A COMPOSITE OBSERVER STRUCTURE FOR ADAPTIVE FOURIER ANALYSIS[1]

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Abstract: In this paper a new composite observer structure is presented for the efficient implementation of an adaptive Fourier analyzer. This structure is based on a proper combination of the concepts of polyphase filtering, the fast Fourier transformation (FFT) algorithm and a phase-locked loop (PLL) like adaptation procedure. The inherent parallelism of this structure enables very high speed practical implementations. This new structure can be extremely useful in practical system identification problems where periodic, multi-frequency perturbation signal are applied. In this case the "tunability" is of key importance, since "good" measurements can be performed only if the Fourier analyzer is "locked" to the fundamental frequency of the periodic perturbation signal. In the case of the adaptive operation there is no need for external synchronization.

Keywords: adaptive algorithms, recursive filters, observers, polyphase networks, frequency tracking.

I. INTRODUCTION

The Adaptive Fourier Analyzer (AFA) (Nagy, 1992; Nagy, 1993) is an adaptive filter-bank structure strongly related to the recursive Discrete Fourier Transformation (DFT) suitable for the calculation of the Fourier coefficients and/or components in a sliding-window mode with a computational complexity proportional to the window size for every input sample (Rabiner and Gold, 1975). The classical version of the recursive DFT based on the Langrange interpolation has been replaced by an observer structure (Hostetter, 1980; Péceli, 1986), and resulted in a common structural framework for signal processing algorithms (Péceli, 1986; Péceli, 1989; Padmanabhan and Martin, 1991). This latter consists of parallel resonators in a common feedback loop, operates as a single-input multiple-output (SIMO) filter-bank, and offers an efficient solution to implement arbitrary recursive transformations. For the DFT this structure is also mentioned as spectral observer. The observer approach can serve also as a conceptual link between the control and signal processing communities providing benefits for both groups. In (Nagy, 1992; Nagy, 1993) an adaptive procedure has been added to the recursive DFT for the analysis of periodic signals with unknown frequency. This adaptation procedure "locks" the fundamental frequency component of the periodic signals like a PLL and locates the harmonically related resonator pole positions correspondingly. The resonator poles generate zeros for the transfer functions of the channels within the SIMO filter-bank in such a way that each channel will suppress all the harmonic components except one corresponding to the channel number.

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This paper presents a new alternative for the implementation of this AFA. The novelty of this approach is the combination of a fast recursive DFT algorithm (Várkonyi-Kóczy, 1995a) with an adaptive "frequency-sampling" method (Rabiner and Gold, 1975). In Section II this recently suggested alternative for the recursive DFT will be introduced, while Section III is devoted to the adaptive part. This latter presents the utilized form of the "frequency-sampling" method, and describes the frequency adaptation algorithm. Finally, some considerations concerning the practical implementation are given.

II. THE POLYPHASE DECOMPOSITION OF THE RECURSIVE DFT

In this Section a possible development of the polyphase DFT structure is presented. The recursive (or sliding window) DFT techniques can be divided in two groups depending on the nature of their outputs. The members of the first group produce Fourier coefficients similarly to the basic, "block-oriented" DFT procedure (i.e. they are real transformations), while the members of the second one operate as filter-banks, and give the Fourier components of the sequence to be transformed. However, it is always true that the actual samples of these Fourier components are equal to the Fourier coefficients calculated for the last data record, i.e. there is no big difference between the two representations. In this paper we will consider both approaches. The basic idea of the polyphase decomposition, however, will be described in details only for the basic transformer structure, since its generalization for filter-banks is straightforward.

The DFT of a sequence of N samples is defined as

$$X_m = \frac{1}{N}\sum_{n=0}^{N-1} x_n e^{-j\frac{2\pi}{N}mn}, \qquad (1)$$

where

- N -the size of the DFT,
- x_n -nth input sample, n=0,1,...,N-1, or running,
- n -discrete "time" index,
- X_m-mth DFT coefficient, m=0,1,...,N-1,
- m - discrete "frequency" index.

The factor of $1/N$ is a simple normalizing factor used typically with the inverse transform (see e.g. Oppenheim and Schafer, 1975), but preferred in (1) in the case of recursive implementation. As a first step to the development of the algorithm, following the "decimation in time" principle (Oppenheim and Schafer, 1975), the input sequence of N samples is decomposed into L blocks of size M:

$$\begin{aligned} X_m &= \frac{1}{N}\sum_{k=0}^{M-1} x_{kL} e^{-j\frac{2\pi}{N}mkL} + \\ &+ \frac{1}{N}\sum_{k=0}^{M-1} x_{kL+1} e^{-j\frac{2\pi}{N}m(kL+1)} + \ldots \\ &+ \frac{1}{N}\sum_{k=0}^{M-1} x_{kL+L-1} e^{-j\frac{2\pi}{N}m(kL+L-1)} = \\ &= \frac{1}{L}\sum_{l=0}^{L-1} \mathrm{DFT_M}(m,l) e^{-j\frac{2\pi}{N}ml}, \end{aligned} \qquad (2)$$

where l denotes the identifier index for blocks of size M, l=0,1,...,L-1, k is the running index within blocks of size M, k=0,1,...,M-1, and DFT(m,l) stands for the mth "frequency" component of the lth M-size DFT (N=ML). From this decomposition follows the basic idea of the algorithm i.e. a DFT of size N can be evaluated as L separate DFTs of size M together with a "phase compensation" according to (2).

This decomposition can be utilized also for the recursive DFT. The input sequence to be transformed is fed to the input of the L separate recursive DFT transformers. An input stage DFT block gets a new sample in every Lth instant and the time-indices of the samples of the lth block are (l modulo L) (due to the polyphase realization). In the same way, the parallel output channels of each transformer produce a new output sample in every Lth time instant. It also follows from the polyphase nature of the operation that the evaluation of (2) can be performed only after collecting of the L subsequent samples from the (m modulo M)th channels of the separate recursive DFT units. These samples will appear at the rate of the original sampling frequency, which means that a recursive evaluation of (2) would also be of considerable interest. By applying the "decimation-in-frequency" principle (Oppenheim and Schafer, 1975) a new set of recursive DFTs gives the solution, since (2) can be rewritten as

$$\begin{aligned} X_m &= \frac{1}{L}\sum_{l=0}^{L-1} \mathrm{DFT_M}(m,l) e^{-j\frac{2\pi}{N}ml} \\ &= \frac{1}{L}\sum_{l=0}^{L-1} \mathrm{DFT_M}(m,l) e^{-j\frac{2\pi}{LM}ml}, \end{aligned} \qquad (3)$$

from which

X_m= DFT$_L${ DFT$_M$(m,l)},
if m=0, M,...,(L-1)M
DFT$_L${ DFT$_M$(m,l)exp[-j(2π/N)l]},
if m=1, M+1,...,(L-1)M+1
•
•
•
DFT$_L${ DFT$_M$(m,l)exp[-j(2π/N)(M-1)l]},
if m=M-1, 2M-1, ...,LM-1

This means, that the evaluation of (2) requires a "phase compensation", and the operation of M DFT filter-banks of size L (see Fig.1).

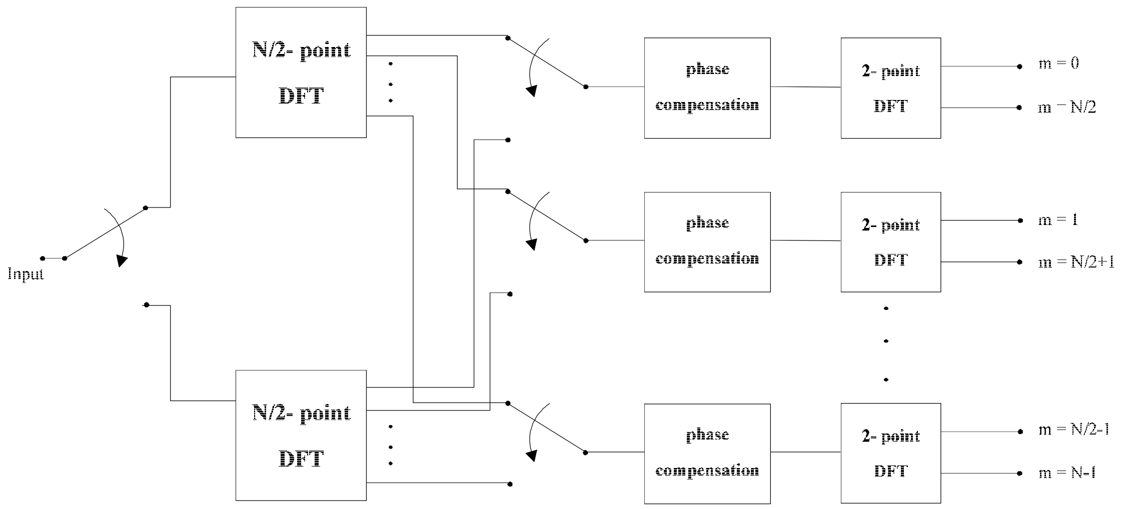

Fig. 1. Polyphase decomposition for M=N/2, and L=2.

If the numbers M and/or L are not prime the corresponding DFT transformers can also be decomposed similarly to the case of the FFT (see e.g. (Oppenheim and Schafer, 1975)). If N is an integer power of 2 the computational complexity can be significantly reduced. In this case the overall structure consists of uniform DFT blocks of size 2. These blocks can be evaluated by a simple division of 2, and an addition and a subtraction, so the number of complex multiplications are equal to the number of "phase compensations".

In the case of the polyphase DFT filter-banks the decomposition is quite similar, the slight difference is due to the fact that within the filter-bank structure in contrast to Fourier coefficients, Fourier components are derived. It is interesting to note that with the introduction of additional complex modulators/demodulators the DFT transformer and filter-bank building blocks can be combined within the overall structure.

If the DFT transformer/filter-bank blocks utilized within this decomposition are not elementary 2*2 DFT units, then for practical implementation it worth mentioning the observer-based approach (Hostetter, 1980; Péceli, 1986; Péceli, 1989 Padmanabhan and Martin, 1991), as well. This observer structure can be used as multi-purpose building block for different signal processing operations. With such blocks the polyphase recursive DFT structure can be generalized towards IIR filter-banks (Várkonyi-Kóczy, 1995b) .

The computational complexity of this polyphase DFT structure is rather similar to that of the tree-structures (Vetterli, 1983; Bruun, 1978) suggested for different orthogonal transformations. If it operates in block-oriented mode, i.e. all the DFT units are evaluated only once during their own period, and thus the whole DFT will give new results only in every Nth time instant, then the complexity figures will correspond to that of the FFT algorithms.

The decomposition technique described above can be utilized also for implementing arbitrary orthogonal transformations. A possible proof can be derived from (Vetterli, 1983). Beside the recursive DFT, the recursive Walsh-Hadamard transformation can play an important role in practical applications due to its simplicity. As it will turn out from the next Section this latter can be successfully utilized also in the case of the adaptive Fourier analyzer.

III. THE NEW ADAPTIVE FOURIER ANALYZER

The AFA described in (Nagy, 1992; Nagy 1993) applies the general transformer structure suggested in (Péceli, 1986), where for the case of Fourier analysis the SIMO filter bank is a set of parallel complex demodulator/modulator pairs with a two-channel discrete integrator in between together with a common feedback resulting in a recursive DFT structure. Two-channel discrete integrators are applied: one channel for the real part and another for the imaginary part of the modulated signal. This structure produces Fourier coefficients at the output of the discrete integrators, and Fourier components at the output of the complex modulators. The frequency adaptation algorithm proposed in (Nagy, 1992) is based on the phase variation of the Fourier coefficient related to the fundamental frequency, and like a PLL, directly controls the actual frequency of the complex demodulator/modulator pairs. The very same frequency adaptation algorithm can be used if instead of the demodulator/modulator pairs and the discrete integrators a completely equivalent resonator set is applied. The only difference is that in the case of resonators the phase variations of the coefficients aren't directly accessible. An additional complex demodulation is needed to get the proper form of information for the one-step frequency prediction algorithm.

The recursive DFT in its original form is an FIR filter-bank which becomes an IIR one if the modulating frequencies differ from the frequencies dictated by (1) unless a proper phase compensation is applied. This phase compensation can be calculated from an explicit formula (see e.g. (Péceli, 1989)), however, its real-time evaluation requires intensive calculations if N is large. For this very reason the AFA structure in (Nagy, 1992) attempts to approximate the phase relations of the DFT by applying always as many channels as many can be accommodated within the frequency range up to the one half of the sampling frequency. This means that it is a structurally adaptive system with an actual order depending on the actual ratio of the sampling frequency and of the basic frequency of the input signal. This approach is proved to be very efficient and robust (Nagy, 1992; Nagy 1993).

With the application of the adaptive "frequency sampling" method (Bitmead and Anderson, 1981), the polyphase decomposition described in Section II can be utilized also adaptive filtering applications. The AFA is a special adaptive filter-bank, where the center frequencies of the filter channels are harmonically related, and there are as many channels as many can be accommodated within the frequency range up to the one half of the sampling frequency. This behaviour can be generated by the fast DFT structure in such a way that the size of the DFT (N) is determined by the smallest possible value of the fundamental frequency to be detected, and to this transformer a set of adapted linear combiners is connected. At the output of the linear combiners the separated signal components will appear. The adaptation consists of two different types of calculations. The first one is the iterative determination of the fundamental frequency. This can be done following the algorithm described in (Nagy, 1992; Nagy, 1993). According to this the actual value of the basic harmonic's angular frequency is calculated recursively from the previous one by adding a correcting term, which equals to the phase variation of the Fourier coefficient (related to the fundamental frequency) divided by the actual order (N_1) of the system.

The second step is the calculation of the coefficients for the linear combiners. The filters to be implemented by the recursive DFT and the linear combiners can be characterized by the following transfer function:

$$H_m(z) = K_m \cdot \prod_{n=0,n\neq m}^{N_1-1}\left(1 - z_n z^{-1}\right), \qquad (4)$$

where $m = 0,1,\dots,N-1$, $z_0 = 1$, $z_{N_1/2} = -1$, $z_n = e^{jn\Phi_0}$, $n = 1,2,\dots N_1-1$, $n \neq N_1/2$. Φ_0 stands for the relative angular frequency of the basic harmonic. The K_m serves as normalizing factor, which can be derived by evaluating (4) at $z = z_m$. N_1 is the actual "order" of the linear combiners in such a way that $N_1\Phi_0 < 2\pi$. The weights for the linear combiners are calculated by "sampling" (4) in the frequency domain at the locations dictated by the Nth roots of unity. This latter requires only the evaluation of some formulas derived from (4), however, these calculations become very intensive if N is large. The "on-line" calculation of the weights can be avoided by a proper tabulation for the frequency range to be covered. For not tabulated frequencies simple (possibly linear) interpolation techniques can be applied. It is important to note that these AFA filters are FIR filters producing minimum response time. The frequency-sampling technique can be utilized in the case of other discrete transformations, as well. As it was already mentioned at the end of Section II, the fast recursive Walsh-Hadamard Transformation (WHT) can be also a good candidate for the transformer part in this new AFA. This change affects the weights of the adaptive linear combiners, however, the modification required is easy to calculate, and finally tabulate, if needed.

The outputs of the linear combiners can be calculated in every time instant, however, this is not always necessary for every channel. The whole system can be operated both in sliding-window, and block-oriented modes. For this latter the outputs are calculated only in every Nth time instant, i.e. only once for each block. This latter can be acceptable if the system is "tuned" by an external frequency signal. In the adaptive case the phase calculation of the basic harmonic is critical, however for slowly varying signals slower adaptation is also acceptable. The original form of the AFA (Nagy, 1992; Nagy, 1993), due to its feedback structure, requires a complete evaluation in every time instant. The necessary computational power is determined by the maximum value of N_1 , which corresponds to the order of the system to be implemented. This recursive, and adaptive system is replaced here by a fast, and recursive transformer structure ($N \geq N_1$) with computational complexity proportional to that of the FFT, and by a set of linear combiners. The weights of the linear combiners can be calculated on-line, or tabulated for the frequency range to tracked. This new system can be advantageous if not all the outputs are needed in every time instant, minimum response time is required, and if both the true DFT and the AFA outputs are to be calculated. As an illustration of the proposed method, on Figures 2, 3, and 4 a simple example is presented. A triangular

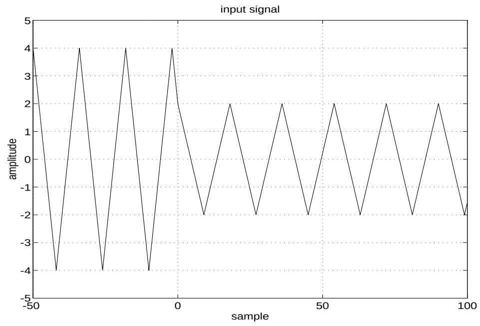

Fig. 2. Triangular waveform input signal: Frequency and amplitude modified at zero sampling time.

waveform consisting of N = 16 samples is modified at time instant zero to another one with smaller magnitude and frequency. The frequency change is approximately 10% (see Fig. 2). For the samples before the modification an N = 16 point fast recursive DFT (RDFT) produces outputs as it can be seen on Fig. 3. On the left-hand side of the individual plots, following a 16-point transient, the corresponding signal components can be observed. After the modification at time instant zero a 16-point transient comes again, and the subsequent samples represent (as it is expected) "composite" waveforms. Fig. 4. gives the idea of the tuning or adapting the frequency. Here a fixed adaptive filter-bank is operated for N = 16. Before the modification, the AFA produces the very same outputs as the RDFT, because their fundamental frequencies are equal. The modification of the input signal is followed by a one-step change of the basic component's angular frequency. This change directly affects the weights of the adaptive linear combiners, and results in the appropriate separation of the signal components.

IV. CONCLUSIONS

In this paper a new Adaptive Fourier Analyzer has been described. The frequency adaptation algorithm of this new AFA is the same as that of the previous one (Nagy, 1992), but the signal processing part has a more efficient formulation. The first part of this formulation is the utilization of the fast recursive Fourier transformation, while the second one is the implementation of adaptive frequency-sampling filters. These latter are bandpass filters with harmonically related center frequencies. The frequency of the basic harmonic component is calculated by recursive adaptation, or can be tuned by an external frequency signal. The weights of the frequency-sampling filters are given by explicit formulas, or can be tabulated for the signal processor at least for those basic frequencies, which are expected during operation. Computer simulations has shown the proposed AFA structure to operate correctly.

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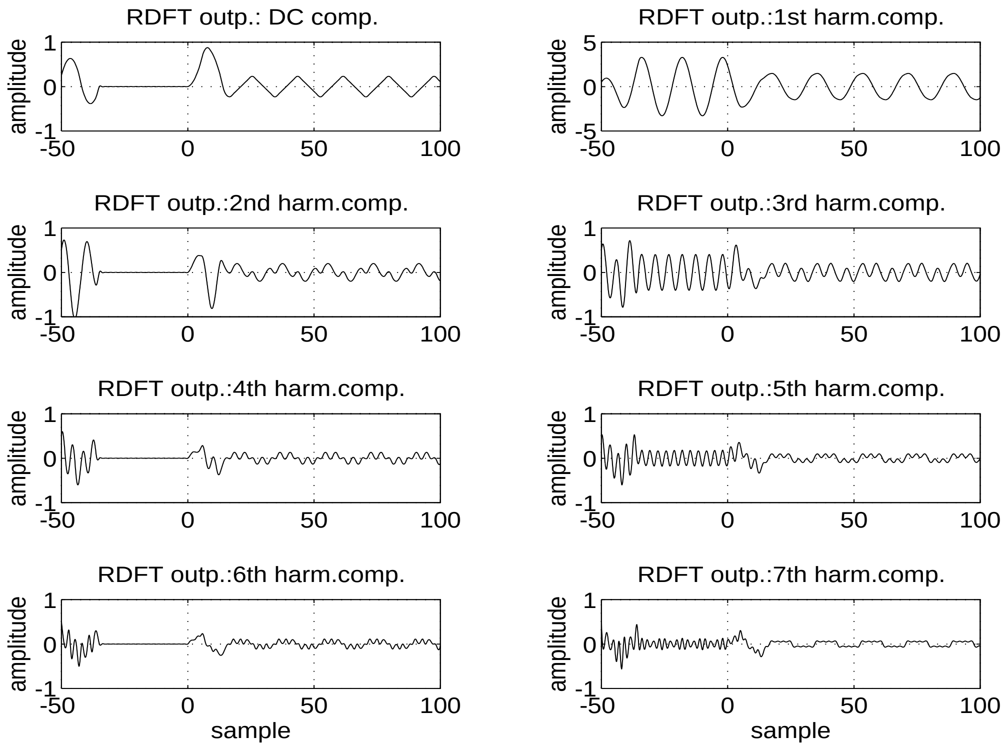

Fig 3. Fourier components on the outputs of the recursive DFT unit for a triangular input signal modified at time instant zero.

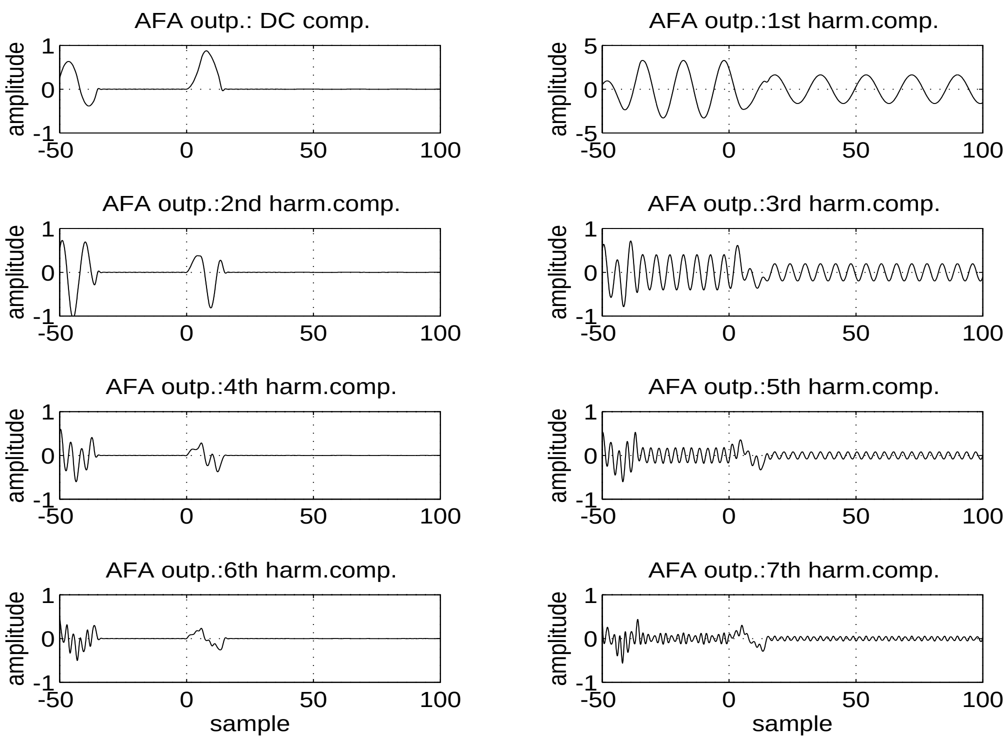

Fig. 4. Fourier components on the outputs of the new AFA unit for a triangular input signal modified at time instant zero.