstracted* for problem solving.
- (Abstract) state = set of real states.
- (Abstract) action = complex combination of real actions.
  - e.g. Arad → Zerind action is a complex set of possible routes, detours, rest stops, etc.
  - The abstraction is valid if the path between two states is passable in the real world.
- (Abstract) solution = set of real paths that are real solutions in the real world.
- Abstract problem should be “easier” than the real problem.

Example: (an abstract) Romania



Problem formulation

- A problem is defined by:
  - (1) An initial state, e.g. '*at Arad*'
  - (2) Successor function (an operator to move in state-space)

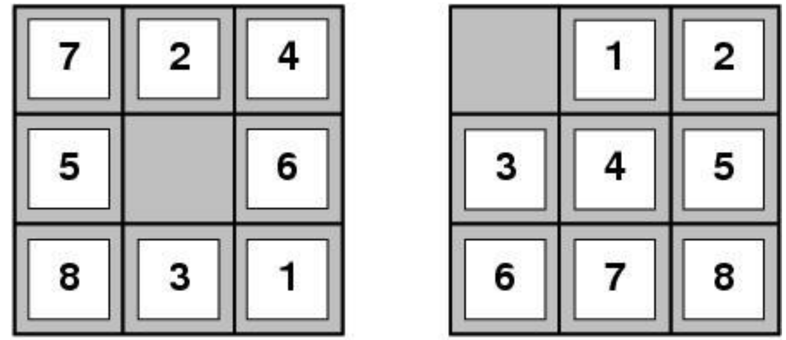
    S(X)= set of action-state pairs

    - e.g. $S(Arad)=\{<Arad \rightarrow Zerind, Zerind>,\ldots\}$

    intial state + successor function = state space as a graph
  - (3) Goal test, here can be:
    - *x='at Bucharest'*
  - (4) Path cost (additive)
    - e.g. sum of distances, number of actions executed, …
    - *c(x,a,y)* is the step cost (petrol?!), assumed to be >= 0

A solution is a sequence of actions from initial to goal state.
Optimal solution has the lowest path cost.

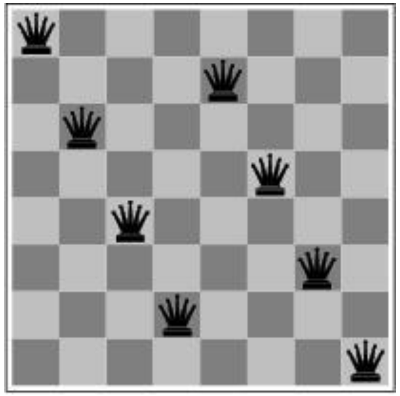
Examples: 8-puzzle



Start State Goal State

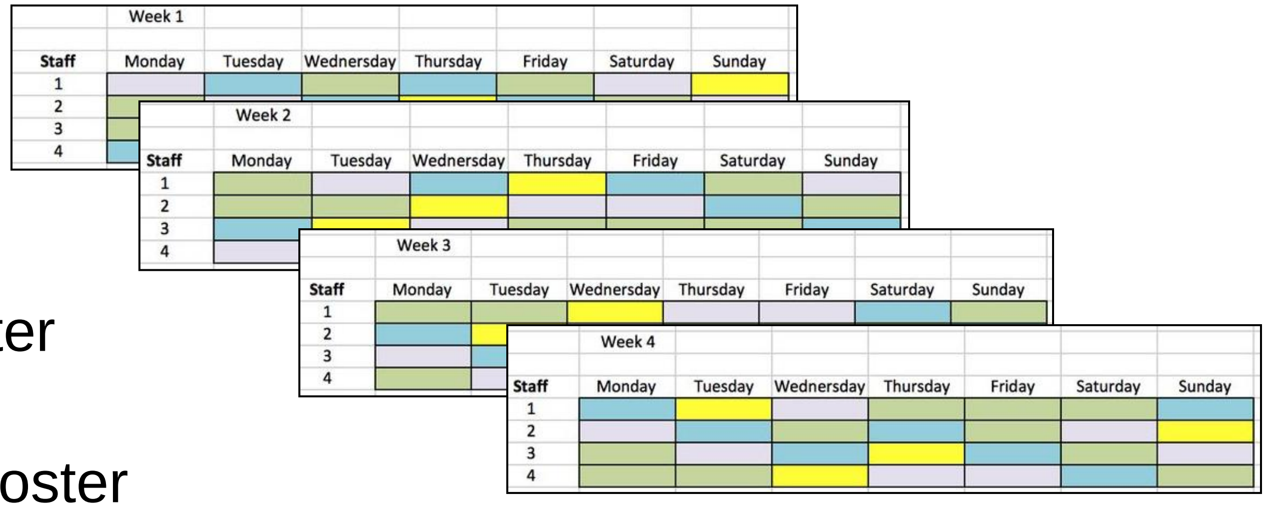
- States
  - The location of the eight tiles, and the blank.
- Initial state
  - {(7.0),(2,1),(4,2), …, (8,6),(3,7),(1,8)}
- Actions (operators)
  - 4 actions (blank moves Left, Right, Up, Down)
- Goal test
  - Is a given state a goal state = {(_,0),(1,1),…,(7,7),(8,8)}?
- Path cost
  - Each step costs 1

Examples: 8-queens problem



- States
  - Complete-state: Any arrangement of all 8 queens
  - Incr/1-state: Any arrangement of 0 to 8 queens
  - Incr/2-state: *n* ($0 \leq n \leq 8$) queens on the board, one per column in the *n* leftmost columns with no queen attacking another.
- Initial state
  - Complete-state: Any arrangement of all 8 queens on the board
  - Incr/1, Incr/2: no queens on board
- Actions
  - Complete-state: move a queen to an empty square
  - Incr/1: Add a queen to an empty square
  - Incr/2: Add a queen in the leftmost empty column, not attacking others
  - From 3×10^{14} to 2057 possible sequences to investigate, depending on problem formulation.
- Goal test
  - All 8 queens on board and none attacked
- Path cost
  - None

Examples: Police precinct shift roster



- States
  - Weekly roster
- Initial state
  - Last week roster
- Action (operator)
  - Moving shifts, moving weekdays, scheduled holidays, participating in special events
- Goal test
  - Captain input
- Path cost
  - Complicated: paid overtime, burden of late shifts, personal incompatibility, paid skipped holidays, …

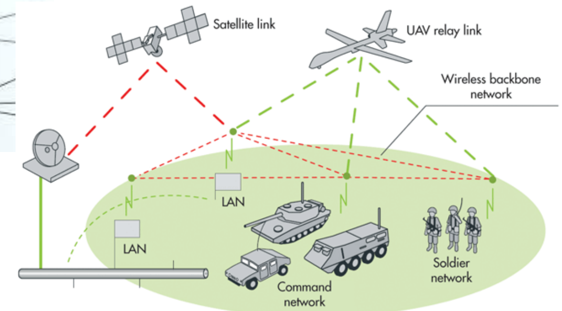
Examples: Route Finding Problems

Car Navigation

Air Travel Planning

Routing in Computer Networks

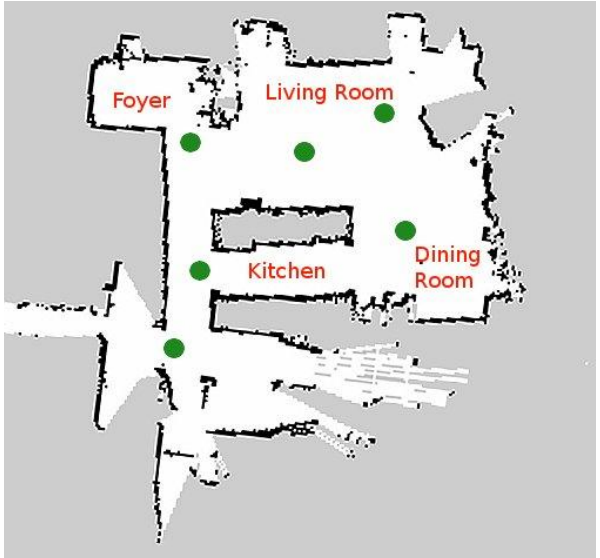
Military Operation Planning



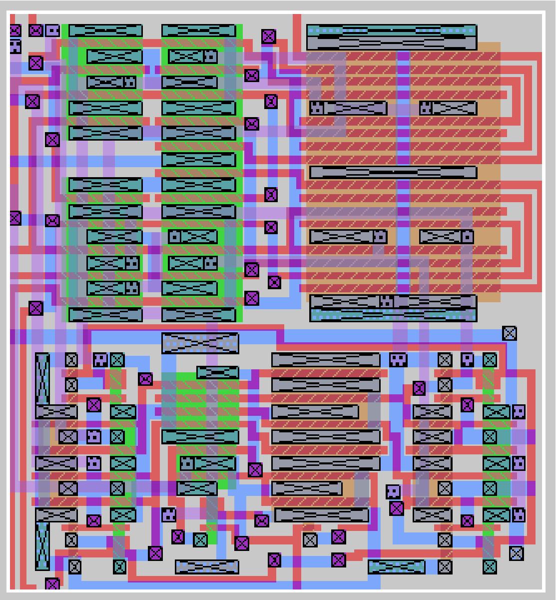
- States
  - Locations
- Initial state
  - Starting point
- Action (operator)
  - Move from one location to another
- Goal test
  - Arriving at a certain location
- Path cost
  - May be quite complex, money, time, travel comfort, scenery, …

Examples: Robot Navigation

- States
  - Locations, Position of actuators
- Initial state
  - Starting poisition (task?)
- Action (operator)
  - Movement, actions of actuators
- Goal test
  - Task-dependent
- Path cost
  - May be quite complex distance, energy consumption, …



Examples: VLSI layout problem



- States
  - Positions of components, wires on chip
- Initial state
  - Incremental: no components placed
  - Complete-state: all components placed (e.g. randomly, manually)
- Action (operator)
  - Incremental: place components, route wire
  - Complete-state: move component, move wire
- Goal test
  - All coponents placed. Components connected as specified
- Path cost
  - May be quite complex, distance, capacity, number of connections per components, …

Basic search algorithms

How do we find the solutions of previous problems?

- Traversal of the search space (a tree or a graph)
- From the initial state to a goal state
- Legal sequence of actions as defined by successor function

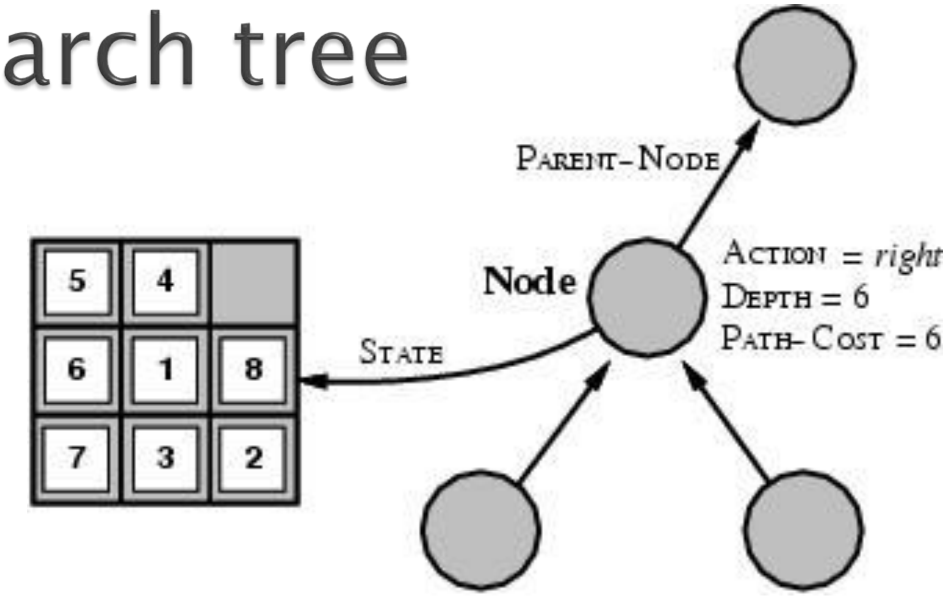
General procedure

- Check for goal state
- Expand the current state
- Determine the set of reachable states
- Return „failure” if the set is empty
- Select one from the set of reachable states
- Move to the selected state

A search tree is generated

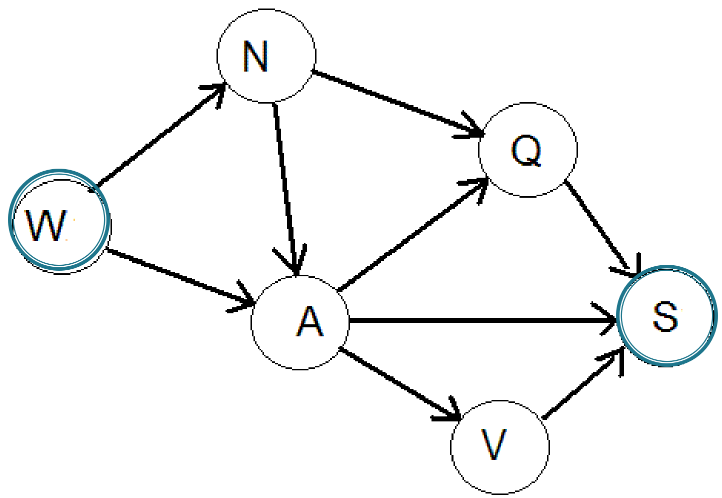
- Nodes are added as more states are visited
- The tree specifies possible paths through the search space

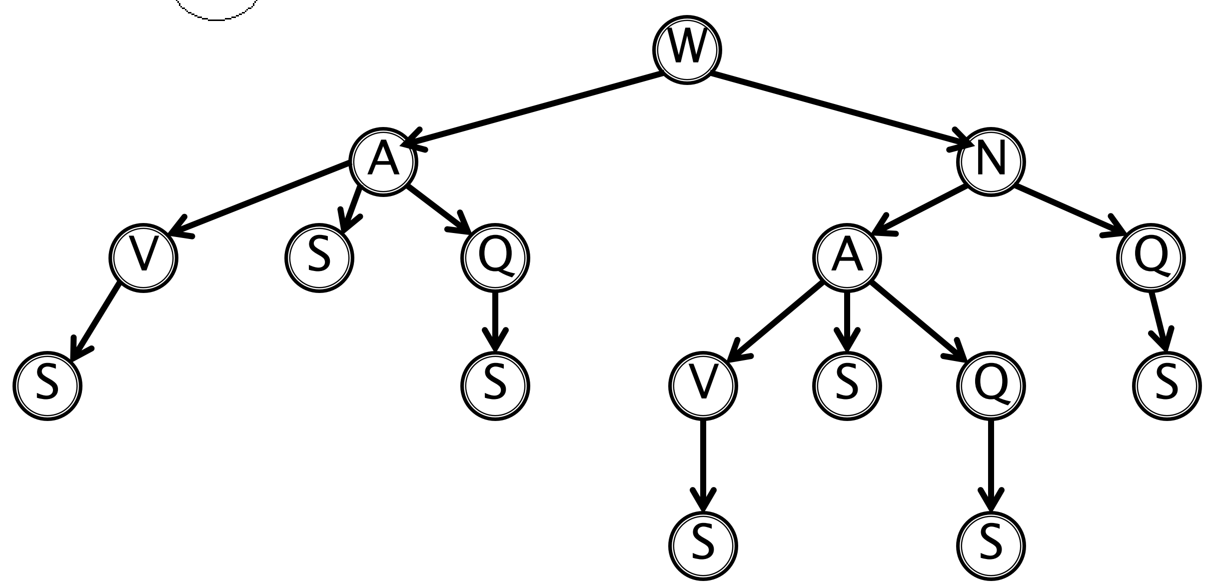
State space vs. search tree



- A ***state*** is a (representation of) a physical configuration
- A ***node*** is a data structure belonging to a search tree
  - A node has a ***parent***, ***children***, … and includes ***path cost***, ***depth***, …
  - Here *node= <state, parent-node, action, path-cost, depth>*
  - ***FRINGE***= contains generated nodes which are not yet expanded.

State space vs. search tree





Tree search algorithm (1)

function TREE-SEARCH(*problem,fringe*)
return a solution or failure

  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[*problem*]), *fringe*)
  loop do
    if EMPTY?(*fringe*) **then return** failure
    node ← REMOVE-FIRST(*fringe*)
    if GOAL-TEST[*problem*] applied to STATE[*node*] succeeds
      then return SOLUTION(*node*)
    fringe ← INSERT-ALL(EXPAND(*node*, *problem*), *fringe*)

Tree search algorithm (2)

function EXPAND(*node,problem*) **return** a set of nodes

successors ← the empty set

for each <*action*, *result***> in** SUCCESSOR**[***problem***]**(STATE[*node*])

do

s ← a new NODE

STATE[*s*] ← *result*

PARENT-NODE[*s*] ← *node*

ACTION[*s*] ← *action*

PATH-COST[*s*] ← PATH-COST[*node*] + STEP-COST(*node, action,s*)

DEPTH[*s*] ← DEPTH[*node*]+1

add *s* to *successors*

return *successors*

Graph-search – handling repeated states and loops, later …

Search strategies

- A strategy is defined by picking the order of node expansion.
- Problem-solving performance is measured in four ways:
  - Completeness: *does it always find a solution if one exists?*
  - Optimality: *does it always find the least-cost solution?*
  - Space Complexity: *number of nodes stored in memory during search?*
  - Time Complexity: *number of nodes generated/expanded?*
- Time and space complexity measure problem difficulty and are defined by:
  - *b - maximum branching factor of the search tree*
  - *d - depth of the least-cost solution*
  - *m - maximum depth of the state space (may be ∞)*

Uninformed search strategies

- (a.k.a. blind search) = use only information available in problem definition.
  - When strategies can determine whether one non-goal state is better than another $\rightarrow$ informed search.

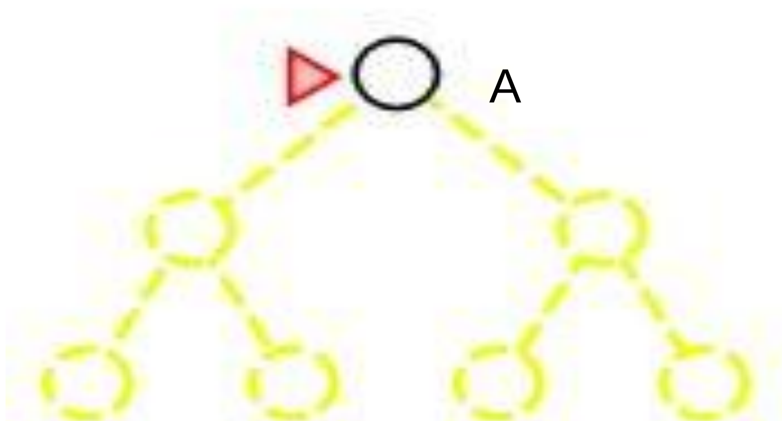
    We do not have this information here!

- Search methods defined by node expansion algorithm:
  - Breadth-first (BF) search
  - Uniform-cost search
  - Depth-first (DF) search
  - Depth-limited search
  - Iterative deepening (ID) search
  - Bidirectional search

BF-search, an example

- Expand *shallowest* unexpanded node
- Implementation: *fringe* is a FIFO queue

to be expanded yet
on fringe
expanded
deleted from memory



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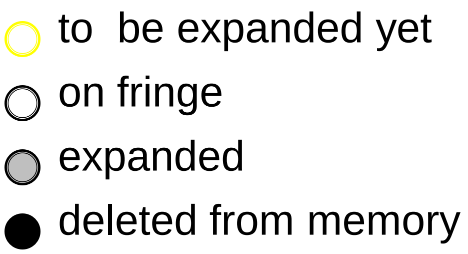

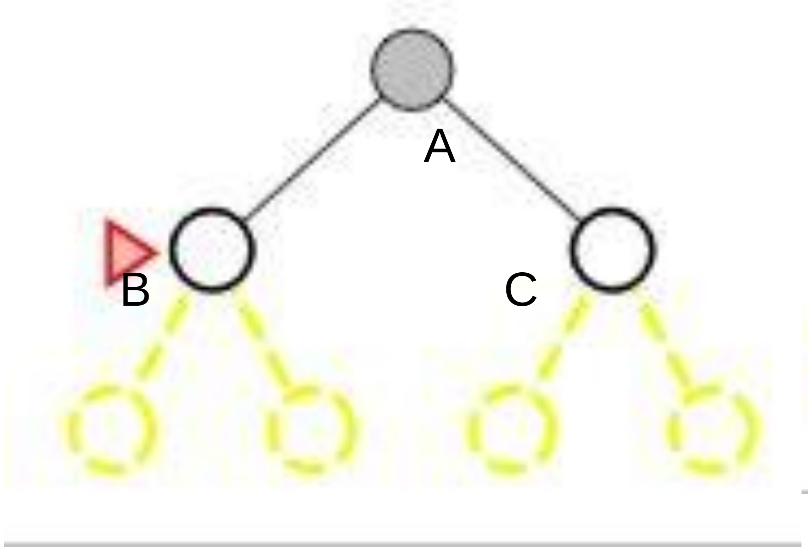

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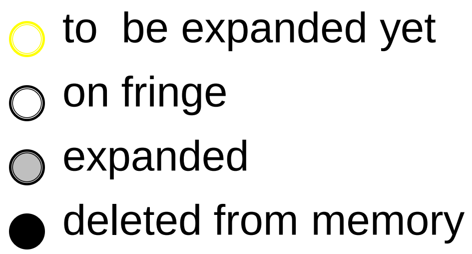

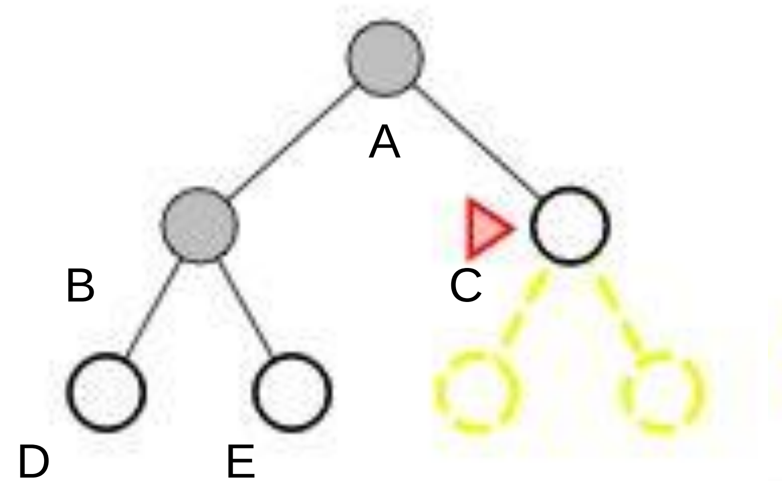

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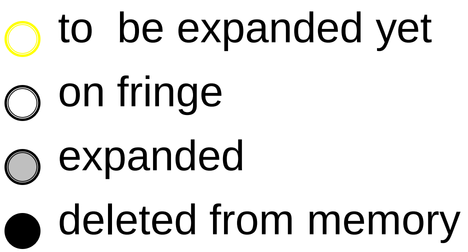

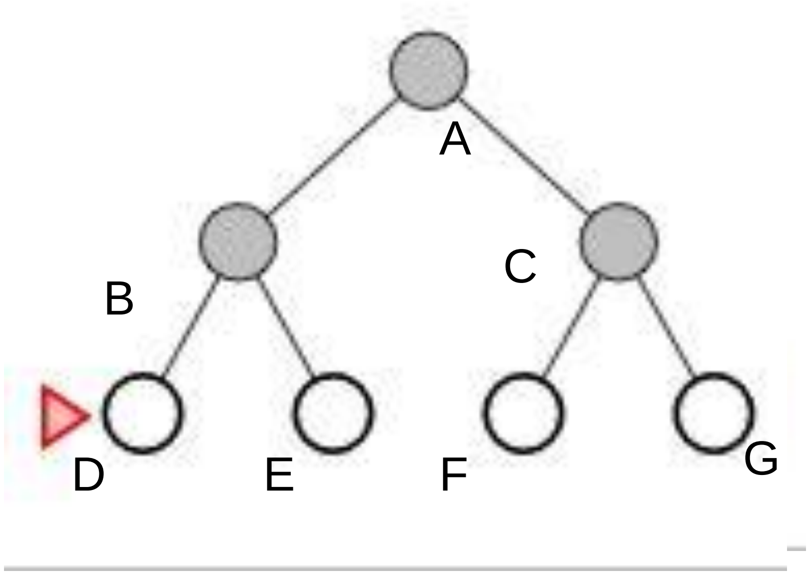

BF-search; evaluation

- Completeness:
  - *Does it always find a solution if one exists?*
  - YES
    - If the shallowest goal node is at some finite depth d
    - Condition: If *b* is finite
      - (maximum num. of succ. nodes is finite)

BF-search; evaluation

- Completeness:
  - YES (if b is finite)
- Time complexity:
  - Assume a state space where every state has b successors.
    - the root has b successors, each node at the next level has again b successors (total b^2), …
    - Assume solution is at depth d
    - Worst case; expand all but the last node at depth d
    - Total number of nodes generated:

$$1 + b + b^2 + b^3 + \ldots + b^d + (b^{d+1} - b) = O(b^{d+1})$$

BF-search; evaluation

- Completeness:
  - YES (if *b* is finite)
- Time complexity:
  - Total number of nodes generated:
- Space complexity:
  - Idem, because each node is retained in the memory

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BF-search; evaluation

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  - YES (if b is finite)
- Time complexity:
  - Total number of nodes generated:

$$1+b+b^2+b^3+...+b^d+(b^{d+1}-b)=O(b^{d+1})$$

- Space complexity:
  - Idem, because each node is retained in the memory
- Optimality:
  - *Does it always find the least-cost solution?*
  - In general YES
    - unless actions have different cost.

BF-search; evaluation

- Two lessons:
  - Maintaining large memory is a bigger problem than the execution time.
  - Exponential complexity search problems cannot be solved by uninformed search methods for any but the smallest instances.

BF-search; evaluation

b = 10 **branching factor** (Chess app. 35 !)

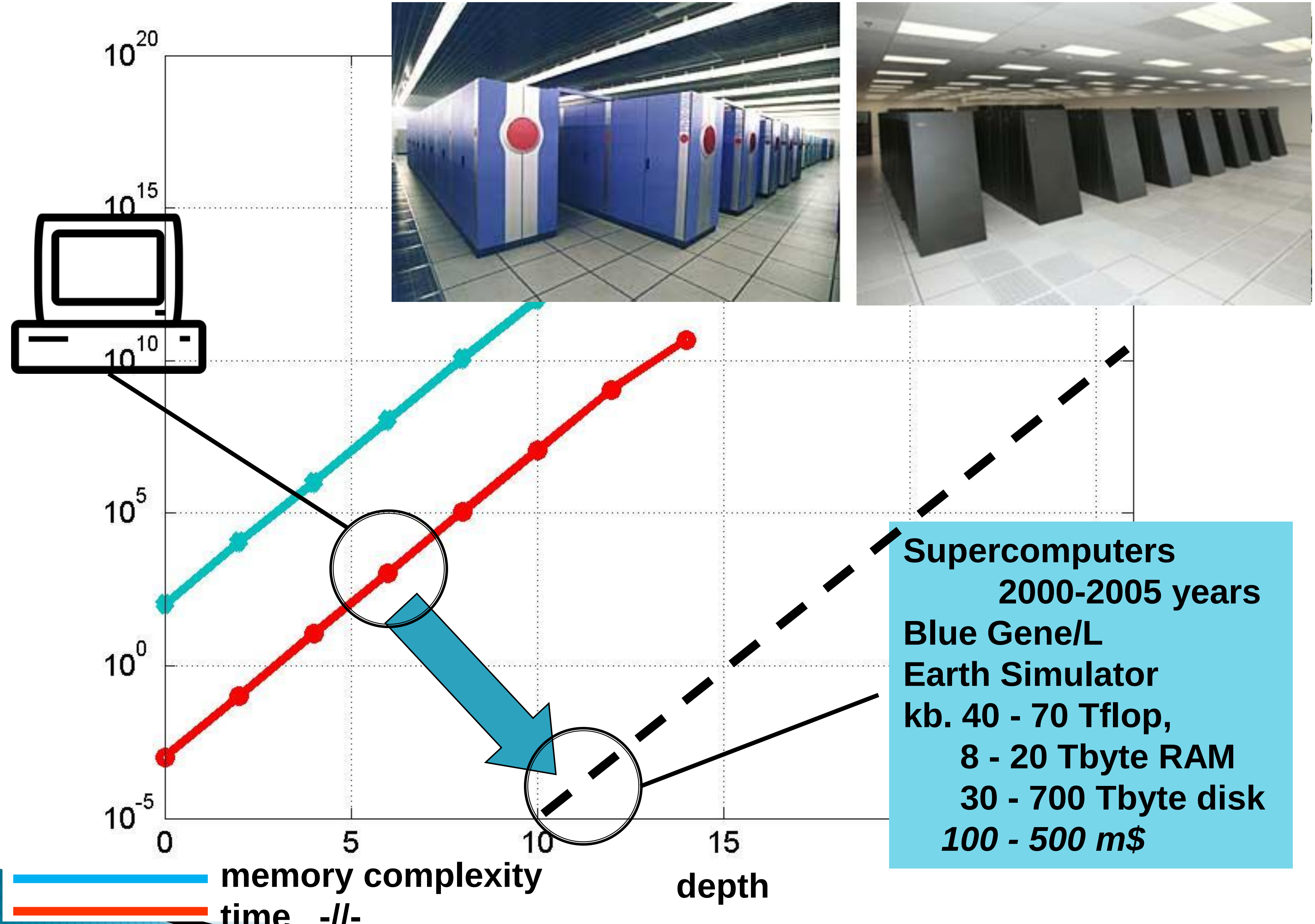
1000 decision / sec ≅ 1kflop
1 decision information: 100 byte

1 byte ≅ 1 letter

Depth	Decisions	Time Demand	Memory Demand
0	1	0.001 sec	100 byte
2	111	0.1 sec	11 kbyte
4	11111	11 sec	1 Mbyte
6	10^6	18 minutes	111 Mbyte
8	10^8	31 hours	11 Gbyte (PC)
10	10^{10}	128 days	1 Tbyte
12	10^{12}	35 years	111 Tbyte
14	10^{14}	1500 years	11111 Tbyte

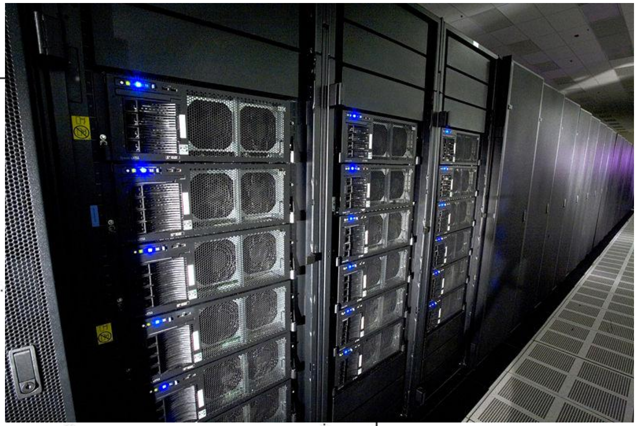
k = 10^3, M = 10^6, G = 10^9, T = 10^{12}, P = 10^{15}

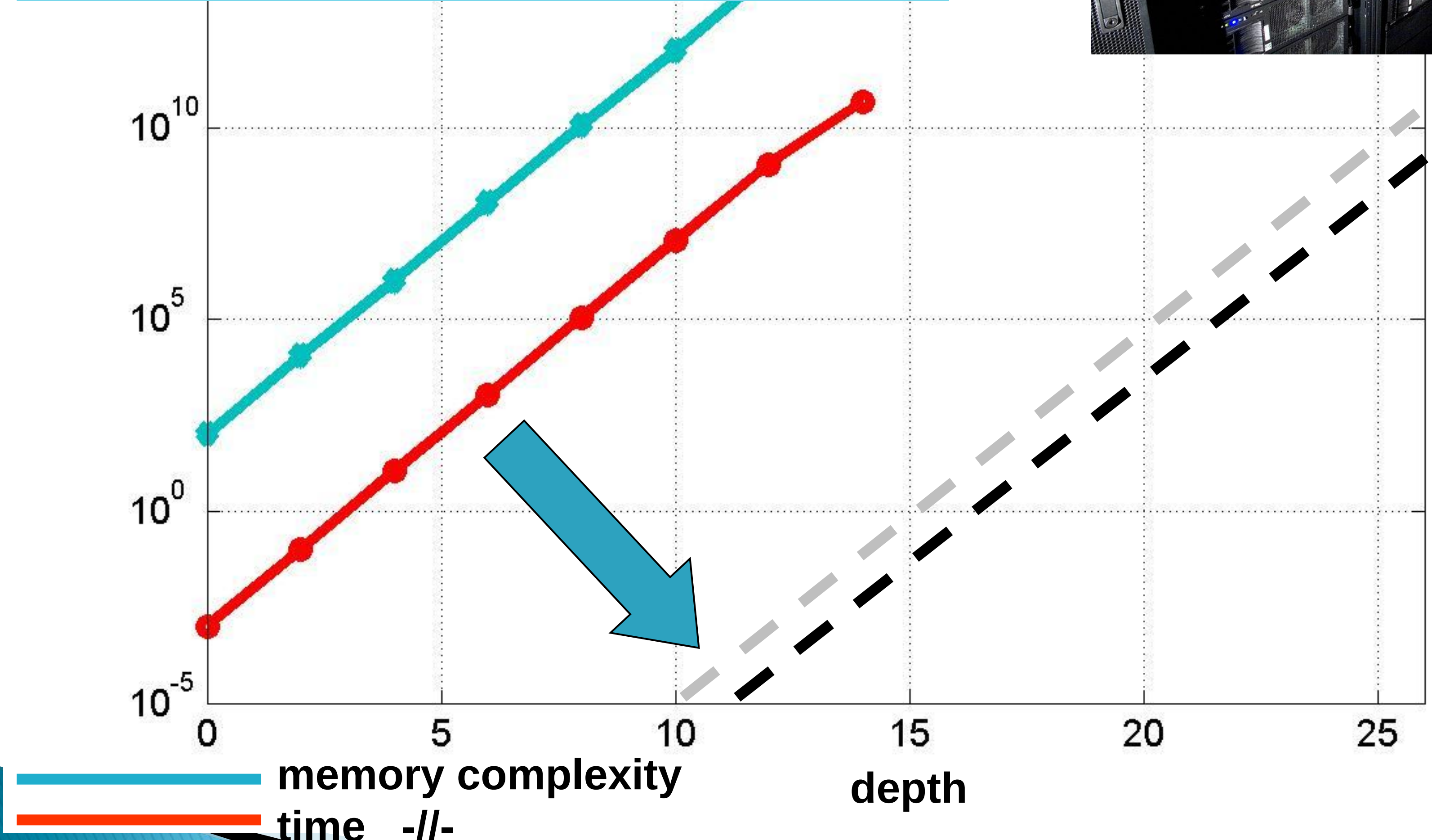
11111 Tbyte ≅ 3 milliard human libraries



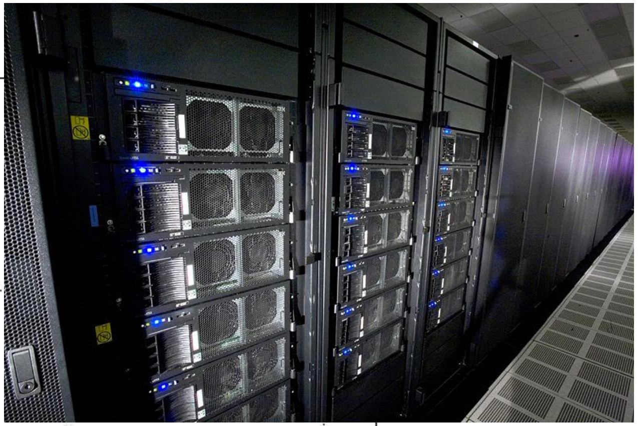
10^{20}
10^{15}
10^{10}
10^{5}
10^{0}
10^{-5}
0
5
10
15
depth
memory complexity
time -//-
Supercomputers
2000-2005 years
Blue Gene/L
Earth Simulator
kb. 40 - 70 Tflop,
8 - 20 Tbyte RAM
30 - 700 Tbyte disk
100 - 500 m$

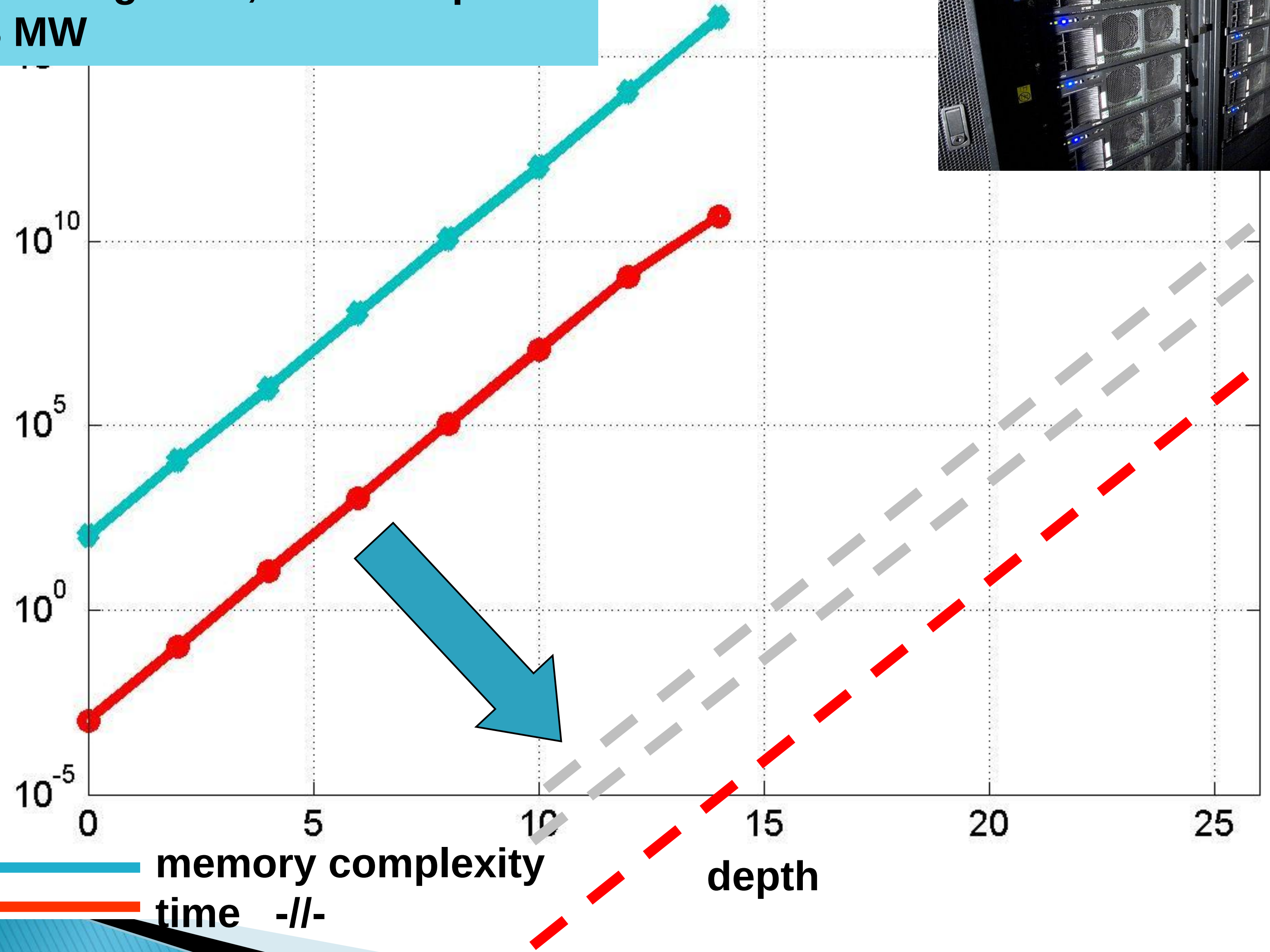
Supercomputers 2007-2008 years
IBM Blue Gene/L, upgrade
Lawrence Livermore Nat Lab, 478 Tflop
IBM Roadrunner, Los Alamos Nat Lab
1.026 Pflop





Supercomputers 2018 year
IBM Summit
Oak Ridge Lab, 1223 Pflop
13 MW





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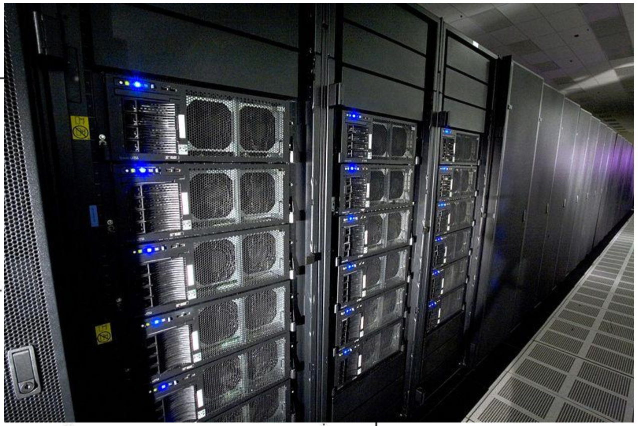
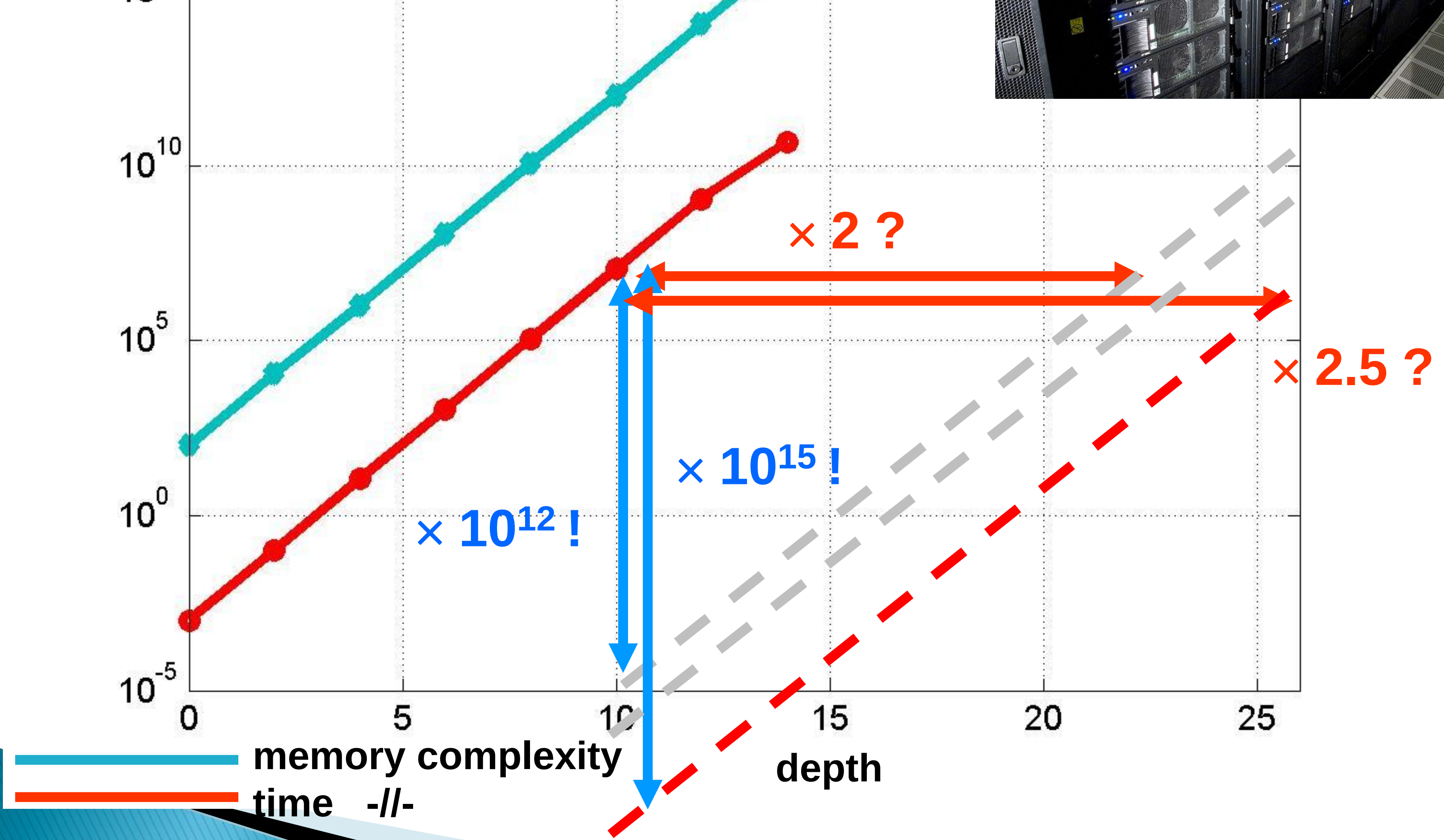




Uniform-cost search

- Extension of BF-search:
  - Expand node with *lowest path cost*
- Implementation: *fringe* = queue ordered by path cost.

- UC-search is the same as BF-search when all step-costs are equal.

Uniform-cost search

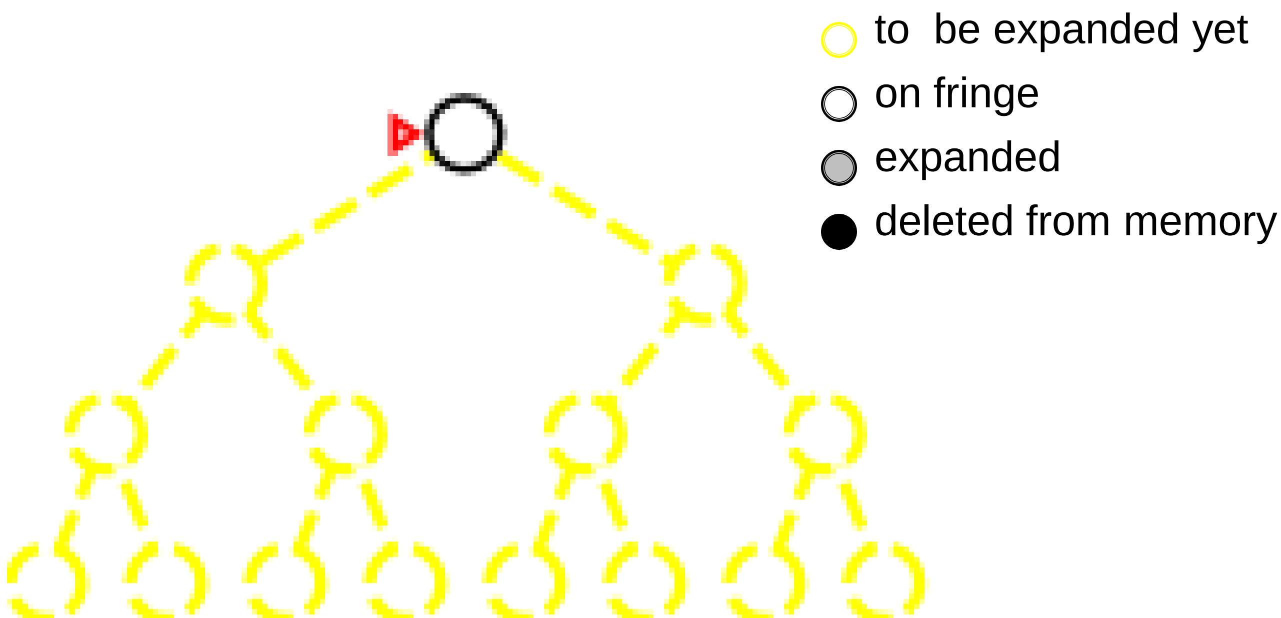
- Completeness:
  - YES, if step-cost > ε (smal positive constant)
- Time complexity:
  - Assume C* the cost of the optimal solution.
  - Assume that every action costs at least ε
  - Worst-case:

$$O(b^{C^*/\varepsilon})$$

- Space complexity:
  - Idem to time complexity
- Optimality:
  - nodes expanded in order of increasing path cost.
  - YES, if complete.

DF-search, an example

- Expand *deepest* unexpanded node
- Implementation: *fringe* is a LIFO queue (=stack)



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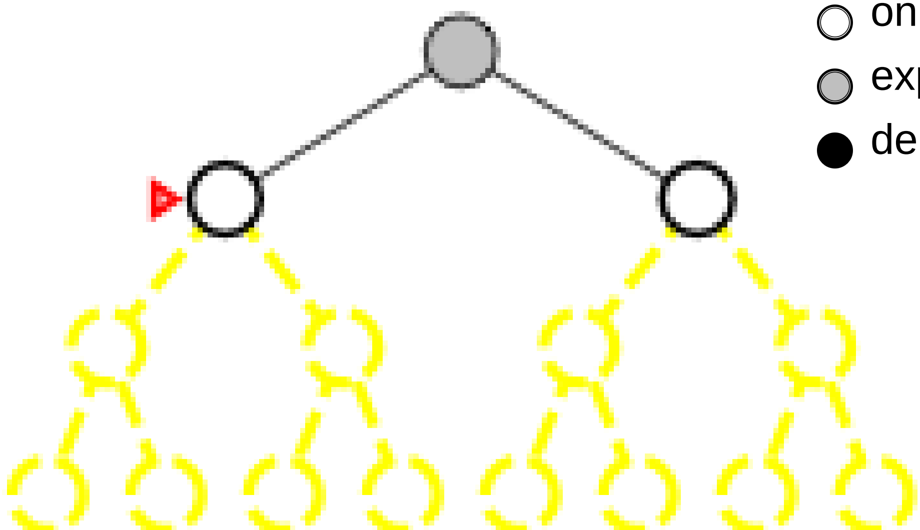

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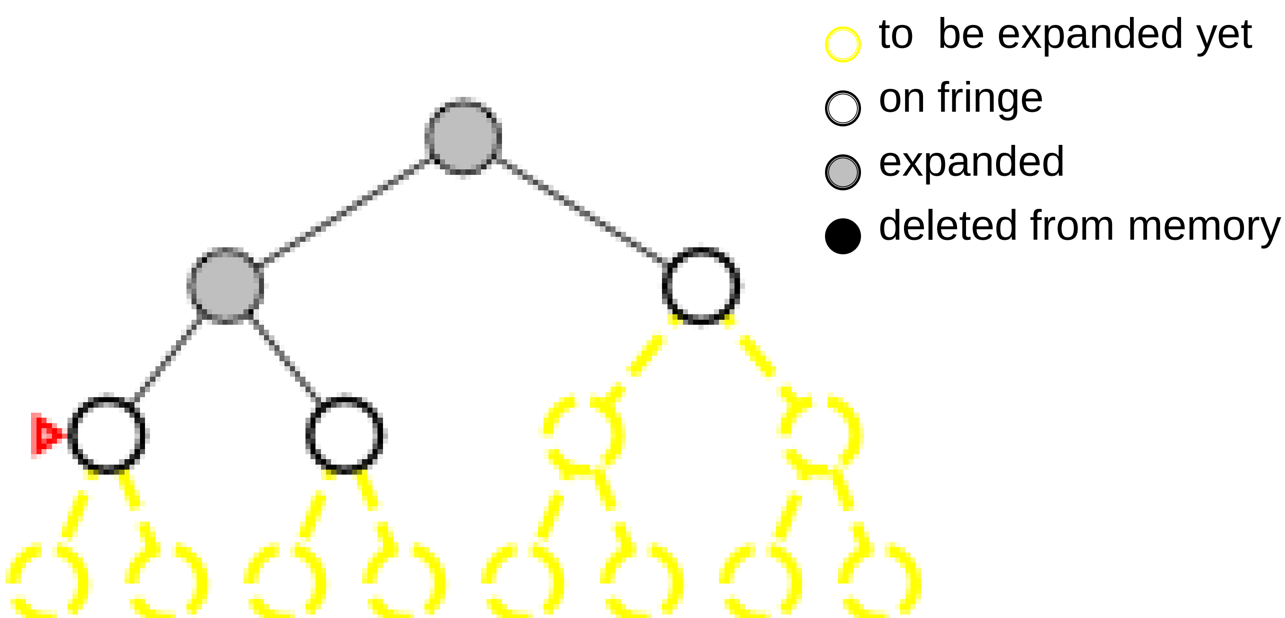

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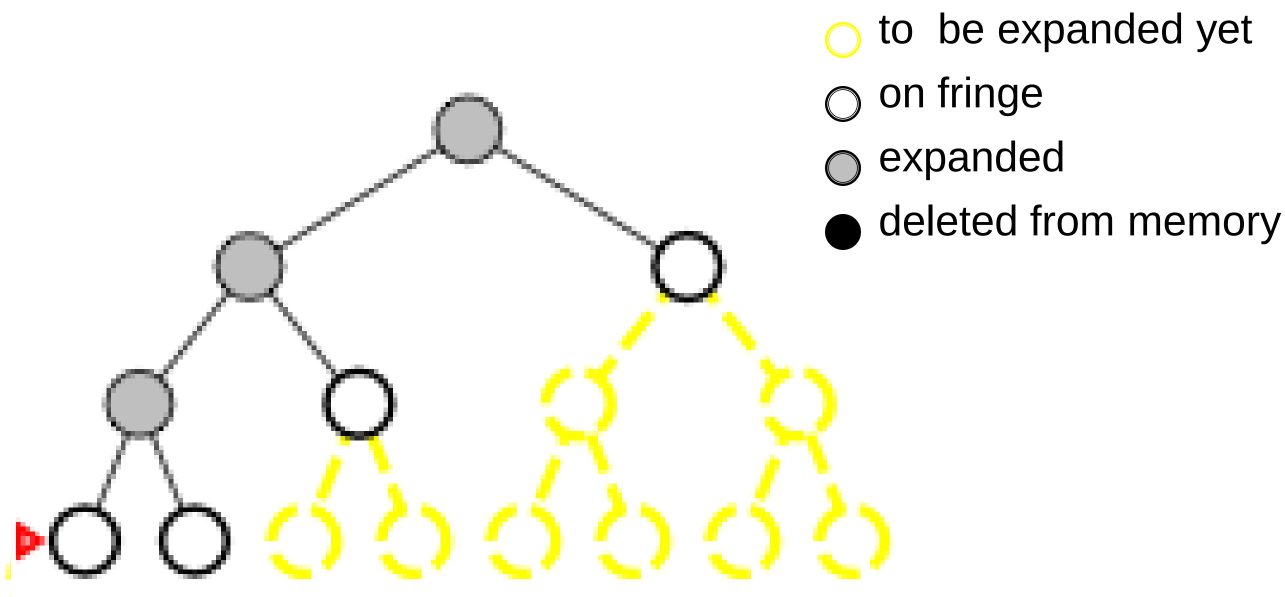

DF-search, an example

- Expand *deepest* unexpanded node
- Implementation: *fringe* is a LIFO queue (=stack)

to be expanded yet
on fringe
expanded
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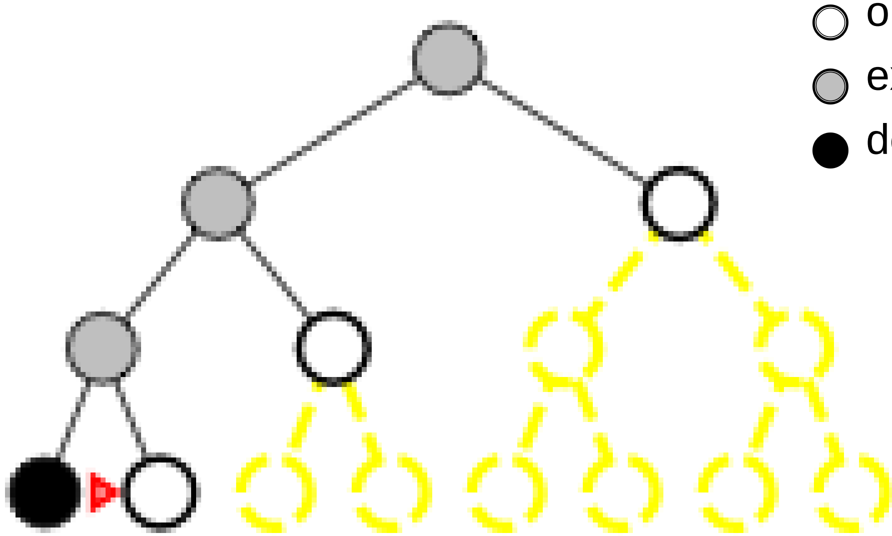

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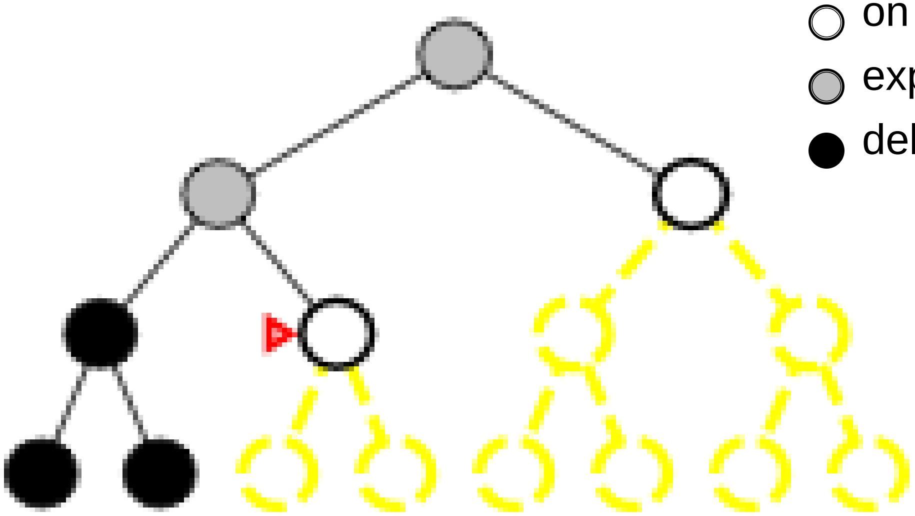

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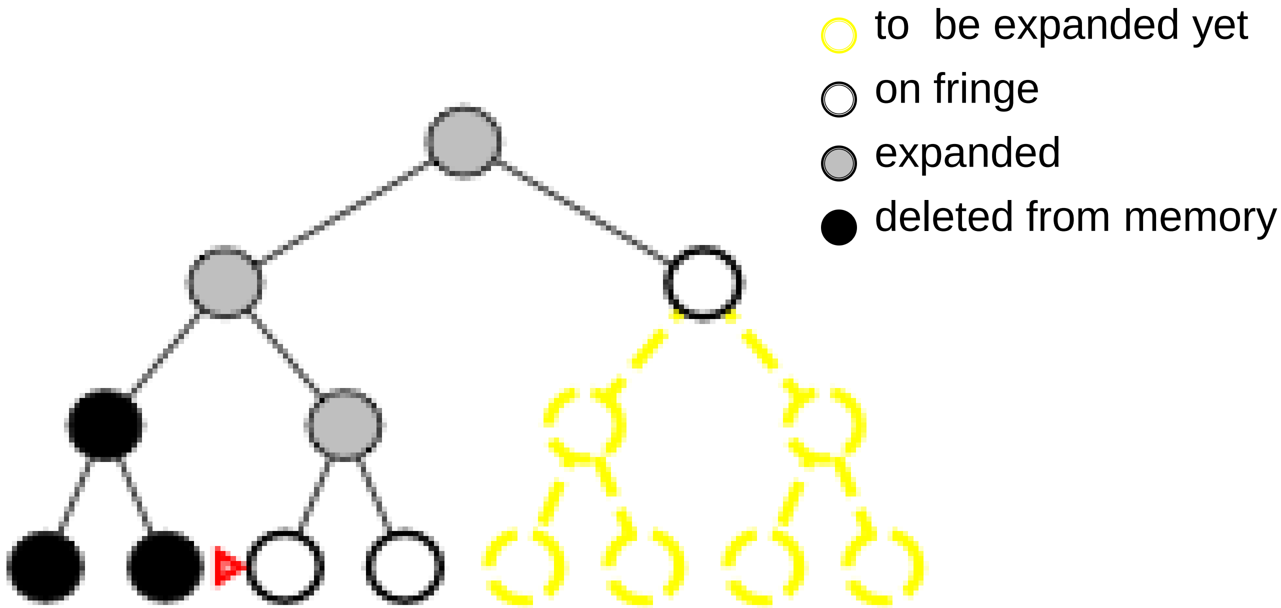

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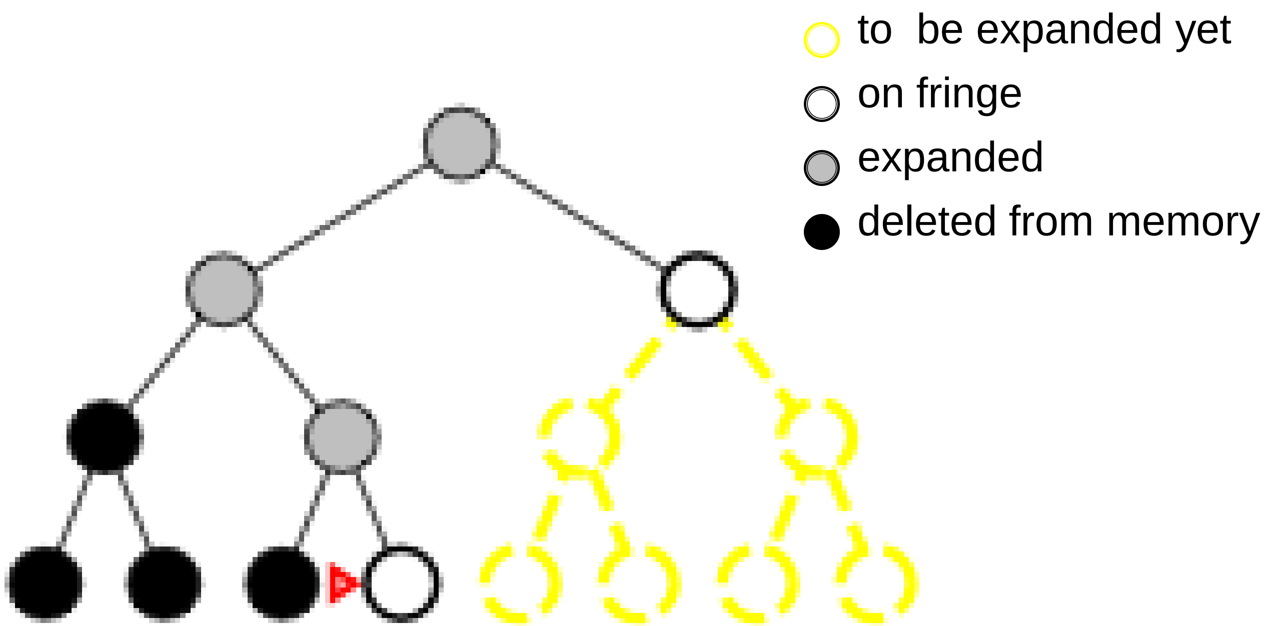

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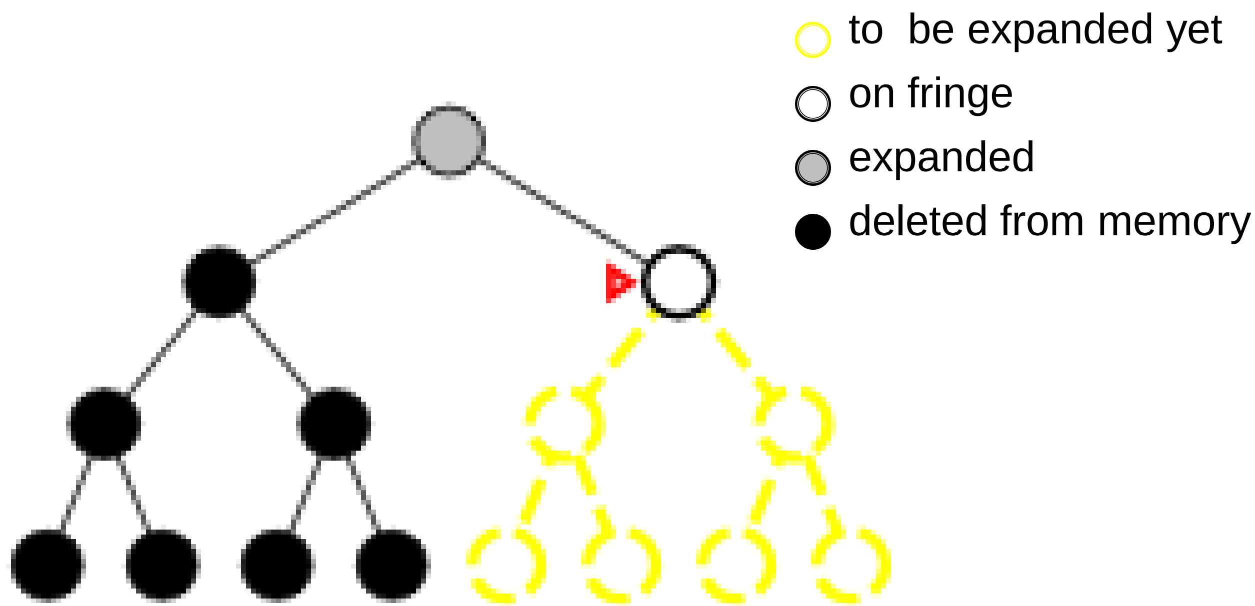

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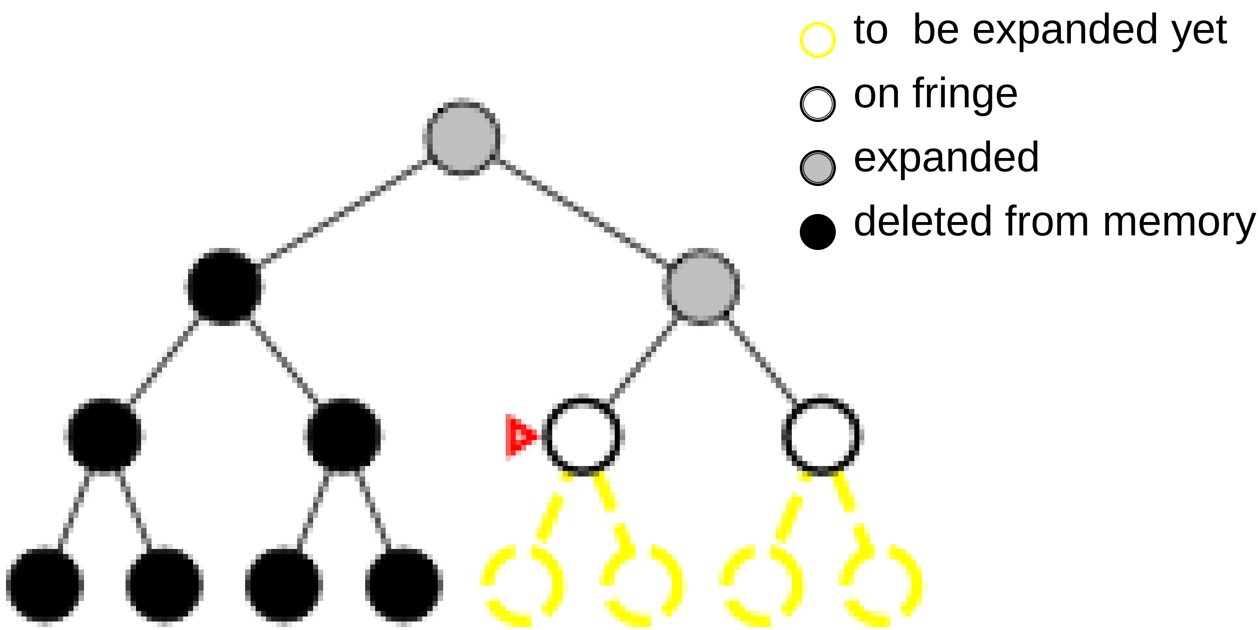

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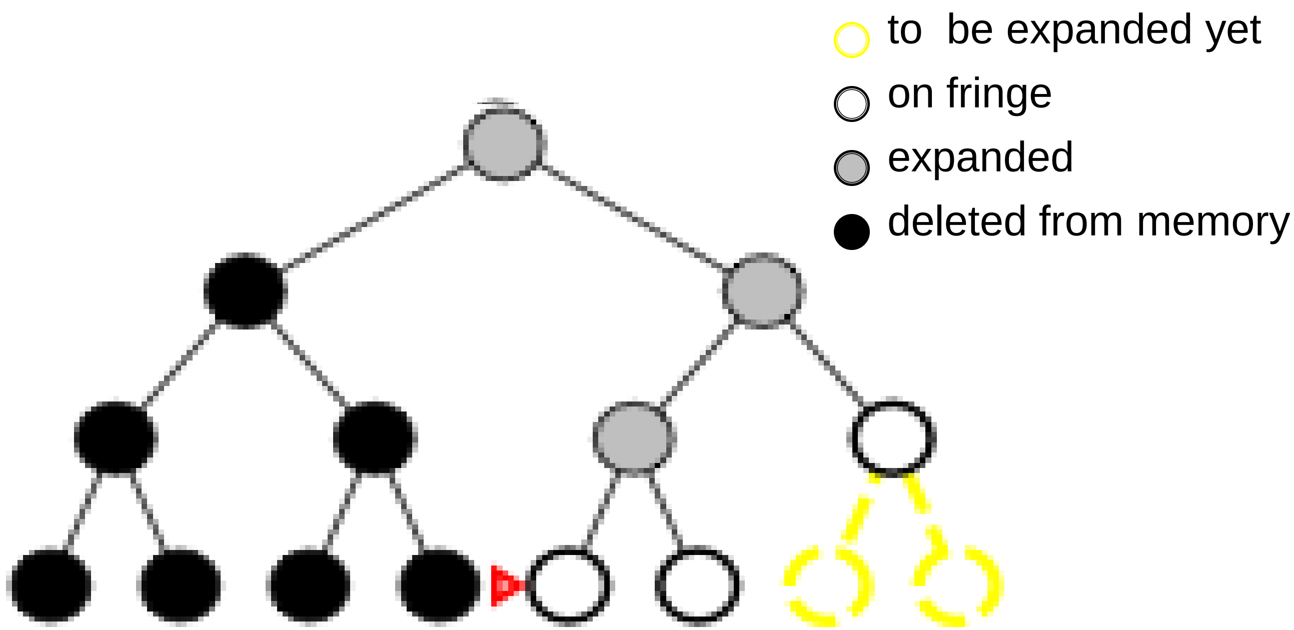

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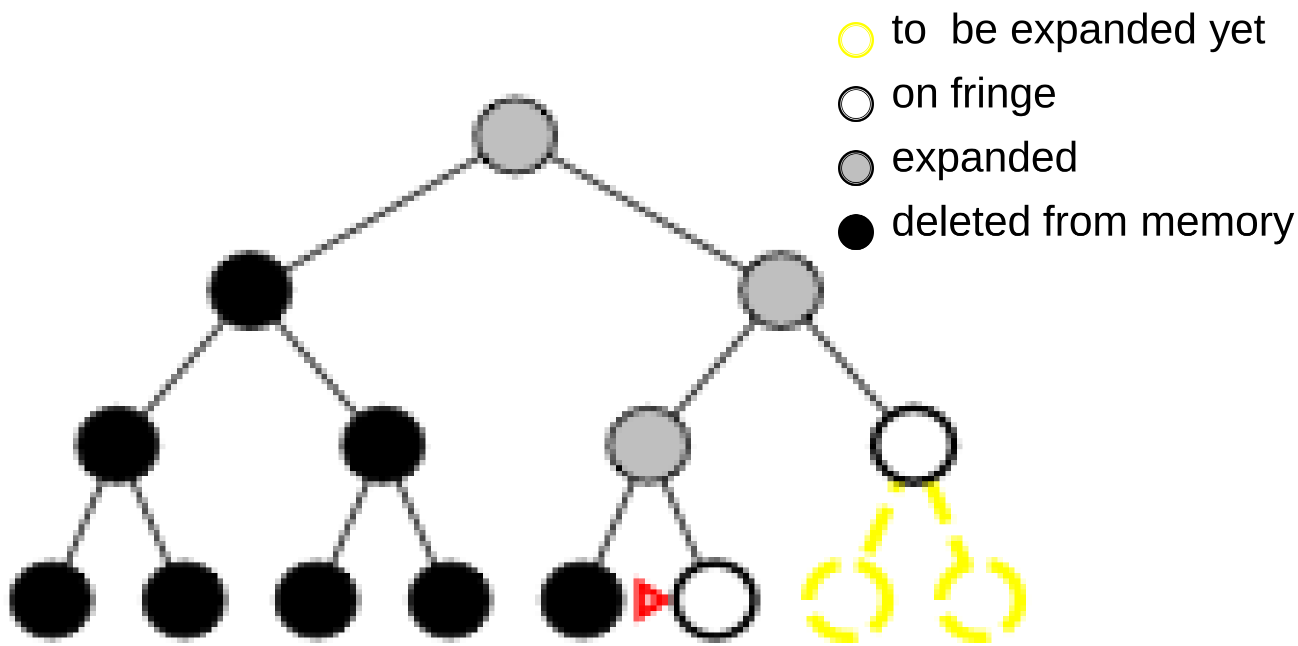


DF-search; evaluation

- Completeness;
  - *Does it always find a solution if one exists?*
  - NO
    - *unless* search space is finite and no loops are possible.

DF-search; evaluation

- Completeness;
  - NO unless search space is finite. $O(b^m)$
- Time complexity;
  - May be terrible if *m* is much larger than *d* (depth of optimal solution)
  - But if many solutions, then faster than BF-search

DF-search; evaluation

- Completeness;
  - NO unless search space is finite.
- Time complexity; $O(b^m)$
- Space complexity; $O(bm+1)$
  - Backtracking search uses even less memory
    - One successor instead of all *b.*

DF-search; evaluation

- Completeness;
  - NO unless search space is finite.
- Time complexity; $O(b^m)$
- Space complexity; $O(bm+1)$
- Optimality; No
  - Same issues as completeness

Depth-limited search

- DF-search with a depth limit *l*.
  - i.e. nodes at depth *l* are not expaneded for successors.
  - Problem knowledge can be used
- Solves the infinite-path problem, but
- If $l < d$ then incompleteness results.
- If $l > d$ then not optimal.
- Time complexity: $O(b^l)$
- Space complexity: $O(bl)$

Depth-limited algorithm

function DEPTH-LIMITED-SEARCH(*problem, limit*) **return** a solution or failure/cutoff

return RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[*problem*]), *problem*, *limit*)

function RECURSIVE-DLS(*node*, *problem, limit*) **return** a solution or failure/cutoff

cutoff_occurred? ← false

if GOAL-TEST[*problem*](STATE[*node*]) **then return** SOLUTION(*node*)

else if DEPTH[*node*] == *limit* **then** return *cutoff*

else for each *successor* **in** EXPAND(*node*, *problem*) **do**

    result ← RECURSIVE-DLS(*successor*, *problem*, *limit*)

    if *result* == *cutoff* **then** *cutoff_occurred?* ← true

    else if *result* ≠ *failure* **then return** *result*

if *cutoff_occurred?* **then return** *cutoff* **else return** failure

Iterative deepening search

- What it is?
  - A general strategy to find best depth limit *l*.
    - Goal is found at depth *d*, the depth of the shallowest goal-node.
  - Then use Depth-limited search
- Combines benefits of DF- en BF-search

Iterative deepening search

function ITERATIVE_DEEPENING_SEARCH(*problem*)
return a solution or failure

inputs: *problem*

for *depth* $\leftarrow$ 0 to ∞ **do**
  result $\leftarrow$ DEPTH-LIMITED_SEARCH(*problem*, *depth*)
  if *result* $\neq$ *cuttoff*
  then return *result*

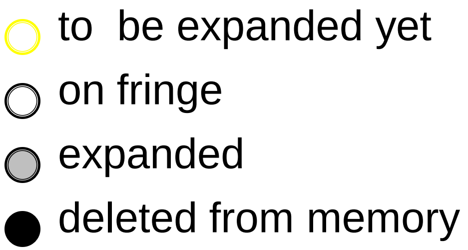
ID-search, example

- to be expanded yet
- on fringe
- expanded
- deleted from memory

- Limit=0

ID-search, example

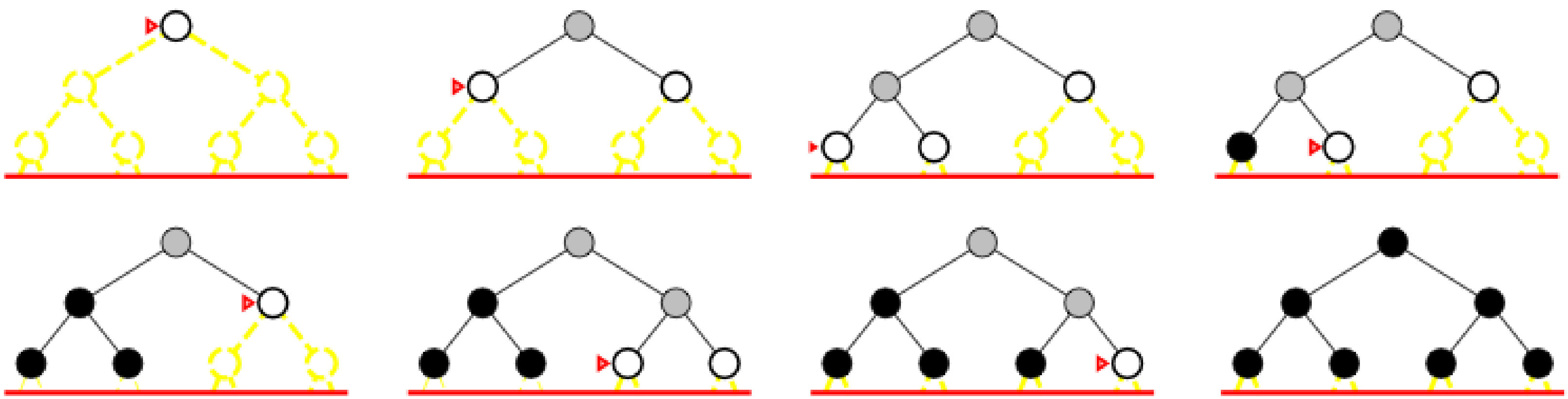
- Limit=1



ID-search, example

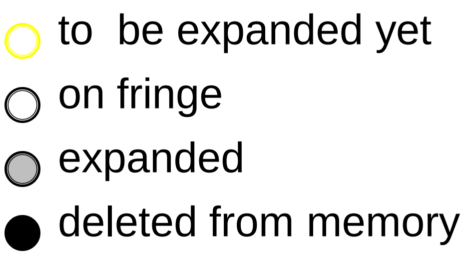
- to be expanded yet
- on fringe
- expanded
- deleted from memory

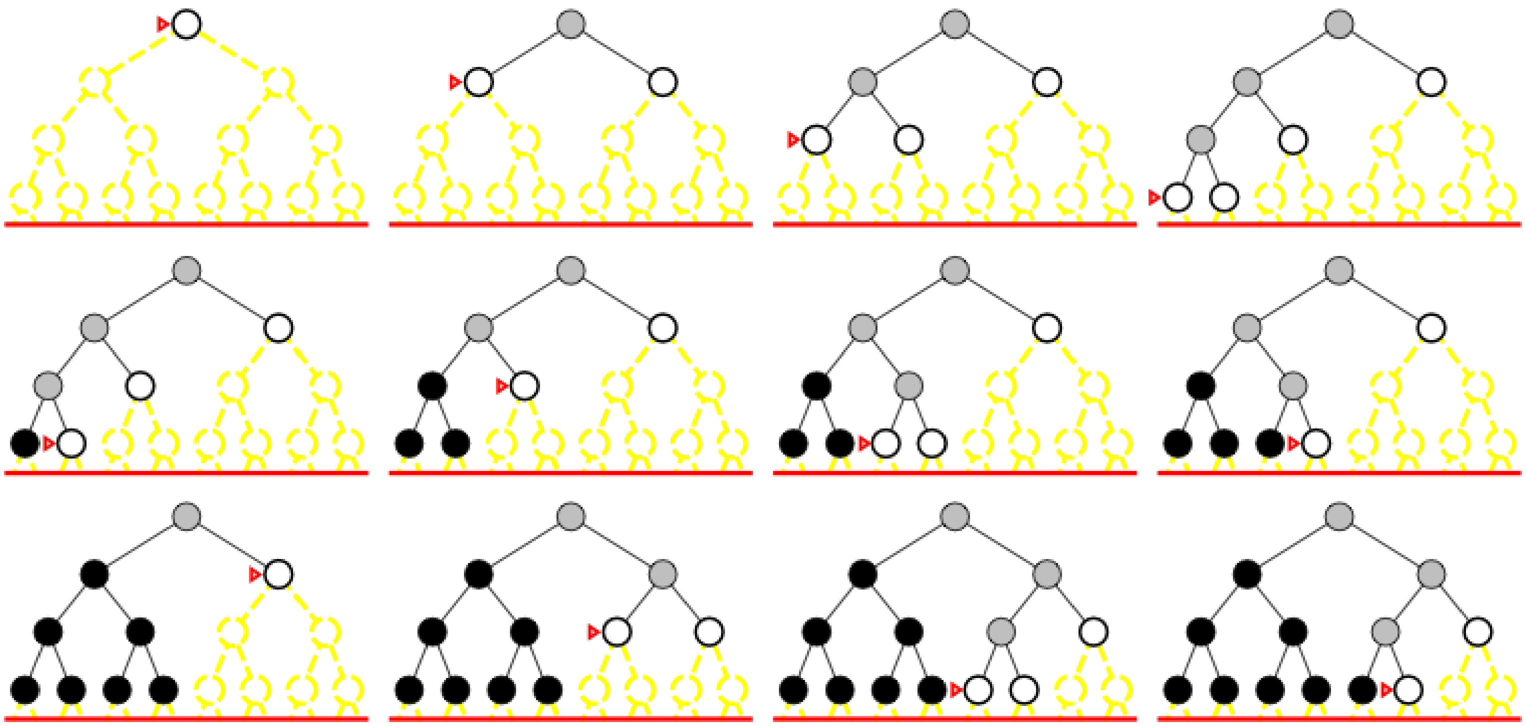
- Limit=2



ID-search, example

- Limit=3





ID search, evaluation

- Completeness:
  - YES (no infinite paths)

ID search, evaluation

- Completeness:
  - YES (no infinite paths)
- Time complexity:
  - Algorithm seems costly due to repeated generation of certain states.
  - Node generation: $O(b^d)$
    - level d: once
    - level d-1: x 2
    - level d-2: x 3
    - …
    - level 2: x (d-1)
    - level 1: x d

$$N(IDS) = (d)b + (d-1)b^2 + \ldots + (1)b^d$$

$$N(BFS) = b + b^2 + \ldots + b^d + (b^{d+1} - b)$$

Compare for b=10 and d=5 (solution at far right)

$$N(IDS) = 50 + 400 + 3000 + 20000 + 100000 = 123450$$

$$N(BFS) = 10 + 100 + 1000 + 10000 + 100000 + 999990$$
$$= 1111100$$

ID search, evaluation

- Completeness:
  - YES (no infinite paths)
- Time complexity: $O(b^d)$
- Space complexity:
  - Cfr. depth-first search $O(bd)$

ID search, evaluation

- Completeness:
  - YES (no infinite paths)
- Time complexity: $O(b^d)$
- Space complexity: $O(bd)$
- Optimality:
  - YES if step cost is 1.
  - Can be extended to iterative lengthening search
    - Same idea as uniform-cost search
    - Increases overhead.

Bidirectional search

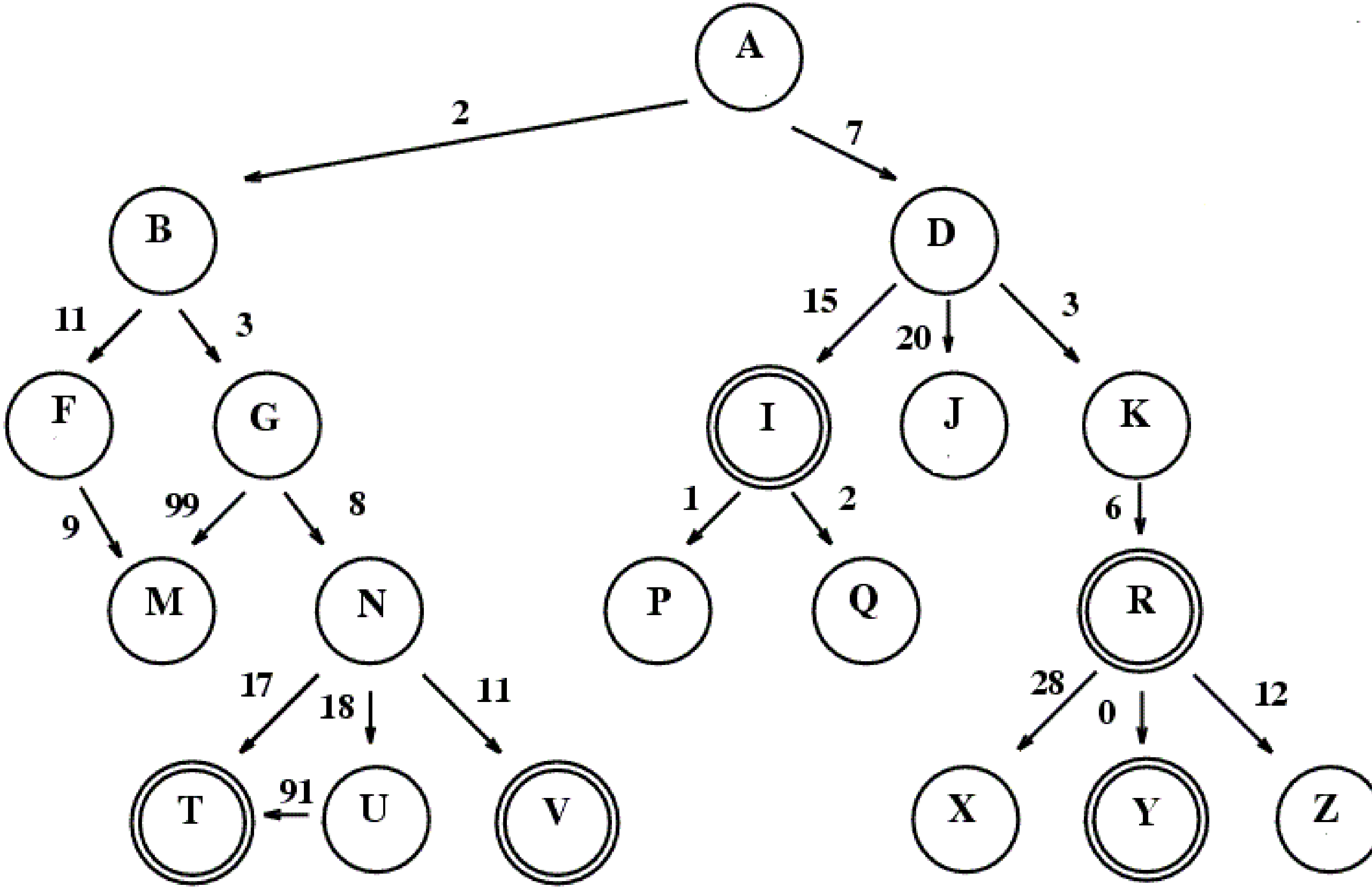
- Completeness:
  - YES, if at least one direction BF-like
- Time complexity: $O(b^{d/2})$
- Space complexity: $O(b^{d/2})$
- Optimality:
  - IF …
  - Complexity of checking for a node in the other search tree
  - Doing search „backwards” from the goal
  - …

Summary of algorithms

Criterion	Breadth-First	Uniform-cost	Depth-First	Depth-limited	Iterative deepening	Bidirectional search
Complete?	YES*	YES*	NO	YES, if $l \geq d$	YES	YES*
Time	b^{d+1}	$b^{C^*/e}$	b^m	b^l	b^d	$b^{d/2}$
Space	b^{d+1}	$b^{C^*/e}$	bm	bl	bd	$b^{d/2}$
Optimal?	YES*	YES*	NO	NO	YES	YES

Summary

- The symbols & search paradigm in AI
- Uninformed search
  - Space complexity: OK!
  - Time complexity: exp. ➔ the knowledge paradigm in AI
- Suggested reading
  - Newel & Simon: Computer science as empirical inquiry: symbols and search, 1975
  - Cognitive architectures: ACT-R
    - http://act-r.psy.cmu.edu/
    - http://act-r.psy.cmu.edu/about/
    - Allen Newell describes cognitive architectures as the way to answer one of the ultimate scientific questions: "How can the human mind occur in the physical universe?
      - http://act-r.psy.cmu.edu/misc/newellclip.mpg



A
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