Frequency Domain Maximum likelihood Estimation of Linear Dynamic Errors-in-Variables Models

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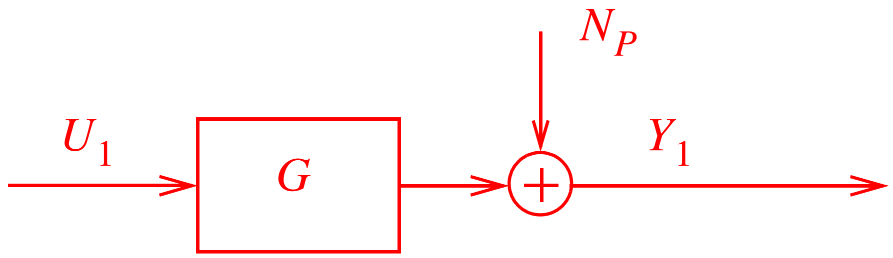
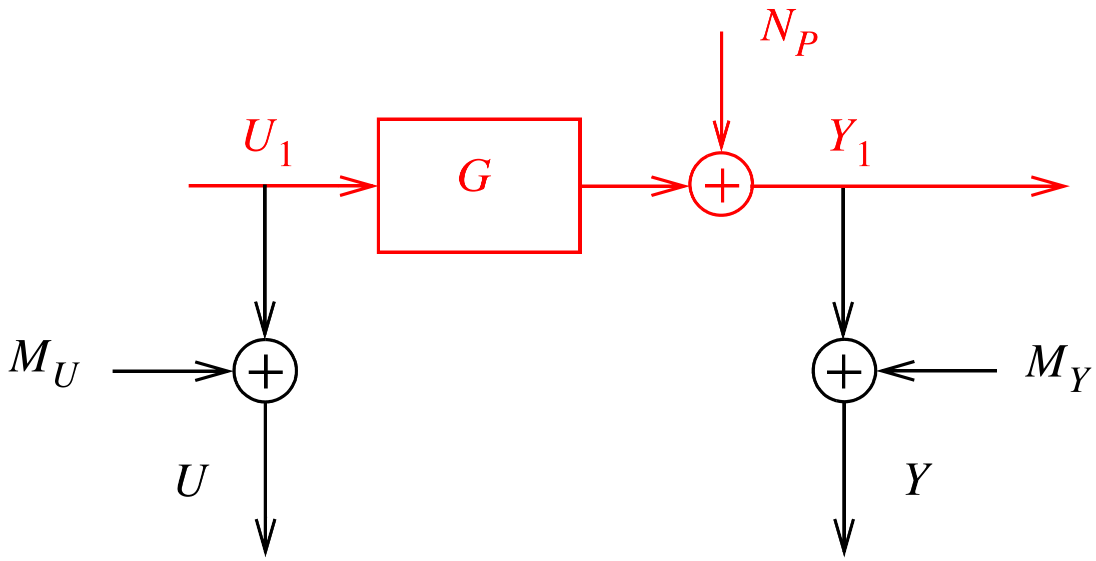
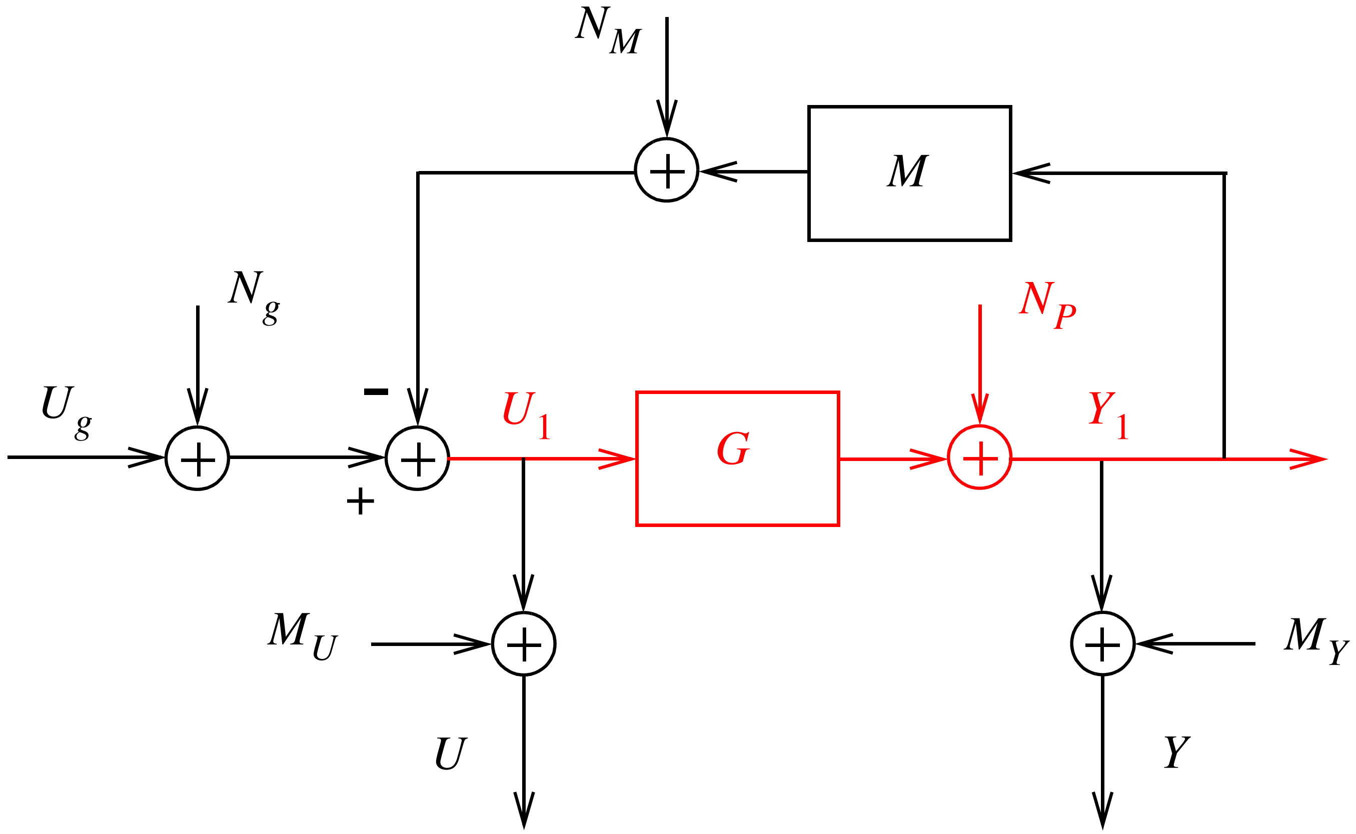
Open Loop Generalised-Output-Error Problem



Open Loop Errors-in-Variables Problem



Closed Loop Errors-in-Variables Problem



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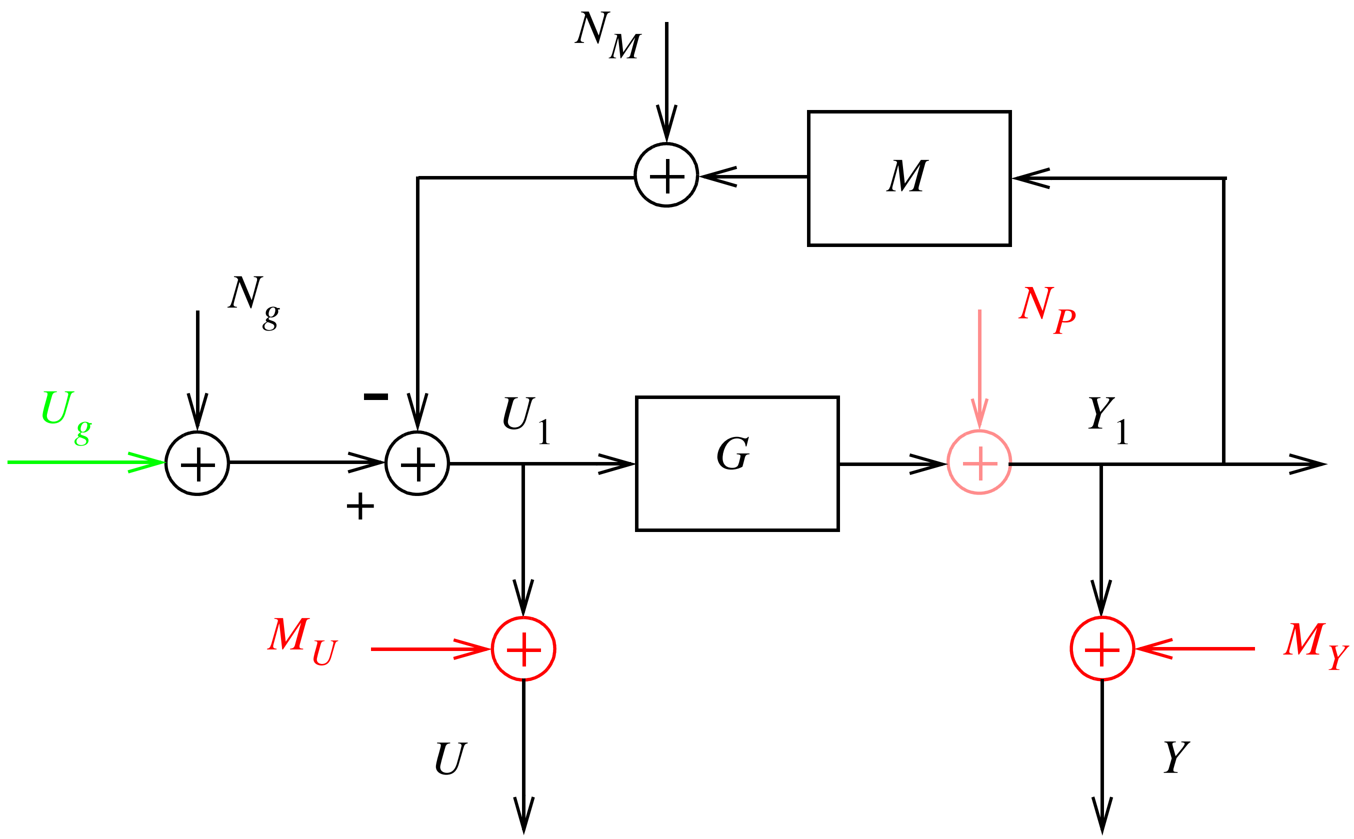
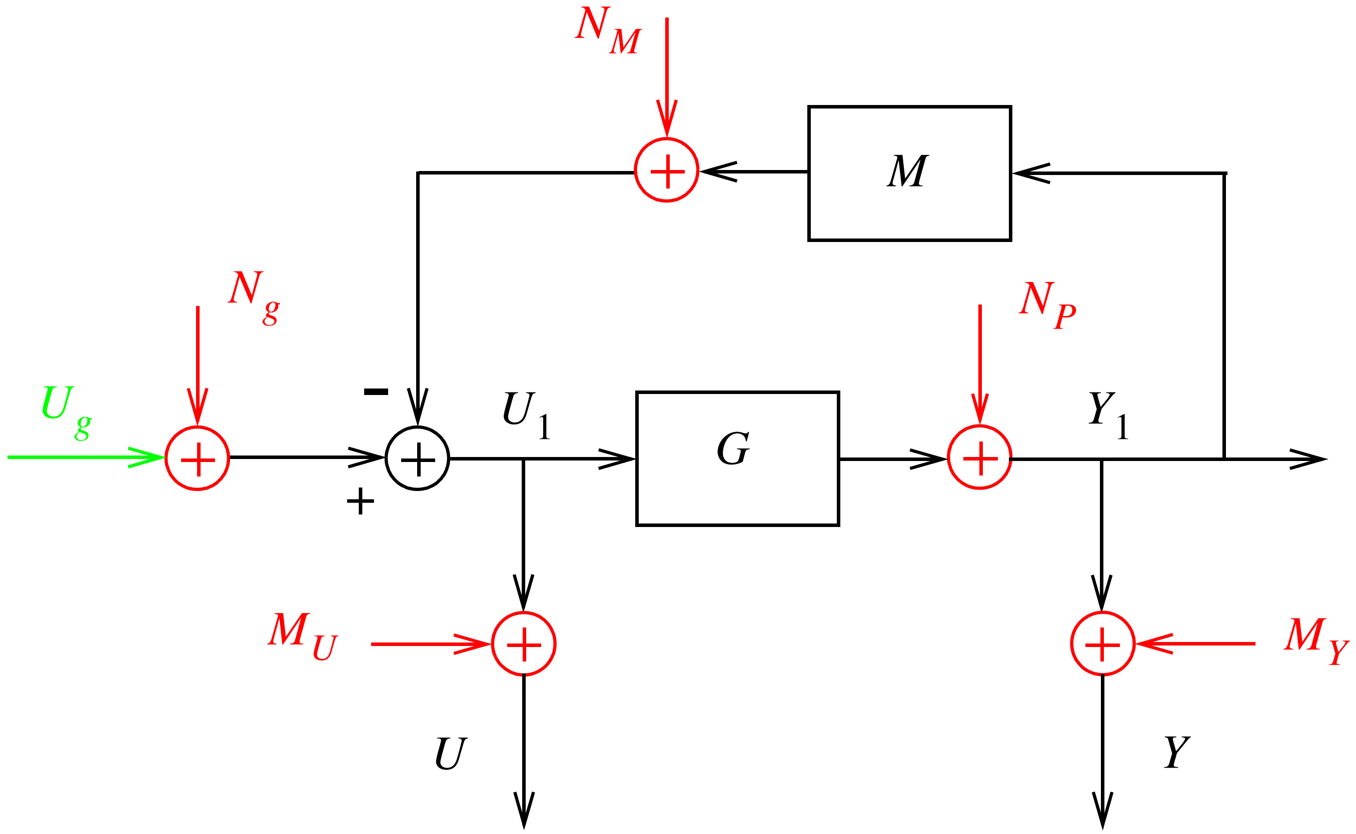


Arbitrary excitation

$$\begin{cases} U_0 = \dfrac{1}{1+GM}(U_g + N_g - N_M - MN_P) \\ Y_0 = \dfrac{G}{1+GM}(U_g + N_g - N_M - MN_P) \end{cases} \text{and} \begin{cases} N_U = M_U \\ N_Y = M_Y + \dfrac{1}{1+GM}N_P \end{cases}$$

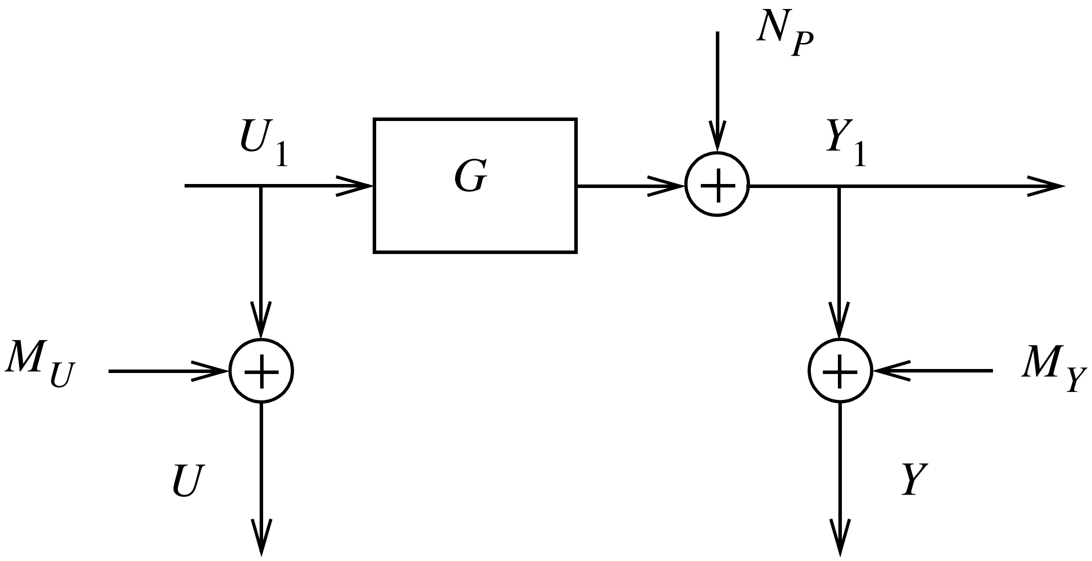
Closed Loop Errors-in-Variables Problem



Periodic excitation

$$\begin{cases} U_0 = \dfrac{1}{1+GM} U_g \\ Y_0 = \dfrac{G}{1+GM} U_g \end{cases} \text{and} \begin{cases} N_U = M_U + \dfrac{1}{1+GM}(N_g - N_M) - \dfrac{M}{1+GM} N_P \\ N_Y = M_Y + \dfrac{G}{1+GM}(N_g - N_M) + \dfrac{1}{1+GM} N_P \end{cases}$$

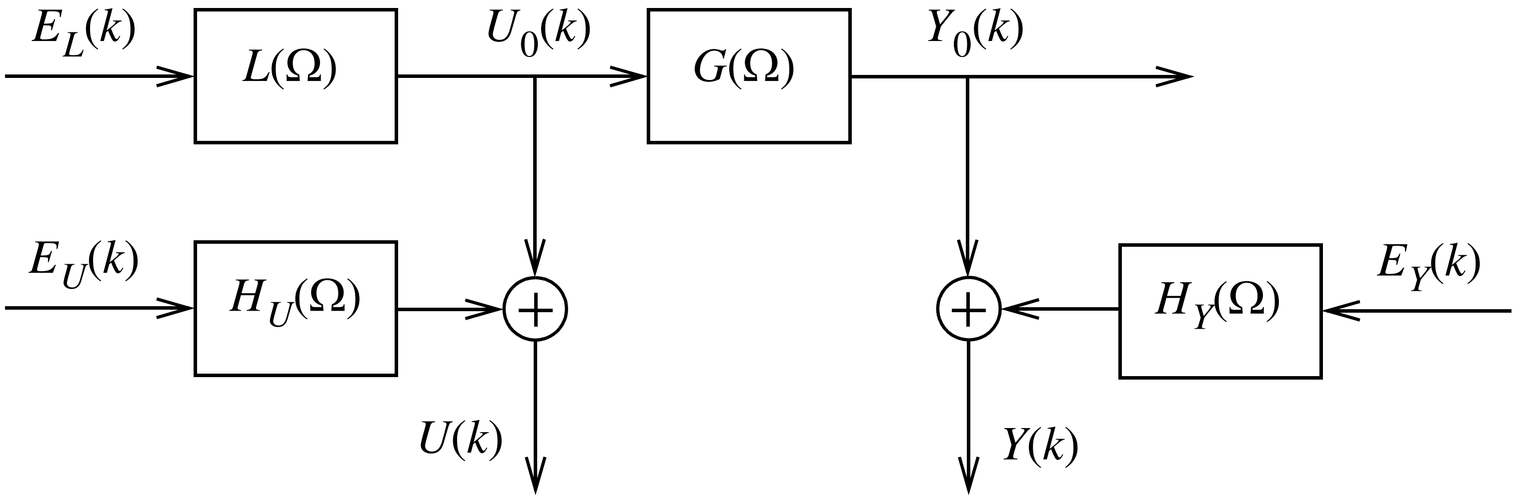
This Paper: Simplified Errors-in-Variables Problem



Assumptions

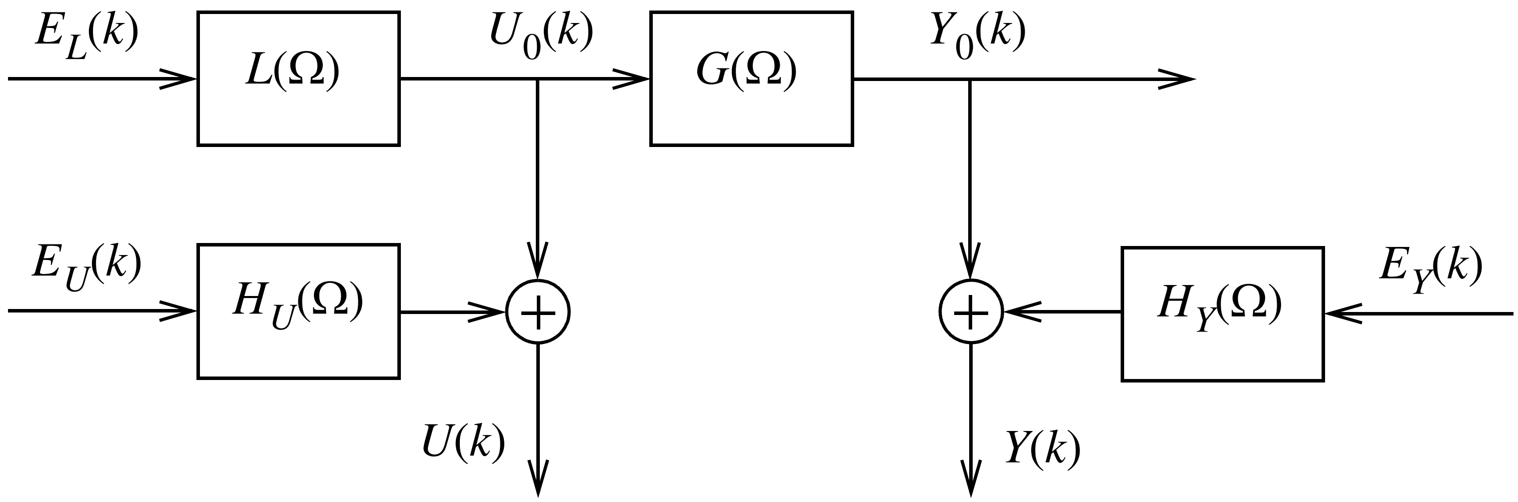
- Open loop
- Filtered white noise excitation
- Independent, filtered white noise input/output errors

Errors-in-Variables Framework



$$\Omega = \begin{cases} z^{-1} & \text{DT} \\ s & \text{CT} \end{cases}$$

Errors-in-Variables Framework

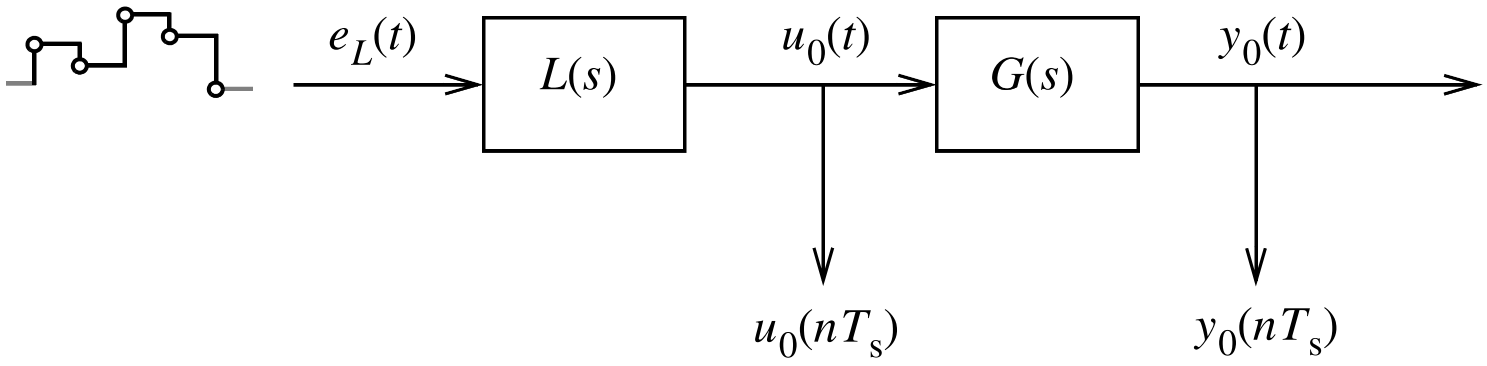


$$\Omega = \begin{cases} z^{-1} & \text{DT} \\ s & \text{CT} \end{cases}$$

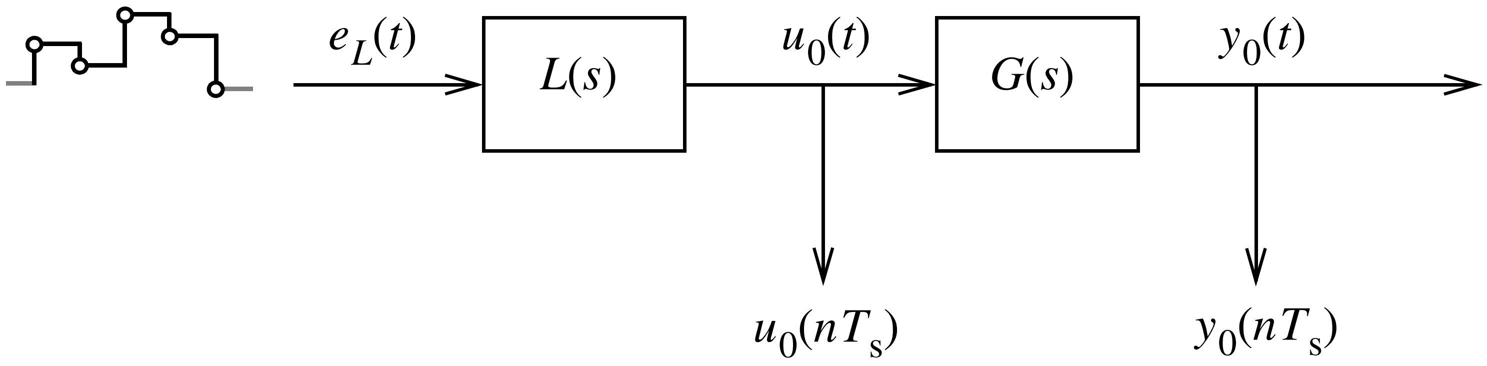
with $E_U(k)$, $E_Y(k)$, and $E_L(k)$

- Mutually independent
- Independent over k
- Circular complex, normally distributed

Errors-in-Variables Framework

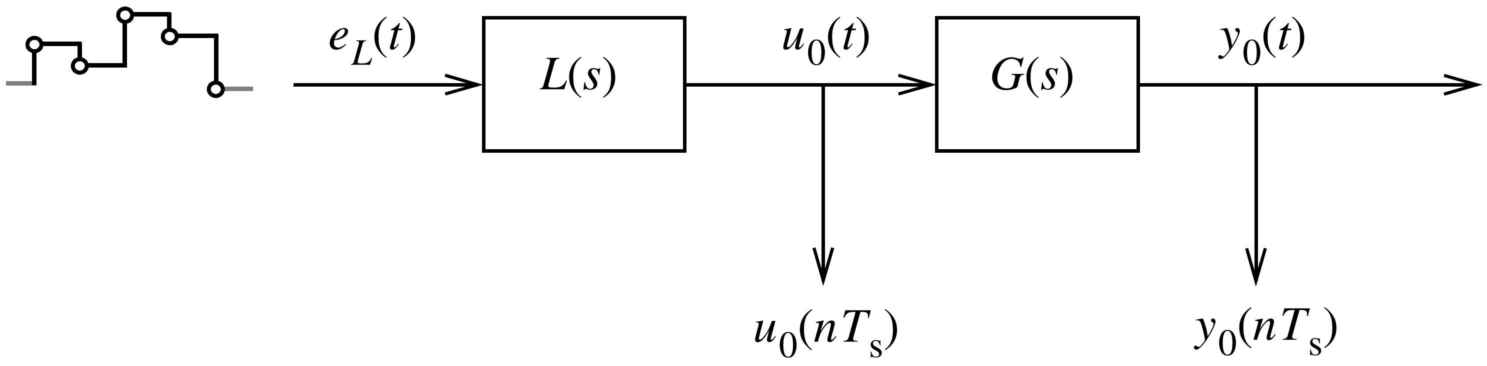


Errors-in-Variables Framework



$$G_1(z^{-1}) = \frac{Z\{y_0(nT_s)\}}{Z\{u_0(nT_s)\}} = \frac{\dfrac{Z\{y_0(nT_s)\}}{Z\{e_L(nT_s)\}}}{\dfrac{Z\{u_0(nT_s)\}}{Z\{e_L(nT_s)\}}} = \frac{(1-z^{-1})Z\{L^{-1}\{L(s)G(s)/s\}\}}{(1-z^{-1})Z\{L^{-1}\{L(s)/s\}\}}$$

Errors-in-Variables Framework



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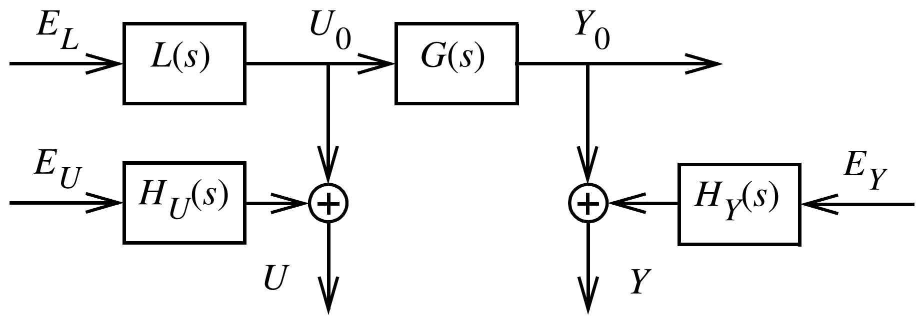
Conclusion

- Exact DT model depends on $L(s)$
- Natural choice = CT modelling

Identifiability Conditions from Second Order Moments



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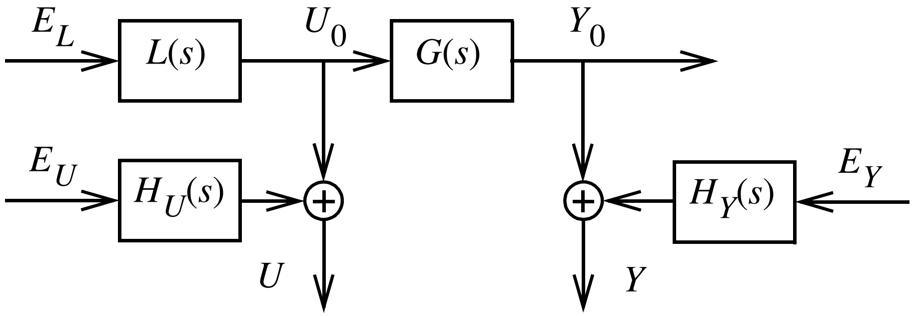
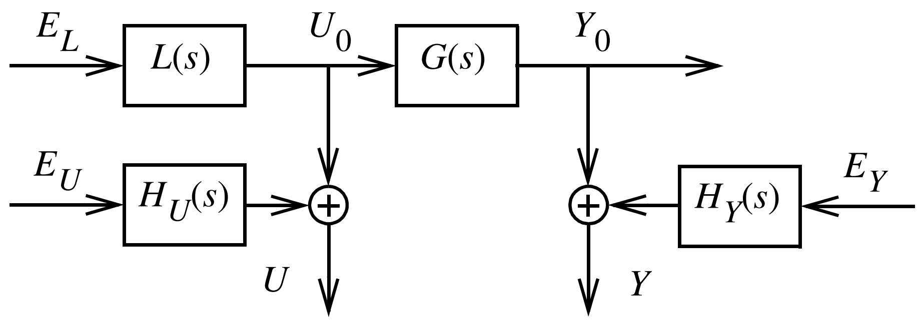


1. $G = \frac{B}{A}$, $L = \frac{P}{Q}$, $H_U = \frac{C_U}{D_U}$, and $H_Y = \frac{C_Y}{D_Y}$ cannot be simplified
2. Monic parametrisation A, P, Q, C_U, D_U, C_Y, and D_Y

Identifiability Conditions from Second Order Moments



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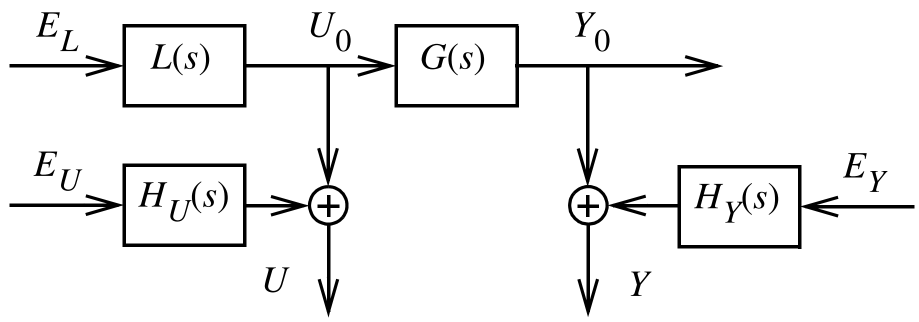
Not related to EIV

Identifiability Conditions from Second Order Moments

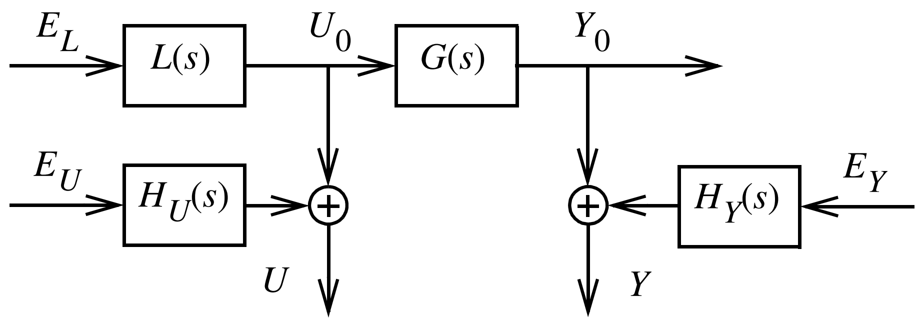


3. $G(s)$ has no quadrant symmetric poles nor zeroes

$$G(s_0) = \begin{cases} 0 \\ \infty \end{cases} \quad \Rightarrow \quad G(-s_0) \neq \begin{cases} 0 \\ \infty \end{cases}$$

4. No pole nor zero of $G(s)$ is respectively a zero or pole of $L(s)L(-s)$

Identifiability Conditions from Second Order Moments



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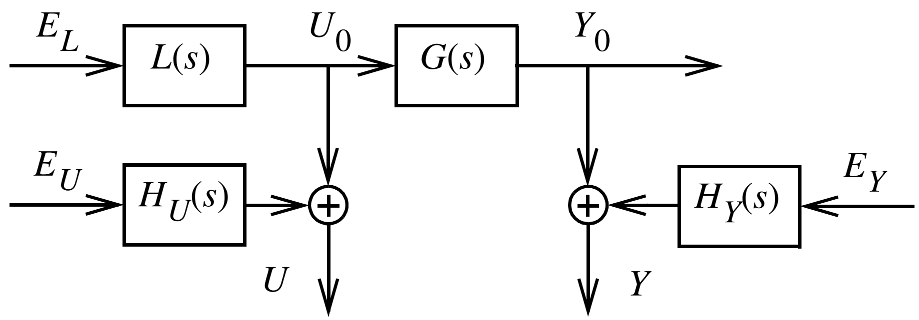
Finite number of solutions with different model structure

Identifiability Conditions from Second Order Moments

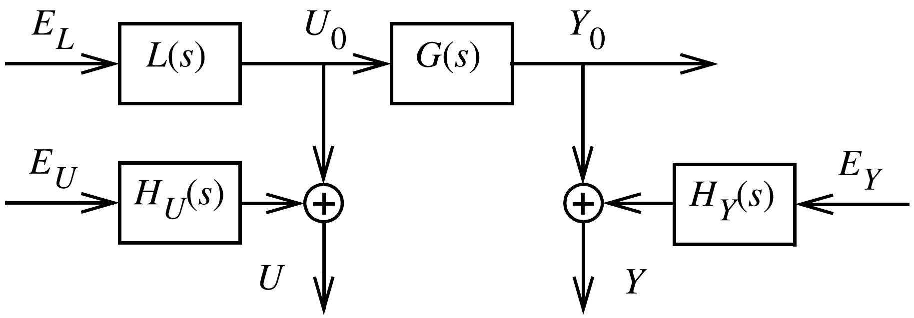


5. One of the following conditions is fulfilled

$$\lim_{s \to s_0} \frac{H_U(s)H_U(-s)}{L(s)L(-s)} = 0 \quad \text{for} \quad s_0 = \infty \quad \text{or pole of} \quad L(s)L(-s)$$

$$\lim_{s \to s_0} \frac{H_Y(s)H_Y(-s)}{G(s)L(s)G(-s)L(-s)} = 0 \quad \text{for} \quad s_0 = \infty \quad \text{or pole of} \quad G(s)L(s)G(-s)L(-s)$$

Identifiability Conditions from Second Order Moments



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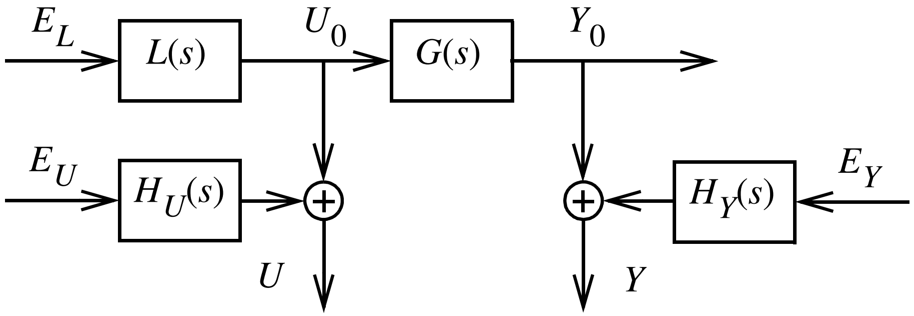
Infinite number of solutions depending on λ_L

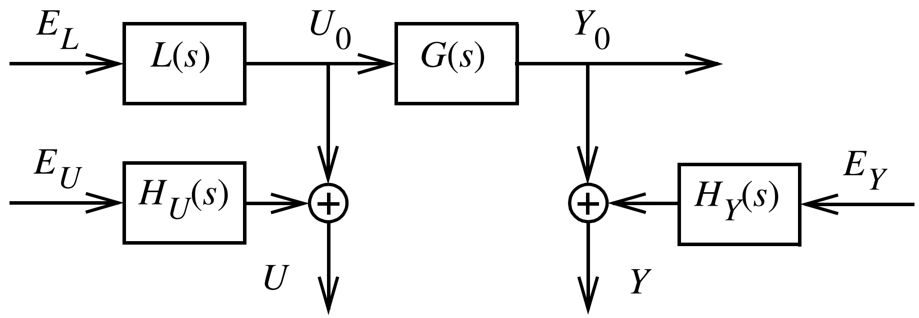
Identifiability Conditions from Second Order Moments



Special cases

1. $L = 1$, $H_U = 1$, and $H_Y = 1 \Rightarrow$ identifiable iff $G(s)$ is dynamic

Identifiability Conditions from Second Order Moments



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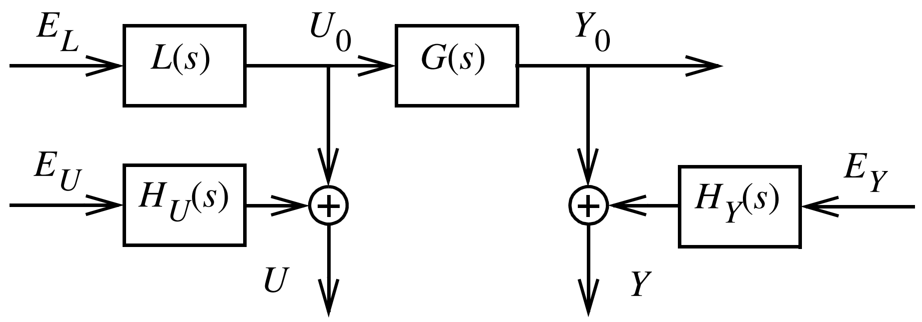
Identifiability Conditions from Second Order Moments



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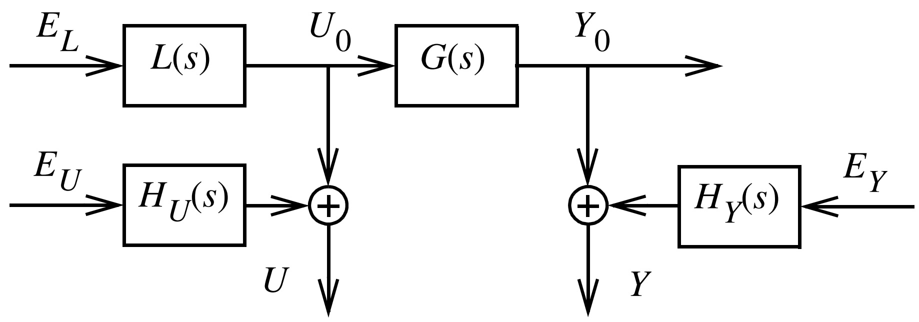
2. $G(s)$ is static $\Rightarrow$ identifiable if either $L(s)$, $H_U(s)$, or $H_Y(s)$ depends on s

3. $H_Y(s)$ may have the same poles as $G(s)$ $\Rightarrow$ ARX and ARMAX

Frequency Domain Gaussian Maximum Likelihood Estimator



Time domain $\rightarrow$ frequency domain

$$Y(k) = \frac{1}{\sqrt{N}}\sum_{t=0}^{N-1} y(t)e^{-2\pi jkt/N}$$

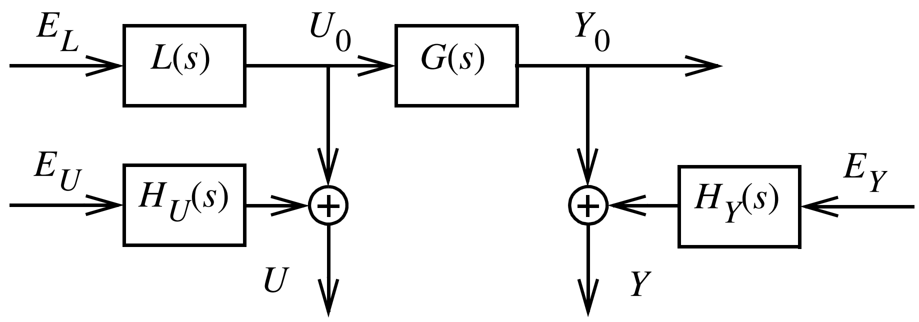
$$U(k) = \frac{1}{\sqrt{N}}\sum_{t=0}^{N-1} u(t)e^{-2\pi jkt/N}$$

Define $Z(k) = \begin{bmatrix} Y(k) & U(k) \end{bmatrix}^T$, and $\Lambda = [\lambda_L, \lambda_U, \lambda_Y]^T$

$$\mathrm{E}\{Z(k) | \theta, \Lambda\} = 0$$

$$\mathrm{Cov}(Z(k) | \theta, \Lambda) = C_{Z(k)}(\theta)$$

Frequency Domain Gaussian Maximum Likelihood Estimator



Negative Gaussian log-likelihood

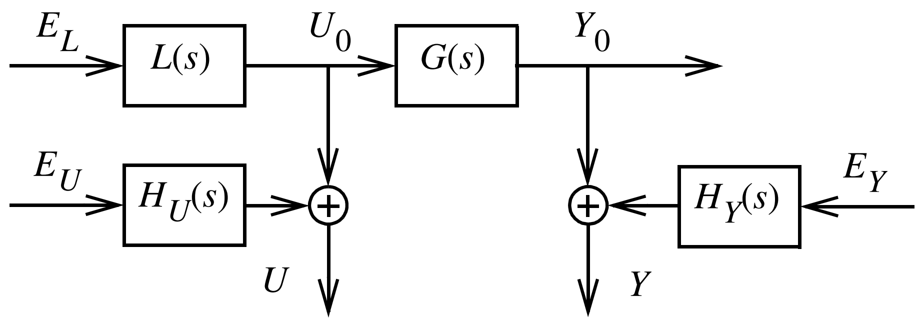
$$V(\theta, \Lambda, Z) = \sum_{k \in \mathbb{K}} \operatorname{logdet}(C_{Z(k)}(\theta)) + \sum_{k \in \mathbb{K}} Z^H(k) C_{Z(k)}^{-1}(\theta) Z(k)$$

with

$$\det(C_{Z(k)}(\theta)) = (|H_Y|^2 \lambda_Y + |GH_U|^2 \lambda_U)|L|^2 \lambda_L + |H_U H_Y|^2 \lambda_U \lambda_Y$$

$$Z^H(k) C_{Z(k)}^{-1}(\theta) Z(k) = \frac{|Y - GU|^2 |L|^2 \lambda_L + |Y|^2 |H_U|^2 \lambda_U + |U|^2 |H_Y|^2 \lambda_Y}{\det(C_{Z(k)}(\theta))}$$

Frequency Domain Gaussian Maximum Likelihood Estimator



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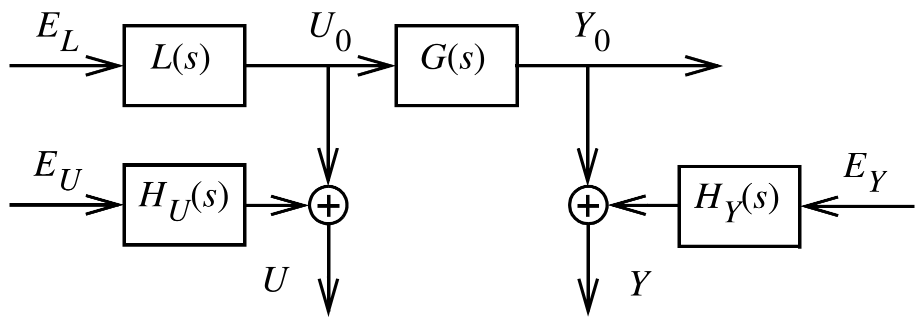
Discussion

- Spectral factorisation = $\sqrt{\det(C_{Z(k)}(\theta))}$

Frequency Domain Gaussian Maximum Likelihood Estimator



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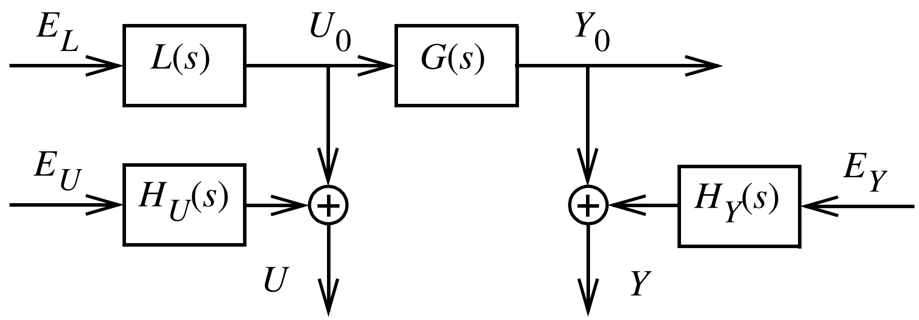
Discussion

- Spectral factorisation = $\sqrt{\det(C_{Z(k)}(\theta))}$
- Numerical stable Gauss-Newton scheme

Frequency Domain Gaussian Maximum Likelihood Estimator



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Discussion

- Spectral factorisation = $\sqrt{\det(C_{Z(k)}(\theta))}$
- Numerical stable Gauss-Newton scheme
- Exact filtering

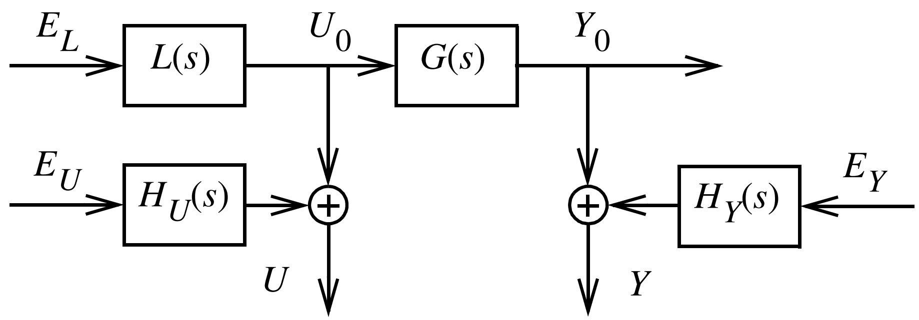
Continuous-Time Simulation Example

$\lambda_L = 1$

$\lambda_U = (0.2)^2$

$\lambda_Y = (0.01)^2$

$N = 20480$



$G(s)$ 5th order

$L(s)$ 1st order

$H_U = H_Y = 1$

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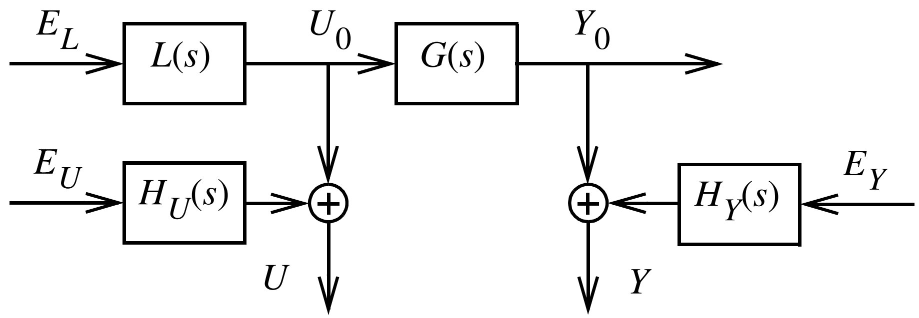

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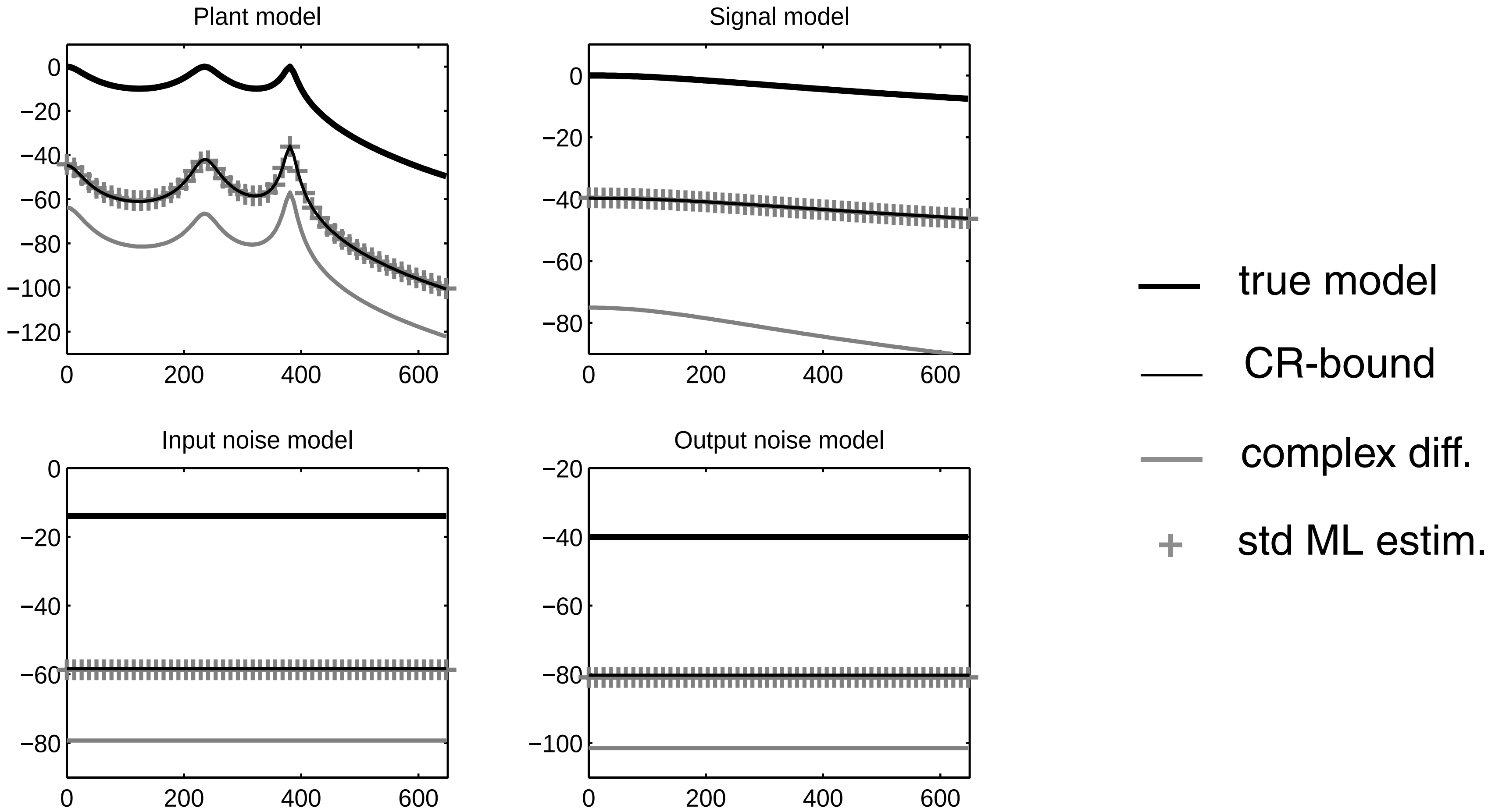
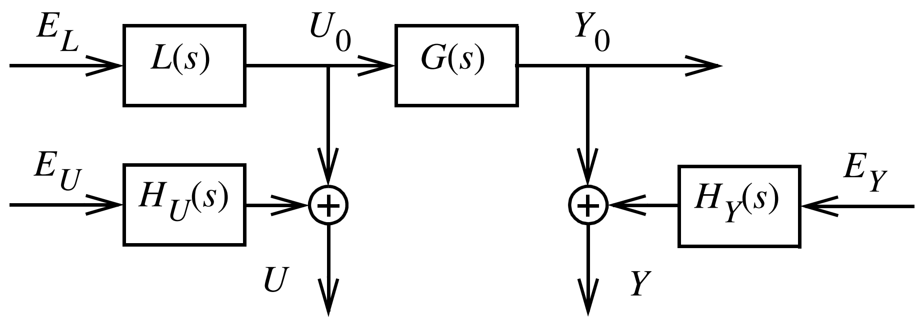
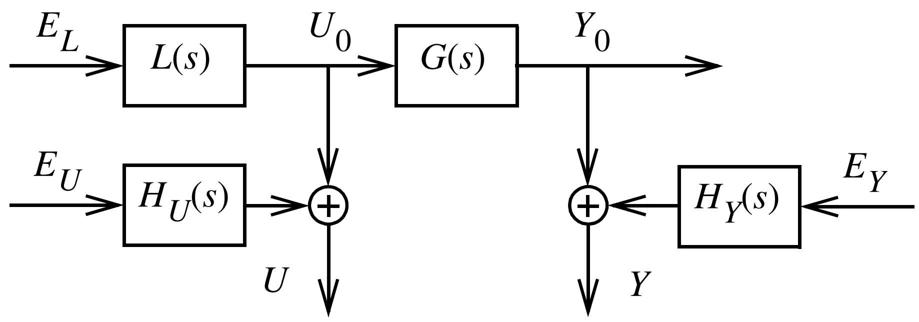


Contributions



1. Frequency domain Gaussian ML estimator
2. Exact filtering
3. Continuous-time modelling
4. Numerical stable Gauss-Newton minimisation
5. Numerical stable calculation Cramér-Rao lower bound

Open Problems

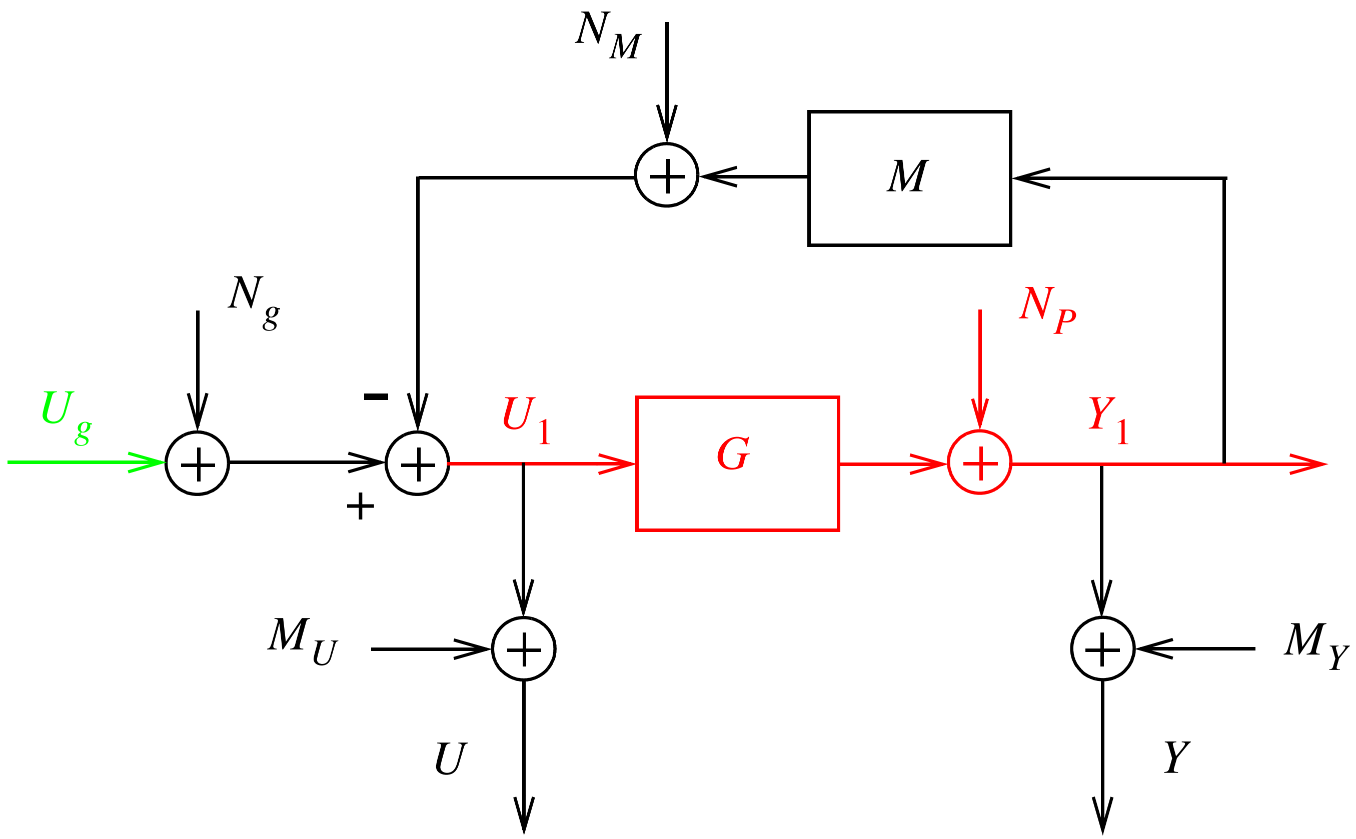


1. High quality starting values for coloured input/output errors

2. Sensitivity to model errors

3. Validation of the identified models

Concluding Remark



Using periodic excitations

Closed loop EIV is as easy as open loop output error