Adapted from AIMA slides

Simple probabilistic models

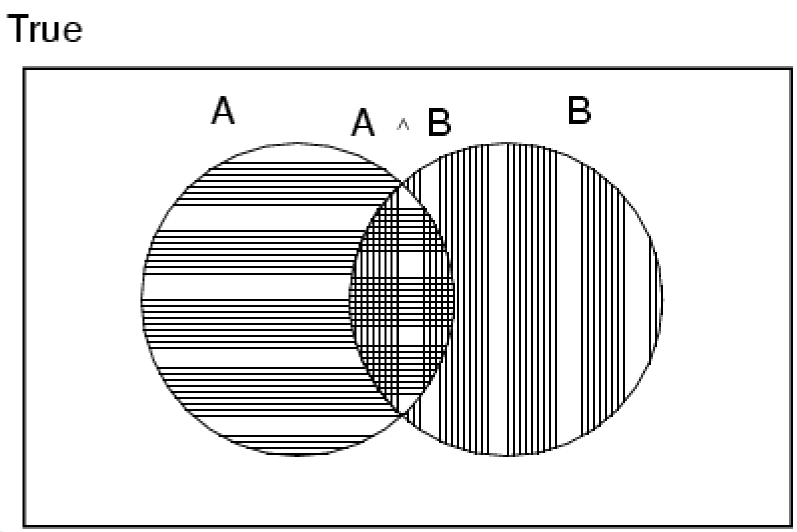
Peter Antal
antal@mit.bme.hu

Outline

- Basics of probability theory
- Relation of two-valued vs probabilistic logic
  - Truth vs belief
  - Proofs vs inference
- Naïve Bayesian networks (N-BN)
- Exercises: SPAM filter construction using N-BNs

Axioms of probability

- For any propositions *A, B*
-
  - $0 \leq P(A) \leq 1$
  - $P(true) = 1$ and $P(false) = 0$
  - $P(A \vee B) = P(A) + P(B) - P(A \wedge B)$
  -



About the event space

- Atomic events are mutually exclusive and exhaustive.
- The single variable case.
  - *Weather* is one of $<sunny, rainy, cloudy, snow>$
  - $P((Weather = sunny) \vee (Weather = rainy))$
- Challenges in the multivariate case.
  - *Weather* is one of $<sunny, rainy, cloudy, snow>$
  - *TemperatureofRain* is one of $<icy, cold, warm>$
    - NONE?

Classical vs probabilistic logic: truth and beliefs

P_1	…	P_3	KB	s	pKB	P(query|evidence)
F	F	F	F	T	.01	.1
F	F	T	T	F	.12	.2
F	T	F	F	T	.35	.3
F	T	T	F	F	..	..
T	F	F	F	T	..	..
T	F	T	T	T	…	…
T	T	F	F	T	..	..
T	T	T	F	T	..	..

Conditional independence

„Probability theory=measure theory+independence"

$I_P(X;Y|Z)$ or $(X \perp\!\!\!\perp Y|Z)_P$ denotes that X is independent of Y given Z:

$P(X;Y|z)=P(Y|z)\ P(X|z)$ for all z with $P(z)>0$.

(Almost) alternatively, $I_P(X;Y|Z)$ iff

$P(X|Z,Y)= P(X|Z)$ for all z,y with $P(z,y)>0$.

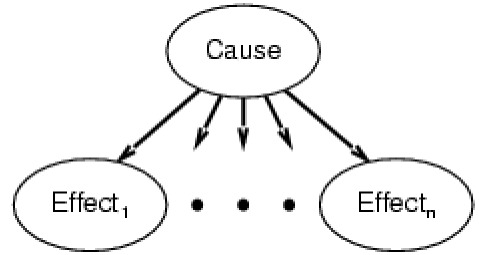
Other notations: $D_P(X;Y|Z)$ =def= $\neg\ I_P(X;Y|Z)$

Contextual independence: for not all z.

Homeworks:

- Intransitivity: show that it is possible that D(X;Y), D(Y;Z), but I(X;Z).
- order : show that it is possible that I(X;Z), I(Y;Z), but D(X,Y;Z).

Naive Bayesian network



Assumptions:

1, Two types of nodes: a cause and effects.

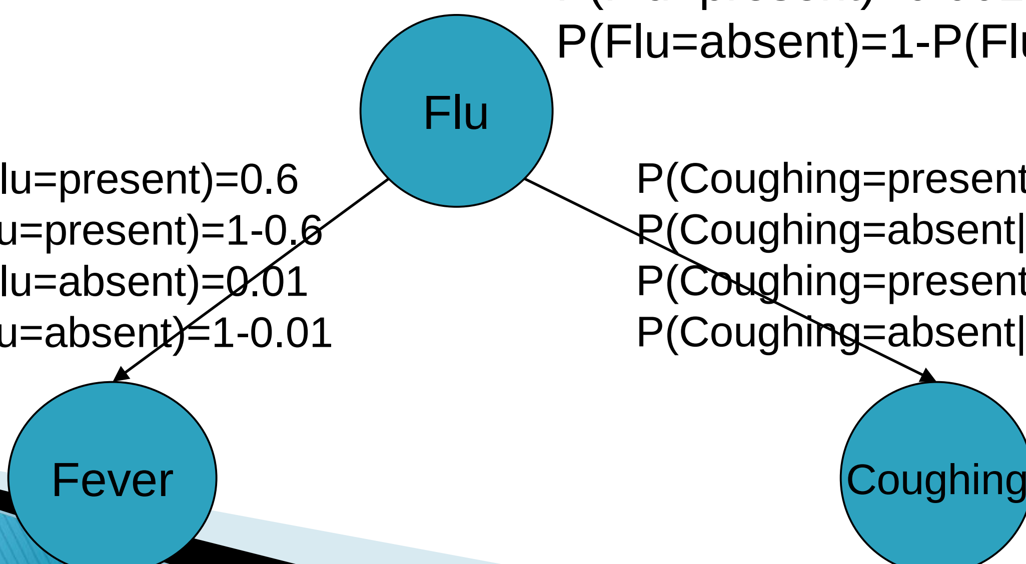
2, Effects are conditionally independent of each other given their cause.

Variables (nodes)

Flu: present/absent
FeverAbove38C: present/absent
Coughing: present/absent

Model

P(Flu=present)=0.001
P(Flu=absent)=1-P(Flu=present)



P(Fever=present|Flu=present)=0.6
P(Fever=absent|Flu=present)=1-0.6
P(Fever=present|Flu=absent)=0.01
P(Fever=absent|Flu=absent)=1-0.01

P(Coughing=present|Flu=present)=0.3
P(Coughing=absent|Flu=present)=1-0.7
P(Coughing=present|Flu=absent)=0.02
P(Coughing=absent|Flu=absent)=1-0.02

Naive Bayesian network (NBN)

Decomposition of the joint:

$P(Y,X_1,..,X_n) = P(Y)\prod_i P(X_i,|Y, X_1,..,X_{i-1})$ //by the chain rule

$= P(Y)\prod_i P(X_i,|Y)$ // by the N-BN assumption

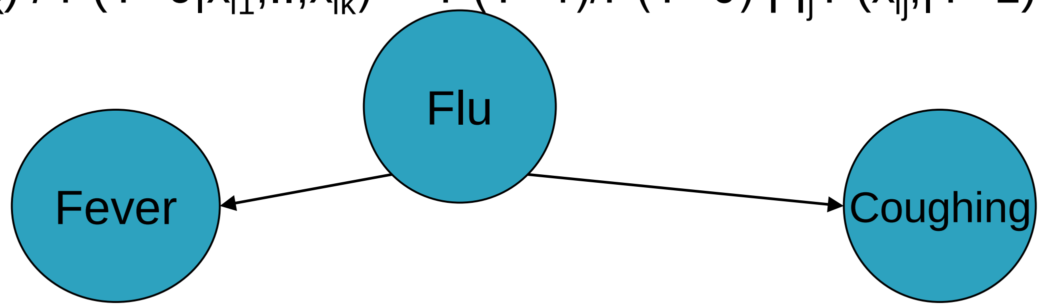
2n+1 parameteres!

Diagnostic inference:

$P(Y|x_{i1},..,x_{ik}) = P(Y)\prod_j P(x_{ij},|Y) / P(x_{i1},..,x_{ik})$

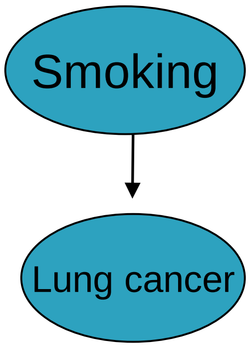
If Y is binary, then the odds

$P(Y=1|x_{i1},..,x_{ik}) / P(Y=0|x_{i1},..,x_{ik}) = P(Y=1)/P(Y=0) \prod_j P(x_{ij},|Y=1) / P(x_{ij},|Y=0)$



$$p(Flu = present \mid Fever = absent, Coughing = present)$$
$$\propto p(Flu = present)\, p(Fever = absent \mid Flu = present)\, p(Coughing = present \mid Flu = present)$$

Conditional probabilities, odds, odds ratios



	$\neg$S	S	
$\neg$LC	P($\neg$S, $\neg$LC)	P(S, $\neg$LC)	P($\neg$LC)
LC	P($\neg$S, LC)	P(S, LC)	P(LC)
	P($\neg$S)	P(S)	

Probability:

P(LC)

Conditional probabilities (e.g., probability of LC given S)**:**

P(LC| $\neg$S)= ??? P(LC| S)= ??? P(LC)

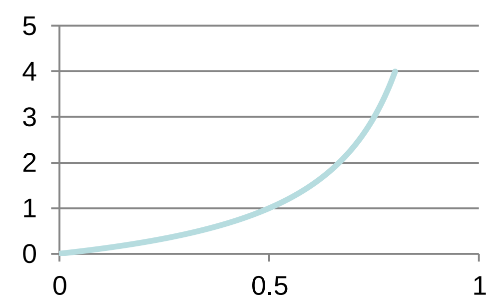
Odds:

$[0,1] \rightarrow [0,\infty]$: Odds(p)=p/(1-p)

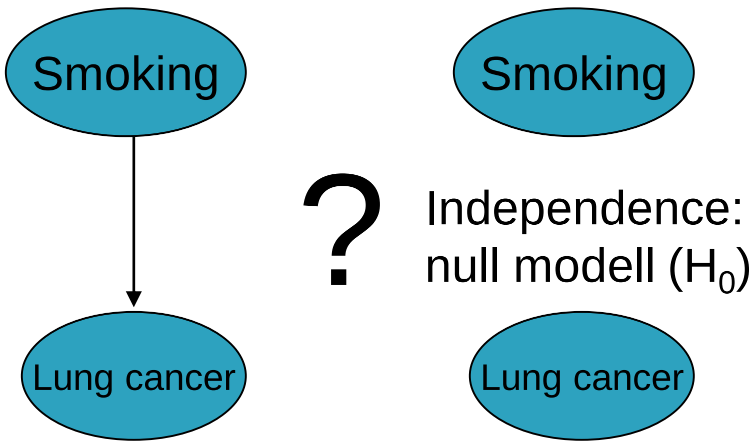
O(LC| $\neg$S)= ??? O(LC| S)

Odds Ratio (OR) Independent of prevalence!

OR(LC,S)=O(LC| S)/O(LC| $\neg$S)



Probabilities, odds, odds ratios



	¬S	S	
¬LC	8	7	15
LC	1	4	5
	9	11	20

Contingency table with marginals

	¬S	S	
¬LC	.4	.35	.75
LC	.05	.2	.25
	.45	.55	

Conditional probabilities:

P(LC| ¬S)=.11 ??? P(LC| S)=.36 ??? P(LC)=.25

Odds:

[0,1] →[0,∞]: Odds(p)=p/(1-p)

O(LC| ¬S)=.12 ??? O(LC| S)=.56

Odds Ratio (OR):

OR(LC,S)=O(LC| S)/O(LC| ¬S)=4.6

BAYES CUBE (~BAYES EYE)

http://bioinfo.mit.bme.hu/

https://www.mit.bme.hu/eng/system/files/oktatas/targyak/10337/BayesCube_manual_en_v31.pdf

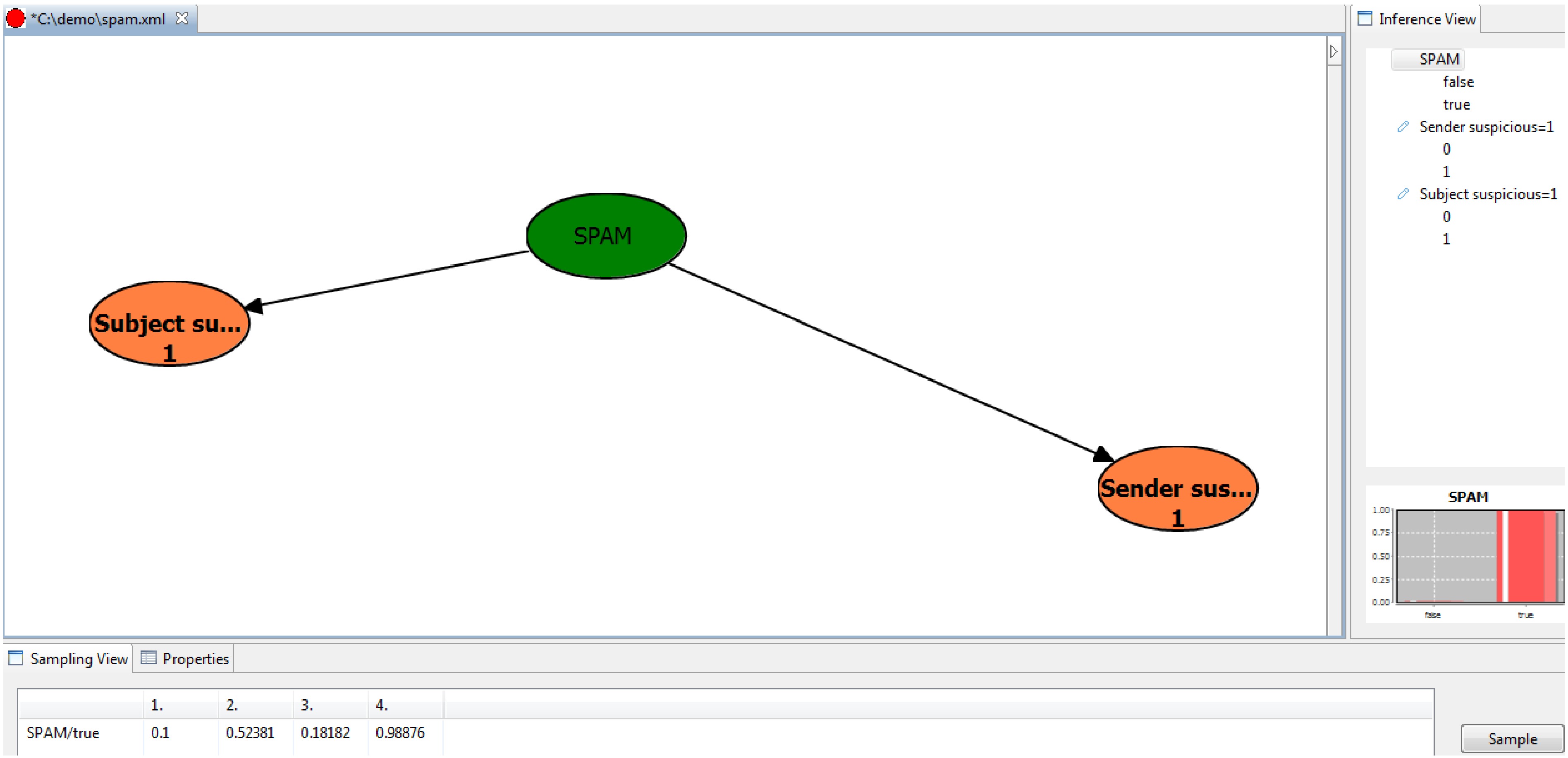
Computational Biomedicine (Combine) workgroup
Intelligent Systems research group,
Department of Measurement and Information Systems,
Budapest University of Technology and Economics

COMBINE
Computational
Biomedicine
Workgroup

Méréstechnika és
Információs Rendszerek
Tanszék

MŰEGYETEM 1782

Example: Construct a spam filter



Summary

- Naïve Bayesian networks (N-BNs) demonstrate the use of independencies to cope with
  - model complexity (~space complexity, number of parameters)
  - inferential complexity (~time complexity).
- The assumption of conditional independence of the effects given their common cause allows
  - the efficient representation of the joint distribution
    - (in the discrete, multinomial case: linear number of parameters instead of exponential),
  - the efficient computation of the diagnostic posterior p(Y|X)
    - (linear number of steps instead of exponential),
- Odds, log odds are popular transformations of probabilities.
- N-BNs are robust knowledge engineering and data analysis tools.
-
- **Suggested reading:**
  - Druzdzell: Building Probabilistic Networks: Where Do the Numbers Come From?, IEEE Transactions on Knowledge and data engineering, 2000