Physics-based Sound Synthesis of Stringed Instruments Including Geometric Nonlinearities

Original Contributions of the Ph.D. Thesis

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1 Introduction and Research Goals

The basic idea of physics-based sound synthesis is that it models the sound generation mechanism of the instrument rather than the generated sound itself, which is more common in sound synthesis. Despite that research on physics-based sound synthesis is going on for three decades, its commercial application is still quite rare, mostly because of its higher computational complexity compared to signal modeling. However, by the increase of computational power and the appearance of better models it is quite probable that physics-based sound synthesis will be able to compete with the most common signal modeling technique, namely, sampling synthesis. Sampling synthesis is based on playing back the recorded samples of instrument sounds. A serious shortcoming of sampling synthesis is that it cannot model the interaction of the different parts of the instrument (e.g., the coupling of different strings). Moreover, all the variations of a single note has to be stored that can be generated by the musician (different bow velocity, bow force, etc.). These problems are automatically avoided in physics-based sound synthesis, where the model blocks correspond to the main parts of the instrument (in the case of string instruments: excitation, string, instrument body). The parameters of the model are physically meaningful (e.g., string length, bow velocity), therefore the control of the virtual instrument is straightforward. A further advantage of physics-based sound synthesis is that it can provide useful information for the acousticians about which are the most important phenomena during sound production and how would the sound of the instrument change by varying its physical properties.

The first step of physics-based sound synthesis is to understand how the instrument works, that is, the equations describing the main parts of the instrument and the interactions of the different parts have to be revealed. Naturally, most of this knowledge is obtainable from the literature, as musical acoustics has a long tradition. However, for some specific parts of the instrument model further investigations are necessary. The resulting precise instrument model can be directly used for sound synthesis after spatial and temporal discretization. However, the required computational complexity of such a model is usually too high for real-time implementation. Therefore, efficient sound synthesis algorithms have to be developed by neglecting the less important features of the precise model. For that, one has to estimate which are those phenomena that are less relevant in producing the characteristic sound of the instrument.

The topic of this thesis is the modeling of stringed instruments. The most important part of these instruments is the string, as the string generates the periodic vibration in the sound. The equation describing the ideal, infinite string is the wave equation

$$\mu \frac{\partial^2 y}{\partial t^2} = T_0 \frac{\partial^2 y}{\partial x^2}, \tag{1}$$

where y is the transverse displacement, x is the position along the string, t is the time, T is the tension, and μ is the mass per unit length [Morse and Ingard, 1968]. In real strings losses and dispersion also occur, which can be modeled by adding further terms. The solution of the wave equation can be calculated by spatial and temporal discretization, i.e., by substituting the derivatives with differences. This is the finite difference method [Hiller and Ruiz, 1971].

A much more efficient approach to string modeling is the digital waveguide [Smith, 1992]. The time-domain solution of the wave equation is the superposition of two functions

$$y(x,t) = y^+(t - x/c) + y^-(t + x/c), \tag{2}$$

where y^+ and y^- can be considered as two traveling waves, which retain their shape during their movement. The function y^+ is the wave going to the right and the function y^- is the wave going to the left direction, and c is the propagation speed. If the spatially sampled values of the two components (y^+ and y^-) are stored in two vectors, then the next state can be computed by shifting the two vectors to the right and to the left. This corresponds to two delay lines, which can be efficiently implemented by circular buffers. The losses and dispersion of the string are modeled by a digital filter inserted between the delay lines.

The third most common string modeling technique is modal synthesis, where the motion of the string modes are computed and the shape of the string is calculated by the summation of these modes as

$$y(x,t) = \sum_{k=1}^{\infty} y_k(t) \sin\left(\frac{k\pi x}{L}\right) \tag{3}$$

where $\sin(k\pi x/L)$ is the modal shape of mode k and L is the length of the string. The instantaneous amplitudes of the modes are given by, which are typically exponentially decaying sinusoidal functions implemented by second-order resonators in discrete-time.

The string gains energy from the excitation, which can be impulse like (plucking, striking) or continuous (bowing). It is common for all the cases that the interaction of the string and the exciter is bidirectional, i.e., the excitation force is a function of string shape [Fletcher and Rossing, 1998]. Modeling of the excitation is carried out by the discretization of the (generally zero dimensional) differential equation of the excitation. As the excitation is nonlinear in most of the cases, the discretization is nontrivial and often leads to numerical instabilities.

The string cannot efficiently radiate, because its radiation impedance is not in the same order as the impedance of the air. The role of the instrument body is providing an impedance match, thus, increasing the efficiency of sound radiation. The most time-consuming operation in physics-based sound synthesis is body modeling, because here the calculation of a two- or three-dimensional vibration is necessary, contrary to the string (one dimension) and the excitation (zero dimension). Therefore, it is common to model the effect of the body as a force-pressure transfer function instead of a precise physical model.

One part of the research goals belongs to existing modeling structures. The most common type of string modeling techniques is the digital waveguide modeling, where the decay times of the partials generated by the model are determined by the loss filter [Smith, 1992]. My goal was to develop robust and simple filter design algorithms that minimize the deviation of the resulting and prescribed decay times of the partials. Other such field is increasing the efficiency of physics-based models, as their heavy computational load is their major shortcoming. Here I have concentrated on efficient modeling of beating and two-stage decay (an amplitude modulation of the partials), which is coming from the coupling of strings, and on efficient instrument body modeling. In the case of nonlinear excitation, my goal was to develop a method for avoiding numerical instability that is simpler than existing techniques.

The second part of the thesis is about the nonlinear vibration of strings. The work concentrates on the geometric nonlinearities of strings, while the nonlinearity of string material is not investigated. Geometric nonlinearity appears on the string because the string length cannot be assumed constant above a certain amplitude of vibration. Therefore, the tension also varies leading to the excitation of longitudinal motion and to the appearance of new transverse components. The investigation of the phenomenon is necessary because the phantom partials [Conklin, 1999] and longitudinal modes present in real instruments cannot be modeled by existing string models. As the generation mechanism of phantom partials has not been explained, I have aimed at developing a general theory that can both help in the understanding of the phenomenon and can found the base of efficient sound synthesis algorithms. After the development of the precise model, my goal was to investigate how the existing string modeling techniques can be made capable of modeling the phenomenon and which simplifications can be applied for efficient implementation.

2 Research Methods

The main goal of my research is realistic sound synthesis. Therefore, I have aimed at developing such models that include only those parts which are absolutely necessary for a given sound quality. As an example, it is not investigated how the different parts of the instrument body vibrate if it is sufficient to know the transfer function of the body for sound synthesis. In musical acoustics we are mostly interested in the frequencies, amplitudes, and decay times of the components that build up the tone. Consequently, parameter estimation of the models is carried out in a way that these parameters of the synthesized tone should be near to the corresponding parameters of real tones. Generally, I have aimed at the simplification of both modeling and parameter estimation methods. For example, I have rejected those ideas where the efficiency of a method could be slightly increased at the expense of significantly more complex parameter estimation. It is a general feature of these models that they are adaptable to the fields of linear system theory and digital signal processing, which are well known for electrical engineers.

In digital filter design it is usual to minimize the deviation of the transfer function of the filter from a target specification. This cannot be done in the case of digital waveguide modeling, as the errors in decay times are of a nonlinear function of magnitude error. Therefore I have applied a special weighting in filter design. The weighting is a function of the magnitude specification and not of the frequency, which is more common in filter design. For designing the most widely used low order (one-pole) loss filter, I have used the weighted polynomial regression, which is one of the basic methods in the field of parameter estimation.

Multi-rate techniques are used for increasing the efficiency of signal processing algorithms. In methods applying the multi-rate technique the various parts of the system run at different sampling rates, depending on the requirements. I have applied the multi-rate approach for excitation modeling, as in this case the numerical stability can be maintained by increasing the sampling frequency of the excitation model, while for the rest of the model an increased sampling frequency is not necessary. For modeling beating and two-stage decay, only a small number of low frequency components should be modeled besides the basic string model. This allows that these low frequency components are generated by second order resonators running at a lower sampling rate. In the impulse response of instrument bodies the low modes are decaying slower than the high ones. Moreover, the ear is more sensitive to the parameters of the individual modes at low frequencies. Therefore, it is reasonable to model the low frequency part of the body response by a long filter running at lower sampling rate, while the high frequency part by a low order filter running at the sampling rate of the system.

For modeling the geometric nonlinearities of real instrument strings, the existing models that calculate the string response for the first (or first few) partials cannot be applied. Therefore, I have started from the general nonlinear wave equation of strings [Morse and Ingard, 1968] and proposed such simplifications that allow the use of methods known in linear system theory (transfer function, convolution). Namely, only the transverse to longitudinal coupling is considered, while the effect of the longitudinal polarization on the transverse one is neglected. This leads to two linear partial differential equations, where the input of the longitudinal equation is a nonlinear function of the output of the transverse one. It has been found that the model applying these simplifications describes the phenomenon correctly for most of the string instruments. I have applied the modal decomposition for both the transverse and longitudinal motion (as in Eq. (3)). This way, the partial differential equation (the wave equation) is decomposed into a set of second-order ordinary differential equations that describe the evolution of the different modes. Once the impulse responses of these modes are known, their response can be computed for arbitrary input by time-domain convolution. Accordingly, the motion of the transverse modes can be computed for a given external force (striking, plucking, etc.) by linear system theory. Then, the transverse modes determine the input of the longitudinal ones through a simple nonlinear function. When the inputs of the longitudinal modes are known, their response is again computed by convolution. I have decomposed the string tension into a spatially constant (but temporally varying) part and into a space- and time dependent part. This is beneficial because the effects of spatially constant tension variation are well known, and these results can be directly applied. Moreover, the decomposition of tension results in the simplification of the equations describing the string motion.

For sound synthesis of geometric nonlinearities, I have chosen the combination and extension of already existing, well-known string modeling techniques. This has the advantage that an already existing modeling architecture can be simply extended by the components required for nonlinear modeling. As the propagation speed of longitudinal waves is significantly higher compared to that of the transverse ones, only a few (ca. ten) longitudinal partials are in the audible frequency range. I have used the modal synthesis for modeling the longitudinal components because in those cases, where only a few modes have to be modeled, the modal synthesis is the most efficient approach. This is in a good accordance with the theoretical model developed for geometric nonlinearities, which is based on the modal decomposition, too. I have investigated the applicability of the three main approaches (finite difference, digital waveguide, modal approach) for modeling the transverse vibration, with respect to how the excitation force of the longitudinal modes can be computed. Different modeling alternatives are given by the successive simplification of the model, which differ in the accuracy of modeling and in the required computational complexity.

3 Original Contributions

This section summarizes the main results of the thesis in the form of scientific statements.

Statement 1: I have developed new methods for decay time-based design of loss-filters for digital waveguides.

1.1: I have proposed a polynomial regression-based method for designing one-pole loss filters, which is the most common type of loss filters. I have derived a formula for the decay time of a digital waveguide using a one-pole loss filter, and I have established a relationship between the parameters for the one-pole filter and the differential equation of the lossy string.

1.2: I have developed a robust and simple method for designing higher-order loss filters, which minimizes the decay time error through a magnitude-dependent weighting function. The weighting function is derived from the first-order Taylor series approximation for the decay time as a function of filter magnitude.

Statement 2: I have suggested the application of multi-rate techniques for increasing the efficiency of string instrument models.

2.1: I have proposed a new method for maintaining numerical stability within the excitation model. According to the method, the excitation model should operate at a higher (e.g., double) sampling rate than the rest of the instrument model.

2.2: I have shown that the beating and two-stage decay effects can be efficiently modeled by running a few resonators in parallel with the basic string model (e.g., a digital waveguide). The method models the phenomenon only for those partials that are dominated by the effect. The resonators run at a sampling rate lower than that of the string model, which results in considerably lower computational complexity than methods developed earlier.

2.3: I have proposed the multi-rate approach for modeling the force-pressure transfer function of the instrument body. In the lower part of the audible frequency range the body is modeled by a high-order filter running at one fourth or one eighth of the sampling frequency, while for high frequencies a low-order filter approximates the body response. The method requires significantly lower computing power compared to traditional filters, while the degradation in sound quality is marginal.

Statement 3: I have developed a comprehensive model for the nonlinear vibration of metal strings that can be efficiently used for sound synthesis. The model takes into consideration the coupling of transverse and longitudinal polarizations.

3.1: I have introduced a classification for the nonlinear behavior of strings, which estimates from the physical parameters of the string and from the amplitude and frequency content of the transverse vibration which phenomena dominate the vibration. This "nonlinearity map" clearly shows the similarities and differences between the various cases.

3.2: I have determined the closed solution for the nonlinear differential equation of the string for the case where the tension on the string is spatially nonuniform, but the variation of tension has a negligible effect on the transverse vibration. This approximation is valid for highly stretched metal strings used in most string instruments. The proposed modal model describes the free vibration of longitudinal modes and the generation of phantom partials jointly.

3.3: I have derived a relationship between the modal model and the uniform tension approximation by decomposing the tension into a spatially uniform and a space-dependent part. I have shown that the spatially uniform tension approximation, which is the most widely used approach, is a special case of the proposed modal model.

Statement 4: I have extended the most common types of physical string models by making them capable of modeling the longitudinal vibrations, too.

4.1: I have proposed various composite string models where the response for longitudinal modes is calculated by nonlinearly excited second-order resonators. For computing the transverse vibration, both the finite-difference and modal-based approaches are appropriate. The method is capable of modeling the bidirectional coupling of transverse and longitudinal polarizations. The proposed models require significantly lower computational cost than the techniques that compute both polarizations through a finite-difference scheme.

4.2: I have shown that dispersive digital waveguides cannot be used for computing the excitation force of the longitudinal polarization in their original form. The problem can be avoided through distributing the dispersion filter, at the expense of increased computational complexity.

4.3: I have proposed ”physically informed” techniques for modeling the longitudinal vibration, which are even more efficient than the above proposed ones. These use a physics-based transverse string model extended by a signal model whose parameters are computed from the physical parameters of the string.

4 Applications of the Results

The results of this work can be grouped into two categories. The first group correspond to sound synthesis applications. The results corresponding to Statement 1 and 2 can be directly used for enhancing the quality of physics-based synthesizers and reducing their computational complexity. The methods provided in Statement 4 are applicable for extending existing string models for making them capable of modeling the nonlinearly generated longitudinal vibrations.

The second group of the results is connected to Statement 3. As this statement is about the theory of geometric nonlinearities, it can be considered as a fundamental research. Therefore, it can be applied by acusticians and instrument makers to gain a better understanding about string vibration. Moreover, the provided methodology (the modal approach and the decomposition of tension) can be used as a starting point for further investigations about the effect of string terminations and the nonlinear coupling of two transverse polarizations.

5 Publications Related to the Ph.D. Thesis

5.1 Journal Papers

[1] B. Bank and L. Sujbert, “Generation of longitudinal vibrations in piano strings: From physics to sound synthesis,” *J. Acoust. Soc. Am.*, vol. 117, pp. 2268–2278, April 2005.

[2] B. Bank, F. Avanzini, G. Borin, G. De Poli, F. Fontana, and D. Rocchesso, “Physically informed signal-processing methods for piano sound synthesis: a research overview,” *EURASIP J. on Appl. Sign. Proc.*, vol. 2003, pp. 941–952, September 2003.

[3] B. Bank and V. Välimäki, "Robust loss filter design for digital waveguide synthesis of string tones," *IEEE Sign. Proc. Letters*, vol. 10, pp. 18–20, January 2003.

[4] B. Bank, J. Márkus, A. Nagy, and L. Sujbert, "Signal- and physics-based sound synthesis of musical instruments," *Periodica Polytechnica, Ser. Electrical Engineering*, vol. 47, no. 3–4, pp. 269–295, 2003.

5.2 International Conferences

[5] B. Bank and L. Sujbert, "Efficient modeling strategies for the geometric nonlinearities of musical instrument strings," in *Proc. Forum Acusticum 2005*, (Budapest, Hungary), August 2005.

[6] B. Bank and L. Sujbert, "A piano model including longitudinal string vibrations," in *Proc. Conf. on Digital Audio Effects*, (Naples, Italy), pp. 89–94, October 2004.

[7] B. Bank and L. Sujbert, "Modeling the longitudinal vibration of piano strings," in *Proc. Stockholm Music Acoust. Conf.*, (Stockholm, Sweden), pp. 143–146, August 2003.

[8] B. Bank, G. De Poli, and L. Sujbert, "A multi-rate approach to instrument body modeling for real-time sound synthesis applications," in *Proc. 112$^{\text{th}}$ AES Conv., Preprint No. 5526*, (Munich, Germany), May 2002.

[9] B. Bank, "Accurate and efficient modeling of beating and two-stage decay for string instrument synthesis," in *Proc. MOSART Workshop on Curr. Res. Dir. in Computer Music*, (Barcelona, Spain), pp. 134–137, November 2001.

[10] F. Avanzini, B. Bank, G. Borin, G. De Poli, F. Fontana, and D. Rocchesso, "Musical instrument modeling: The case of the piano," in *Proc. MOSART Workshop on Curr. Res. Dir. in Computer Music*, (Barcelona, Spain), pp. 124–133, November 2001.

[11] B. Bank, V. Välimäki, L. Sujbert, and M. Karjalainen, "Efficient physics-based sound synthesis of the piano using DSP methods," in *Proc. 10$^{\text{th}}$ Eur. Sign. Proc. Conf.*, (Tampere, Finland), pp. 2225–2228, 2000.

[12] B. Bank, "Nonlinear interaction in the digital waveguide with the application to piano sound synthesis," in *Proc. Int. Computer Music Conf.*, (Berlin, Germany), pp. 54–58, 2000.

5.3 Hungarian Conferences

[13] B. Bank and L. Sujbert, "Longitudinal waves in piano strings: Do we need to model them?," in *Proc. 10*$^{\text{th}}$ *BUTE Mini-Symposium*, (Budapest, Hungary), pp. 42–43, February 2003.

[14] B. Bank and L. Sujbert, "A multi-rate approach to piano soundboard modeling," in *Proc. 9*$^{\text{th}}$ *BUTE Mini-Symposium*, (Budapest, Hungary), pp. 44–45, February 2002.

[15] B. Bank and L. Sujbert, "Model based synthesis of piano sound," in *Proc. 8*$^{\text{th}}$ *BUTE Mini-Symposium*, (Budapest, Hungary), pp. 52–53, February 2001.

5.4 Technical Report

[16] B. Bank, "Physics-based sound synthesis of the piano," Master's thesis, Budapest University of Technology and Economics, Hungary, May 2000. Published as Report 54 of HUT Laboratory of Acoustics and Audio Signal Processing, URL: `http://www.mit.bme.hu/~bank/thesis`.

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