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System Identification in a Real World

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***Abstract*—In this paper we discuss how to identify a mathematical model for a (non)linear dynamic system starting from experimental data. In the initial step, the frequency response function is measured, together with the properties of the disturbing noise and the nonlinear distortions. This uses nonparametric preprocessing techniques that require very little user interaction. On the basis of this information, the user can decide on an objective basis, in an early phase of the modelling process, to use either a simple linear approximation framework, or to build a more involved nonlinear model. We discuss both options here: i) Identification of linear models in the presence of nonlinear distortions, including the generation of error bounds; and ii) Identification of a nonlinear model. For the latter, a double approach is proposed, using either unstructured nonlinear state space models, or highly structured block oriented nonlinear models. The paper is written from a users perspective.**

***Index Terms*—linear and nonlinear modeling, best linear approximation, nonlinear state space models, block oriented models**

I. INTRODUCTION

The control community makes use of mathematical models intensively to design high-quality controllers. These mathematical models are often obtained from first principles, making use of detailed knowledge about the physical laws that describe systems. The major advantage of such an approach is that it provides detailed physical models that give much physical insight into the problems studied, however, at the cost of a long, difficult, and expensive modeling process. Alternatively, a data-driven approach can be followed, where all information is retrieved from experimental data. These models are called black box models, and it is usually less expensive and less time-consuming to get them. System identification theory addresses the need for good methods to estimate mathematical models from noisy data.

Nowadays, mature and inexpensive tools are available to derive good models for linear dynamic systems [1], [2], [3]. Unfortunately, many real systems are nonlinear and require more advanced modelling tools [4], [26]. Building nonlinear models is much more involved, more expensive, and more time consuming when compared to linear modelling. For that reason it is important to decide at the beginning of the design procedure, whether a simplified linear model or a full nonlinear model is most appropriate. In this paper, we will guide the reader towards a data-driven solution for this problem.

The paper makes three main contributions. In the first part of this presentation it will be shown that it is possible to detect, qualify, and quantify the presence of nonlinear distortions using simple nonparametric preprocessing methods that require very little user interaction. On the basis of this information it can be decided whether an inexpensive linear model will be good enough for the application in mind, or that a more elaborated nonlinear modeling effort should be made. In the second part, the impact of nonlinear distortions on the linear identification framework will be discussed, so that the user gets a better understanding of the potential risks and problems when linear models are used in a nonlinear setting. Eventually, in the third part of the paper, nonlinear modeling strategies will be discussed, considering unstructured and highly structured models.

In this paper, we will focus completely on the basic ideas and illustrate them with some figures and experimental results. We refer the reader for the full mathematical details and formal descriptions to the literature. We also add a set of guidelines to each section to help the reader making proper choices.

II. THE SYSTEM

Consider a dynamic (non)linear, time-invariant system:

$$y_0(t) = g(u_0(t)). \tag{1}$$

with $u_0(t), y_0(t) \in \mathbb{R}$ respectively the input and output signals. For simplicity we consider, without loss of generality, single-input single-output discrete time systems.

In this paper we assume that the input signal is exactly known, while the output measurements are disturbed by process and measurement noise $v(t)$:

$$y(t) = y_0(t) + v(t). \tag{2}$$

The class of nonlinear systems is very general. We will consider two overlapping sub-classes (See Figure 1): the fading memory (FM) systems and the nonlinear feedback (NLFB) systems.

Fading memory systems: Loosely spoken, the dependency of the output of a fading memory system on its past inputs decays towards zero [5] for inputs that are further in the past.

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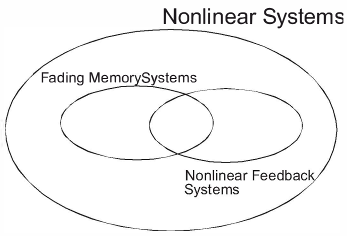

Figure 1. NL systems considered in this study..

We will make intensive use of the following properties of these systems:

- FM-systems have a unique steady state solution, and the response to a periodic input results in a periodic output with the same period.
- FM-systems can be arbitrary well approximated by a nonlinear open-loop representation (no nonlinear sub-system in a feedback loop). This leads to a Volterra representation of the system that can be considered as a (static) polynomial approximation that is extended with a fading memory [5], [6], [7].

FM-systems include hard nonlinear systems like saturation, clipping, and dead-zones. However, amongst others, hysteresis effects and a chaotic behavior cannot appear within this class of systems.

Nonlinear feedback systems: these systems have at least one nonlinear element that is captured in a feedback loop, and they have the following interesting properties used in this discussion:

- NLFB-systems can have multiple solutions for the same input signal. Which solution is actually present depends upon the initial conditions. This complex behavior can eventually lead to a chaotic behavior.
- In general, it is not possible to make an arbitrarily good approximation with open loop models (Volterra series).
- Some NLFB-systems are a FM-system on a restricted input domain. The fading memory property is in that case not a pure system property, it is conditioned on the class of inputs that is considered.

III. Nonlinear distortion analysis

As we explained in the introduction, the first step of the identification process is to determine whether we can use 'simple' linear models, or if we need the more 'complex' nonlinear models. This classification is performed using a distortion analysis. Since the presence and the level of nonlinear distortions is strongly dependent on the nature of the excitation signal, we first have to specify the class of the excitation. Next, we show that it is possible to measure the level of the nonlinear distortions directly with very little user interaction, for the class of fading memory systems.

A. Excitation signals

The choice of the excitation signal is extremely important in a nonlinear framework. The behavior of a nonlinear system depends not only on the power spectrum of the applied excitation signal, also its amplitude distribution has a strong impact [3]. In this paper we will use signals with a Gaussian amplitude distribution.

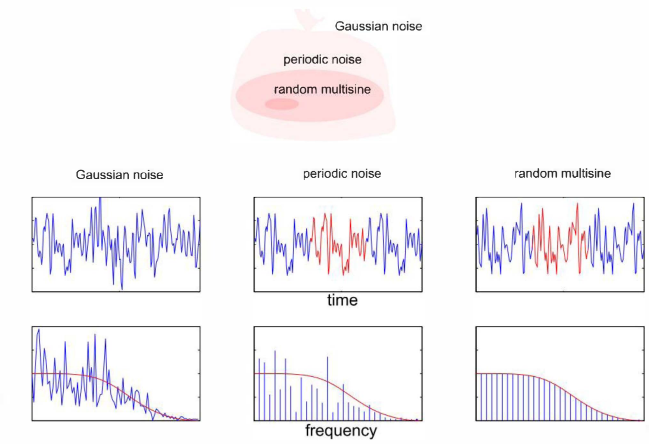

Figure 2. Examples of excitation signals: Top: time domain, Bottom: amplitude spectrum of the actual realization (blue) and the power spectrum (red); Left: Gaussian random noise, Middle: periodically repeated Gaussian noise, Right: random phase multisines.

Gaussian random noise excitations (Figure 2, left) are very popular among practicing engineers, because they seem to be simple to design. However, we prefer periodic excitations because these signals offer significant advantages to make a nonparametric nonlinear distortion analysis. A first possibility to generate a periodic signal is to periodically repeat a finite segment of a random noise sequence (Figure 2, middle). Using a random phase multisine [3], [8] it is possible to do much better. Consider the signal

$$u_0(t) = \frac{1}{\sqrt{N}} \sum_{k=-N/2+1}^{N/2-1} U_k e^{j(2\pi k f_0 t + \varphi_k)} \quad (3)$$

$$= \frac{2}{\sqrt{N}} \sum_{k=-N/2+1}^{N/2-1} U_k cos\left(2\pi k f_0 t + \varphi_k\right) \quad (4)$$

where $\varphi_{-k} = -\varphi_k$ and $U_{-k} = U_k$, $U_0 = 0$, and $f_0 = f_s/N = 1/T$. The sample frequency to generate the signal is f_s, and T is the period length of the multisine. The phases φ_k will be selected independently such that $E\{e^{j\varphi_k}\} = 0$, for example by selecting a uniform distribution on the interval $[0, 2\pi[$. The amplitudes U_k are chosen to follow the desired amplitude spectrum (Figure 2, right). In [9], a detailed discussion about the user choices and the properties of these signals is made. The major advantage of the random phase multisine is that it still has (asymptotically for sufficient large N) all the nice properties of Gaussian noise, while it also has the advantages of a deterministic signal: the amplitude spectrum does not show dips at the excited frequencies as the two other signals do (see Figure 2). At those dips, the measurements are very sensitive to all nonlinear distortions and disturbing noise.

User guidelines:

- Use random phase multisine excitations.
- The spectral resolution f_0 of the multisine should be chosen high enough so that no important resonances are missed [10]. Since $f_0 = 1/T$, it sets immediately the period length of the multisine. A high frequency resolution requires a long measurement time.
- The amplitude spectrum should be chosen such that the frequency band of interest is covered. The signal amplitude should be scaled such that it also covers the input amplitude range of interest.
- In the next section, it will be shown that nonlinear distortions can be easily detected by putting some amplitudes U_k equal to zero for a well selected set of frequencies.
- A detailed step-by-step procedure how to generate and process periodic excitations is given in Chapter 2 of [8].

B. Separation of the signal, the disturbing noise, and the nonlinear noise

In this paper we explain the basic principle of the nonlinear distortion analysis. We refer the reader to [3] for a theoretical analysis, and for extensions that are more robust with respect to non-idealities present in the setup (for example interaction of the actuator and the system).

The basic idea is very simple (see Figure 3): a multisine that excites a well selected set of odd frequencies (odd frequencies correspond to odd values of k in eq. (3)) is applied to the nonlinear system under test. Even nonlinearities show up at the even frequencies, odd nonlinearities are present at the odd frequencies, and become visible at the odd frequencies that were not excited (for example frequency 5 and 7 in Figure 3). By using a different color for each of these contributions, it becomes possible to recognize these in an amplitude spectrum plot of the output signal.

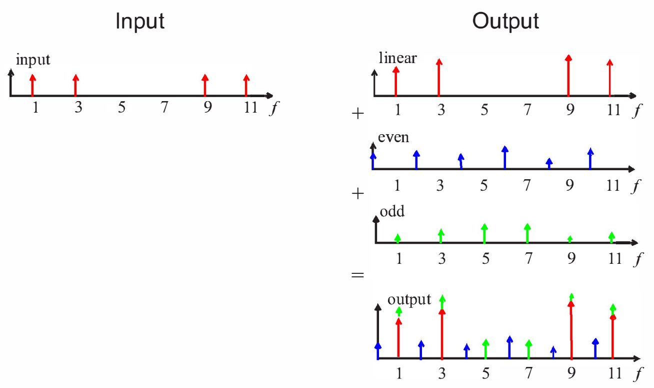

Figure 3. Design of a multisine excitation for a nonlinear analysis. Left: Selection of the excited frequencies; Right: from top to bottom: linear contributions, even and odd contributions, total output.

This method is experimentally illustrated on an electronic circuit that mimics a mass/damper/spring system with a hardening spring (see Figure 4, top), called the silverbox [3], [11], [23]. The circuit is known in the literature as a forced Duffing oscillator, and its properties are intensively studied. Although this is a nonlinear feedback system, it behaves as a fading memory system for sufficiently small input amplitudes. At the right side of the figure, we show not only the evolution of the nonlinear distortions as a function of the frequency, but also the level of the disturbing noise is shown. It can be seen that, for a low excitation level, we can detect the presence of odd nonlinear distortions around the resonance frequency. When the excitation level grows, the odd nonlinear distortions grow faster than the even ones, while the observed disturbing noise level remains almost the same. This figure is very informative for the modeller. For small excitation levels (left side of the figure), the nonlinear distortions are 30 dB below the linear contributions. In that case a linear model can be used if a moderate precision is sufficient. For higher excitation levels (right side of the figure), it is clear that the nonlinear distortions can no longer be neglected as the nonlinear distortions are as large as the linear contributions. In that case a full nonlinear model will be needed. It is important to realize that all this information is directly available from a simple nonparametric nonlinear analysis.

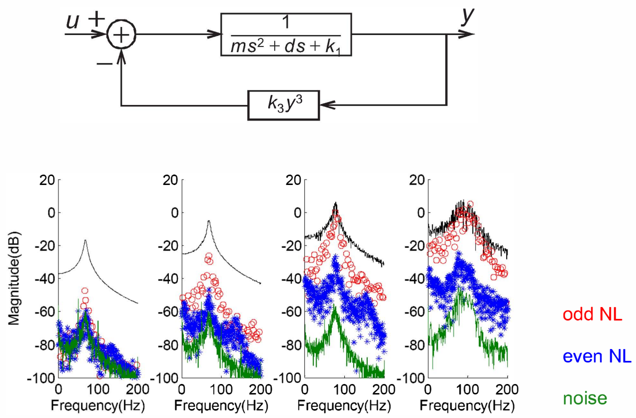

Figure 4. Detection of the level of the nonlinear distortions on a mass/damper/spring system with a hardening spring (simulated by a nonlinear electronic circuit) for different excitation levels. Top: most simple model for a hardening spring system (s denotes the Laplace variable). Bottom: Distortion analysis for an increasing excitation level (from left to right). The thin black line gives the output at the excited frequencies. The level of the even and the odd nonlinearities is given by the blue stars and the red circles respectively. The thin green line gives the disturbing noise level.

User guidelines:

- Design a multisine to detect the presence of nonlinear distortions following the guidelines of Section III-A. To do so, the even frequencies and a set of randomly selected odd frequencies should be put to zero. See [9] for a detailed discussion.
- Make a series of (steady state) measurements with varying amplitudes or offsets of the excitation signal that cover the amplitude range of interest, and make the nonlinear analysis. More advanced signal processing methods can be used to remove transient effects [19].
- If the nonlinear distortions are smaller than the specified level of accuracy of the model to be built, a linear design might be sufficient. This will lead to the best linear approximation of the nonlinear system. In the other case,

a more involved nonlinear model will be needed.

- Be aware that the best linear approximation varies in general as a function of the power spectrum and amplitude distribution of the excitation signal.
- A detailed step-by-step explanation to make a nonparametric nonlinear distortions analysis is given in Section 6.1 of [8].

IV. Linear identification in the presence of nonlinear distortions

In the previous section, it turned out that we have two possibilities to deal with nonlinear distortions: we can use either a linear approximation, or we can built a nonlinear model. In this section, the best linear approximation (BLA) will be studied. First, a short introduction to the concept is given. Next, we measure the frequency response function of the BLA and discuss how to obtain a parametric model. Finally, error bounds on the nonparametric and the parametric BLA-estimates are discussed.

A. The best linear approximation

The best linear approximation G_{BLA} of a nonlinear system is defined as that linear system whose output is as close as possible to the output of the nonlinear system. This is obtained by minimizing the mean squares error [3], [12], [13], [14], [9], [15]:

$$G_{BLA}(q) = \arg\min_{G} E\left\{ \left| y_0(t) - G(q)\, u_0(t) \right|^2 \right\} \qquad (5)$$

with q the shift operator for a discrete time model. In most applications, it is important to remove first the DC-value of the input and output signal. Similar expressions can be given for continuous time models. All expected values $E\{\}$ in this paper are taken with respect to the random input $u_0(t)$ (e.g. random phase multisine or random noise excitation).

The best linear approximation G_{BLA} depends on the amplitude distribution (e.g. Gaussian, uniform) and the power spectrum (RMS value and coloring) of the input [16], [17], [3].

The output of a nonlinear system that is driven by a random excitation (or an equivalent signal [9]) can be split in two classes of contributions, being the coherent contributions Y_B and the non-coherent contributions Y_S:

- *Coherent output*: The relation between the input $U_0(k)$ and the coherent nonlinear contributions $Y_B(k)$ is very similar to the input output behavior of a linear system, and can be also written like that:

$$Y_B(k) = G_B(k) U_0(k).$$

The transfer function $G_B(k)$ of that system depends on the input characteristics (e.g. power spectrum), but not on the actual realization of the random phases in (3). G_B contributes to the FRF of the BLA:

$$G_{BLA}(k) = G_0(k) + G_B(k),$$

where $G_0(k)$ is the transfer function of the underlying linear system (if it exists).

- *Non-coherent output*: The non-coherent output y_S accounts for the difference between the output of the best linear approximation and the actual nonlinear output. For random excitations, it is very difficult for an untrained user to distinguish the nonlinear noise $y_s(t)$ from the additive disturbing output noise $v(t)$. The nonlinear distortions are uncorrelated with $u_0(t)$ because they are the residuals of the solution of a least squares problem. However, $u_0(t)$ and $y_s(t)$ are mutually dependent as there exists a nonlinear relation between both signals, viz.

$$y_S(t) = y_0(t) - G_{BLA}(q)\, u_0(t).$$

An alternative representation of (1) is to write the noise free output $y_0(t)$ as the sum of its best linear approximation plus an error term [16], [17], [3]

$$\begin{aligned} y(t) &= y_0(t) + v(t) \\ y_0(t) &= G_{BLA}(q)\, u_0(t) + y_S(t). \end{aligned} \qquad (6)$$

In the frequency domain this expression becomes

$$\begin{aligned} Y(k) =& Y_0(k) + V(k) \\ =& G_{BLA}(k)\, U_0(k) + Y_S(k) + T(k) + V(k), \end{aligned} \qquad (7)$$

where the transients $T(k)$ represents the initial transients and leakage errors [18], [3]. From now on we assume that we measure under steady state conditions such that we can neglect the transient terms in (7) in what follows. Equation (7) becomes

$$Y(k) = G_{BLA}(k)\, U_0(k) + Y_S(k) + V(k). \qquad (8)$$

The power spectra of Y_s and V can be measured using the methods explained in Section III-B.

B. Nonparametric measurement of G_{BLA}

Exactly the same measurement techniques that were developed for the measurement of the frequency response function (FRF) of a linear system [3], [20] can be used to measure the FRF of the BLA. In Figure 5, the FRF that is experimentally measured on the silverbox (see Figure 4) is shown. We averaged the measurements over 50 realizations of the random phase input, to get a smoother measurement. By changing the excitation from one realization to the other, we reduce not only the disturbing noise, also the stochastic nonlinear distortions will be reduced.

Two observations can be made. The resonance frequency shifts to the right for increasing excitation levels, and the measurements become more noisy. Both effects are completely due to the nonlinear distortions. The impact of the distortions can be evaluated starting from the distortion analysis in Figure 4.

User guidelines:

- Measure the FRF $\hat{G}_{BLA}(k)$ and its variance $\hat{\sigma}^2_G(k)$ using multiple realizations of a random phase multisine, designed following the guidelines of Section III-B. All

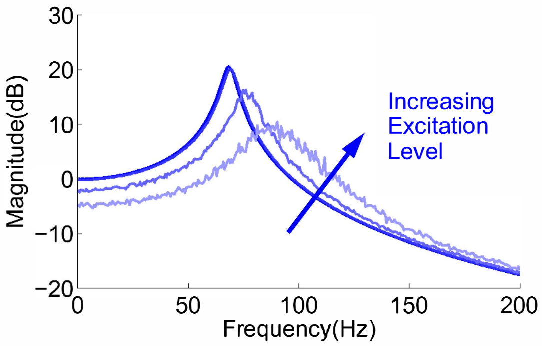

Figure 5. Measurement of the FRF of the silverbox: an electronic simulation of a mass/damper/spring system with a hardening spring. The system is excited with a random phase multisine at the same levels as in Figure 4.

the nonparametric expressions of the linear theory can be used.

- Averaging over multiple realizations reduces the impact of the disturbing noise and the stochastic nonlinearities Y_S. It results in a smoother estimate. However, it does not reduce the systematic contributions G_B of the nonlinear distortions. These result in a systematic contribution to the BLA that cannot be reduced using averaging techniques.
- How to measure the FRF $\hat{G}_{BLA}(k)$ and its variance $\hat{\sigma}_G^2(k)$ is explained in Chapters 3 and 6 of [8].

C. Parametric modeling of G_{BLA}

In many applications, a parametric transfer function model or state space representation of the system is needed. Starting from $\hat{G}_{BLA}(k)$ and $\hat{\sigma}_G^2(k)$, it is possible to obtain such a parametric model by minimizing the following weighted least-squares cost function that comes from the linear system identification theory [1], [2], [3], [16]:

$$V(\theta) = \frac{1}{F}\sum_{k=1}^{F}\frac{\left|\hat{G}_{BLA}(k) - G\left(\Omega_k, \theta\right)\right|^2}{\hat{\sigma}_G^2\left(k\right)}, \quad (9)$$

where Ω_k is the continous- or discrete-time frequency variable. It can be shown that the minimizer $G\left(\Omega_k, \hat{\theta}\right)$ of the cost function (9) is consistent (the estimate converges to the exact value as the number of data points tend to infinity). The linear system identification theory provides also a theoretical estimate of the variance of the estimated model.

However, a detailed study shows that this result is wrong in the presence of nonlinear distortions [21]. The actual sensitivity to the nonlinear distortions Y_S will be much higher than what is predicted by the linear theory. This leads to far too optimistic uncertainty bounds, under-estimation of the actual variance with a factor 7 (about 8 dB) or more occurs.

This is illustrated in Figure 6 on the identification of a Wiener-Hammerstein system (Figure 6, top). A Wiener-Hammerstein system consists of the cascade of a linear dynamic system, a static nonlinear system, and a linear dynamic system. It can be shown, that for Gaussian excitations,

$$G_{BLA}(k) = \alpha G_1(k) G_2(k),$$

with G_1, G_2 the transfer function of the first and second linear system, and α a constant that depends on the nonlinear system and the properties of the excitation signal. From this figure it can be seen that the actual observed error level in the simulations σ_{sim} is significantly larger than the expected level σ_{th} from the linear system identification theory.

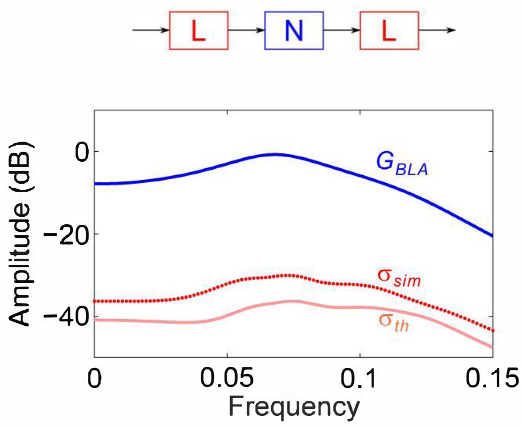

Figure 6. Parametric identification of a Wiener-Hammerstein system (top). The bottom figure shows the theoretical σ_{th} and the actually observed σ_{sim} standard deviation.

User guidelines:

- Measure the FRF $\hat{G}_{BLA}(k)$ and its variance $\hat{\sigma}_G^2(k)$ following the guidelines of Section IV-B and estimate the parametric model. Take care: while the uncertainty bounds of the linear theory could be safely used for non-parametric models, they are NOT valid for the parametric model. There exists, for the moment being, no simple theory to provide better error bounds.
- The BLA can also be directly estimated from the raw input-output data in the time- or in the frequency domain, using the classical linear framework.
- A detailed step-by-step procedure explaining how to identify a parametric estimate of the BLA is given in Chapter 7 of [8].

V. Nonlinear system identification

If the nonlinear distortions are above the error level that can be tolerated, the user has no choice and should start a more expensive nonlinear modeling procedure. In this paper, we comment on the use of unstructured models that are 'easy' to identify (the nonlinear state space models), and highly structured block-oriented models that are more difficult to retrieve from the experimental data (see Figure 9). We can only briefly discuss some modeling and identification issues. Again, we refer the reader to the literature for more information. However, most contributions in the literature deal with simple block oriented model structures. There are today no operational identification methods available for block-oriented models with a very complex structure.

A. *Unstructured nonlinear state space models*

There exist many possible approaches to model nonlinear systems [4], [31], [6], [26]. A very powerful nonlinear model is the nonlinear state space model (NLSS) (see Figure 7). It can be split in a linear and a static nonlinear part as shown in the figure.

$$x_+ = Ax + Bu + F(x,u)$$
$$y = Cx + Du + G(x,u)$$

Figure 7. Nonlinear state space model with a split in a linear (pink) and a static nonlinear (blue) part. x_+ denotes $x(t+1)$, al the other signals are evaluated at t.

To initialize the identification of such a model, we can retrieve the linear part, starting from the BLA estimate of the nonlinear system [11]. Next, the multivariate static nonlinear function can be retrieved while the linear part is fixed to its initial estimate [11], [22]. To model the static nonlinear functions, we can use multivariate polynomials, but also more advanced methods from the machine learning community are available, for example, neural networks or support vector machine modeling can be used [22], [23].

This modeling approach turns out to be very flexible, and it has been tested on many examples, including a few benchmark problems [24].

Here, we show some illustrative results on the silverbox that was discussed before [11], [23]. A good model should be able to describe new data that were not used during the estimation procedure. Such a data set is called the validation data. The validation of a NLSS-model for the silverbox is shown in Figure 8. The model has two state variables (order 2), and a multivariate polynomial of degree 3 is used for the static nonlinear part $F(x,u)$. From the figure, it is seen that the error on the simulated output is below 1% in the frequency band from DC to 150 Hz.

The major disadvantage of the NLSS modelling approach is that for systems with many states, the number of polynomial coefficients to be estimated grows combinatorially. This can lead to very complex models that give little insight to the user about the underlying structure. Retrieving this structure is a major drive to use block-oriented models, as will be discussed in the next section.

User Guidelines:

- A NLSS-model gives a (good) description of the input-output behavior for a wide class of nonlinear systems.
- NLSS-models provide little structural insight, especially for systems with many states.
- It is very important to collect experimental data that are rich enough (see User guidelines of Section III-A). We also advice to add a series of experiments to the estimation data that mimic well the signals that will be later applied. This is illustrated in an industrial case study that models the closing- and coupling phase of an industrial wet clutch [25].

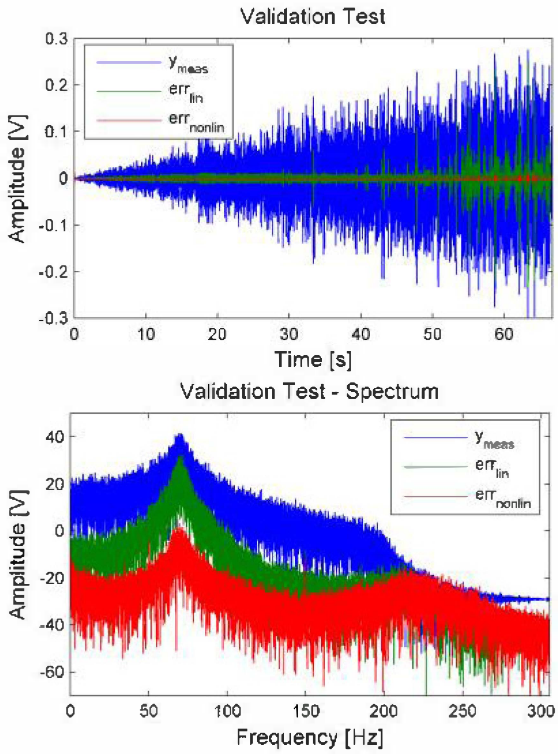

Figure 8. Validation of a NLSS-model of the silverbox. Top: time domain; Bottom: frequency domain. Blue: the measured output, green: the error of the BLA, red: the error of the NLSS-model

B. *Highly structured block-oriented models*

In order to avoid the structure loss, we can use block-oriented models [27], [29]. These consists of a set of connected linear dynamic blocks and static nonlinear blocks [28]. In Figure 9 we give a number of examples. The Hammerstein (H), Wiener (W), and Wiener-Hammerstein (WH) are the most intensively studied structures.

However, there is a strong need for more advanced structures in order to deal with a broad class of real systems. It is known already for a long time that parallel cascade structures can address some of these needs [7], [30], [31], only recently methods are proposed to identify parallel Wiener, parallel Hammerstein, and parallel Wiener-Hammerstein systems [32].

Also the identification of the nonlinear feedback structures is still a huge challenge, but recently interesting advances are discussed in [34], [35], [36]. The WH feedback system and the nonlinear LFR system in Figure 9 are examples of such systems. We refer the reader again to the references for more details on the available identification methods.

In this paper, we highlight some of the structural limitations of the different model classes to prevent that time is wasted on identifying models that are too simple to describe the system.

1) Single branch models W, H, WH: As mentioned before, single branch models are very popular in the literature, mainly

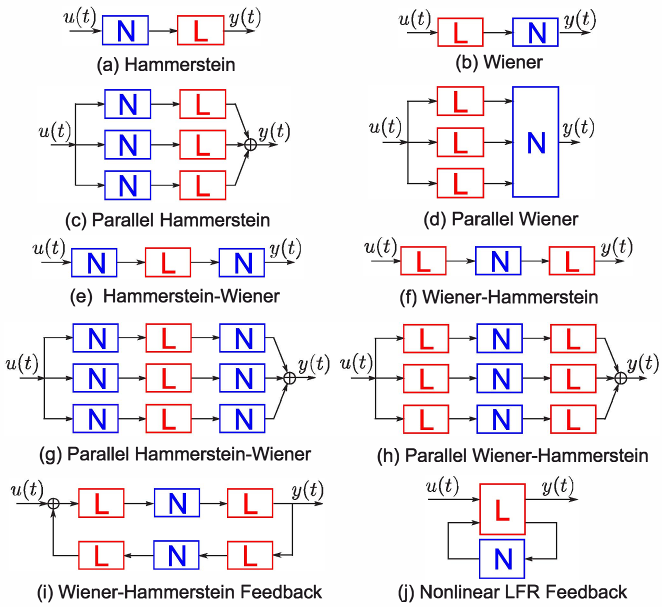

Figure 9. Examples of structured nonlinear block oriented models.

because we are able nowadays to deal with them. Although a Hammerstein system can be used to include actuator nonlinearities in the model, and a Wiener system can cover sensor nonlinearities, the general applicability of these models is rather limited. This can be easily understood by looking at the BLA. In Figure 10, we show a typical example for W, H, or WH systems. It shows that these models can not deal with shifting resonances or varying phase characteristics as it was visible for example in the silverbox results in Figure 5. This is too hard a restriction for many applications. For that reason we need more advanced structures. A deeper theoretic insight in the behaviour of these models, driven by Gaussian excitations, is given in [16], [3], [15].

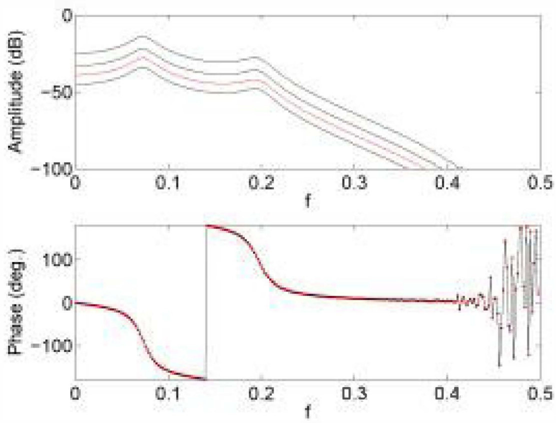

Figure 10. Typical variation of the BLA for Wiener, Hammerstein, or Wiener-Hammerstein systems for varying Gaussian excitation conditions.

2) NLFB systems: Many systems show a BLA with varying resonance frequencies or changing damping ratios for varying experimental conditions (see for example the BLA of a nonlinear feedback structure in Figure 11). As we illustrated before in Figure 10, is impossible to model these systems with single branch block oriented models.

A first possibility to approximate NLFB systems is to use parallel WH structures. These can be considered as an open loop approximation of a closed loop system [7], [31]. This simplifies significantly the identification problem. However, open loop parallel structures can only create shifting zeros, the poles remain fixed. For that reason the application field of these models is still limited, although they can offer a balance between identification complexity and model flexibility. Some initial tools are available to identify such structures [32].

The major challenge is to come up with identification methods for the nonlinear feedback structures. Today, first results are available for the identification of these structures [34], [35], [36], and more methods are expected to be published in the coming years.

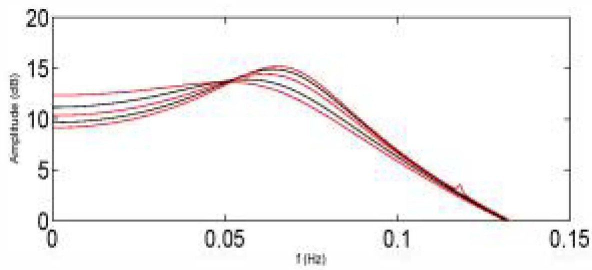

Figure 11. Example of the FRF of a BLA with a varying resonance frequency for a nonlinear feedback structure.

User guidelines:

- The initial selection of the model structure is very important. Too simple models will never be able to capture the complex behavior of many nonlinear systems, while too complex models are very hard to identify.
- Measure the BLA under varying excitation conditions, e.g. increasing amplitude, a varying offset, ... The variations of the BLA give a lot of insight about the required model complexity. For example, shifting poles can only be modeled using nonlinear feedback structures. More information can be found in [38]. This can save a lot of energy.

C. Retrieving the structure of the model

A major problem during the identification of nonlinear systems is the loss of the original structure during the identification process. Consider the systems at the left side of Figure 12. It consists of a number of decoupled branches, each of the nonlinear functions $f^{[i]}$ is a scalar function. Direct identification of a decoupled structure is impossible at this moment. For that reason we propose a two-step procedure that identifies in a first step an unstructured model with coupled nonlinearities (given in Figure 12, right). In a second step, we look for state transformations that decouple the multivariate vector functions ((given in Figure 12, left). Initial results are reported in [33], [32], [37].

The complexity to describe a multivariate coupled function is much higher than it is to describe a set of scalar functions. The coupling results not only in a much larger number of unknown parameters to be estimated, it also reduces the intuitive or physical insight into the behavior of the system. This is another important reason to replace the coupled structure by a decoupled one. A similar problem pops up in the nonlinear state space models in Figure 7. Also for these models it is highly desirable to decouple the nonlinear function F for exactly the same reasons.

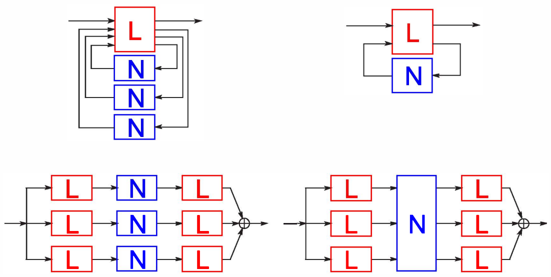

Figure 12. Replacing a coupled nonlinear structure (right) by a decoupled structure (left). Top: LFR structure; Bottum: parallel WH.

VI. Conclusions

In this paper we discussed a system identification approach for modeling real world systems. Typical for most problems is that some level of nonlinear distortions is present. However, it is not always advisable to build a full nonlinear model. Only when the errors of a linear approximation are too large for the intended application of the model, it pays off to invest in nonlinear identification procedures. Since the latter are much more involved, it is important to provide tools to the user to detect and quantify the level of the nonlinear distortions. If the nonlinear distortions are smaller than the specified level of accuracy of the model to be built, a linear design might be sufficient. This will lead to the best linear approximation of the nonlinear system. In the other case, a more involved nonlinear model will be needed. In this paper we presented simple nonparametric tools that can provide the requested information with very little user interaction, at a cost of imposing periodic excitations to the user.

The classical linear system identification theory can provide consistent (non)parametric estimates of the best linear approximation. Also the uncertainty bounds on nonparametric FRF models, obtained with the linear theory, are still valid, even in the presence of strong nonlinear distortions. However, in that case, the uncertainty bounds that are provided by the linear theory for parametric models are wrong. The variances that are calculated with the linear theory underestimate the real variance with a factor 7 or more.

In case the linear approximation is not good enough to solve the problem, a nonlinear model is used. We proposed a double approach. On the one hand we can use non-structured nonlinear state space models that are 'easy' to identify, on the other hand we can use highly structured nonlinear block-oriented models that are difficult to initialize. For that reason we look nowadays for a combination of both approaches in a two-step procedure. First, unstructured initial estimates are obtained using, for example, the nonlinear state space approach. Next, we try to retrieve structured models by using recently developed methods like tensor decomposition approaches.

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