

$$(1) \hat{R}_{SA}(N) = \arg \min_R V_{SA}(R, N)$$

$$V_{SA}(R, N) = \sum_{k=1}^N (R(k) - R)^2$$

$$\frac{\partial}{\partial R} V_{SA} = 0$$

$$2 \sum_{k=1}^N (R(k) - R) = 0$$

$$\boxed{\hat{R}_{SA} = \frac{1}{N} \sum_{k=1}^N \frac{v(k)}{r(k)}}$$

$$(2) \hat{R}_{LS} = \arg \min_R V_{LS}(R, v, i)$$

$$V_{LS}(R, v, i) = \sum_{k=1}^N (v(k) - R i(k))^2$$

$$\frac{\partial}{\partial R} V_{LS} = 0$$

$$2 \sum_{k=1}^N (v(k) - R i(k)) i(k) = 0$$

$$\boxed{\hat{R}_{LS} = \frac{\frac{1}{N} \sum_{k=1}^N v(k) i(k)}{\frac{1}{N} \sum_{k=1}^N i^2(k)}}$$

$$(3) \hat{R}_{EV} = \arg \min_R \left[\underbrace{\sum_{k=1}^N (v(k) - v_p)^2 + \sum_{k=1}^N (i(k) - i_p)^2}_{\text{CRITERION}} + \underbrace{\lambda (v_p - R i_p)}_{\text{CONSTRAINT}} \right]$$

(CONSTRAINT OPTIMIZATION)

LAGRANGE MULTIPLIER

$$\frac{\partial}{\partial R} V_{EV} = -\lambda \cdot R i_p = 0 \quad \lambda = 0$$

$$\frac{\partial}{\partial i_p} V_{EV} = 2 \sum_{k=1}^N (i(k) - i_p) - \lambda R = 0$$

$$i_p = \frac{1}{N} \sum_{k=1}^N i(k)$$

$$\frac{\partial}{\partial v_p} V_{EV} = 2 \sum_{k=1}^N (v(k) - v_p) + \lambda = 0$$

$$v_p = \frac{1}{N} \sum_{k=1}^N v(k)$$

$$\frac{\partial}{\partial \lambda} V_{EV} = v_p - R i_p = 0$$

$$\boxed{\hat{R}_{EV} = \frac{\frac{1}{N} \sum_{k=1}^N v(k)}{\frac{1}{N} \sum_{k=1}^N i(k)}}$$