# Learning to Make Better Decisions: Challenges for the 21st Century

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Based on joint work with:
Yasin-Abbasi Yadkori and Dávid Pál

# Making a difference

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- Autonomous cars: Save lives of people dying on the road

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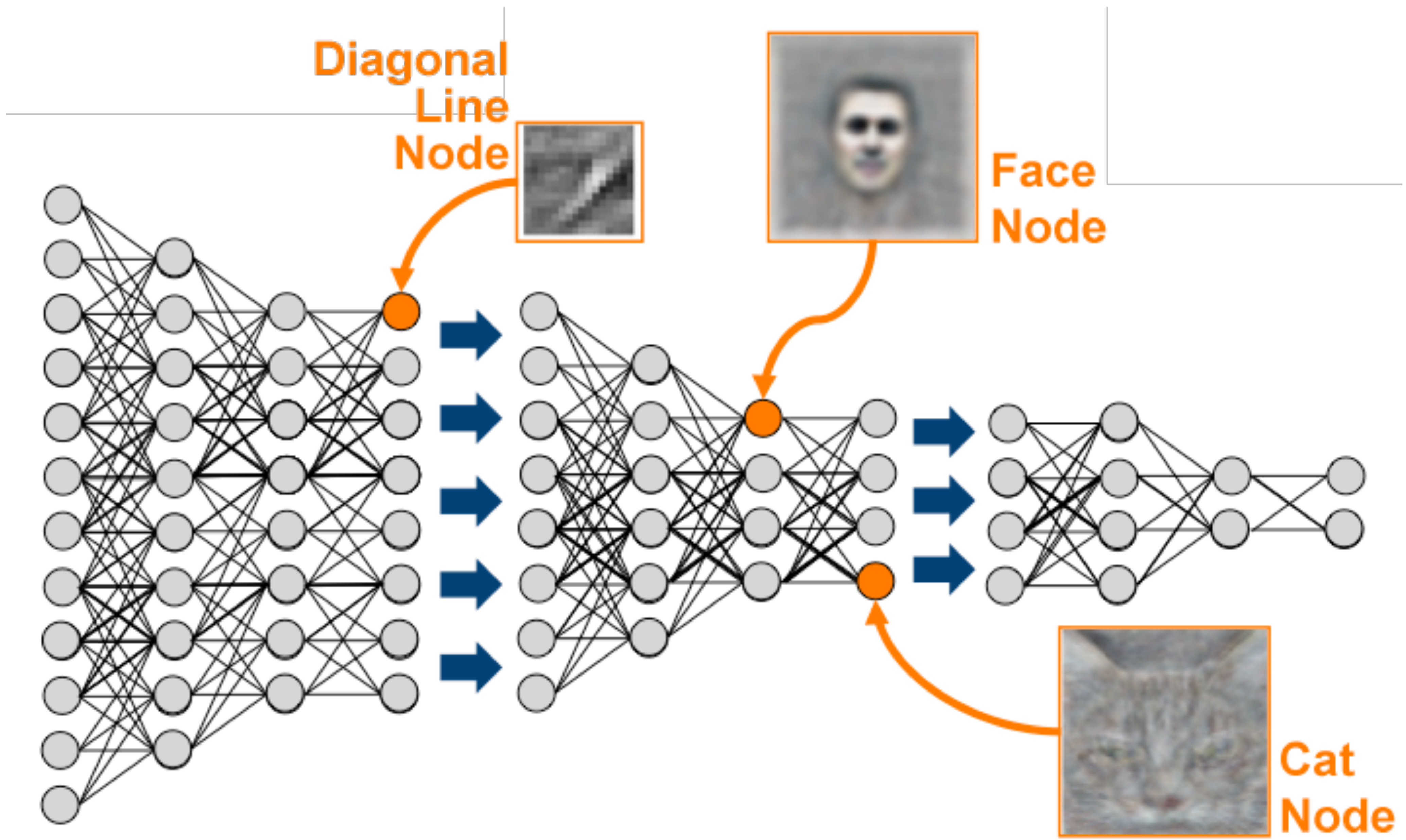
- Autonomous cars: Save lives of people dying on the road
- Voice-user interface systems: Humanizing computer-human interaction
- Dynamic treatment regimes: Save patients. Maximize treatment efficiency while avoiding ill effects
- Intelligent Tutoring: Bring education to the masses while improving it

How?


# Explosion of data


# Computation

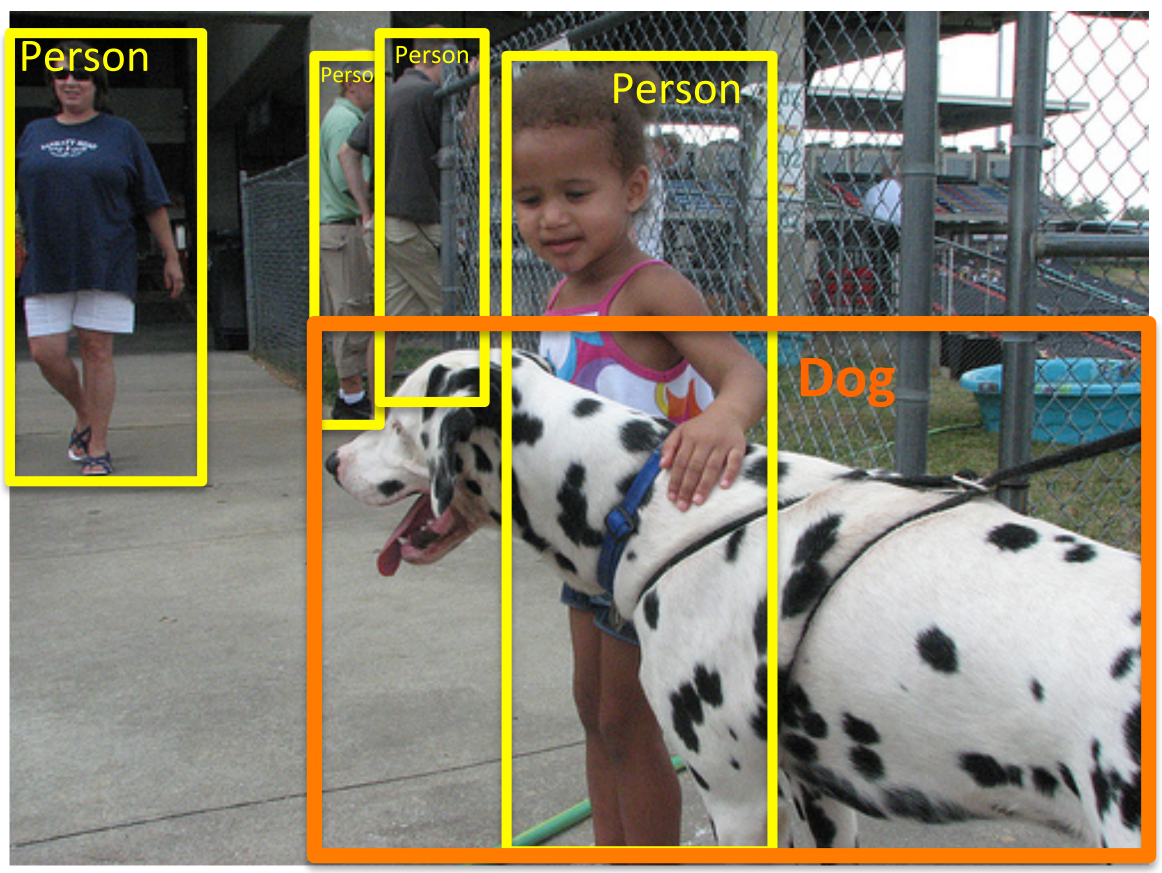



# Improved Learning Methods

How far did we get?

# IMAGENET Large Scale Visual Recognition Challenge (ILSVRC) 2010-2014

**1000 object classes** **1,431,167 images** **CLS-LOC**




http://image-net.org/challenges/LSVRC/

# Evaluation

Steel drum

| **Output:** |
| --- |
| Scale |
| T-shirt |
| <u>Steel drum</u> |
| Drumstick |
| Mud turtle |

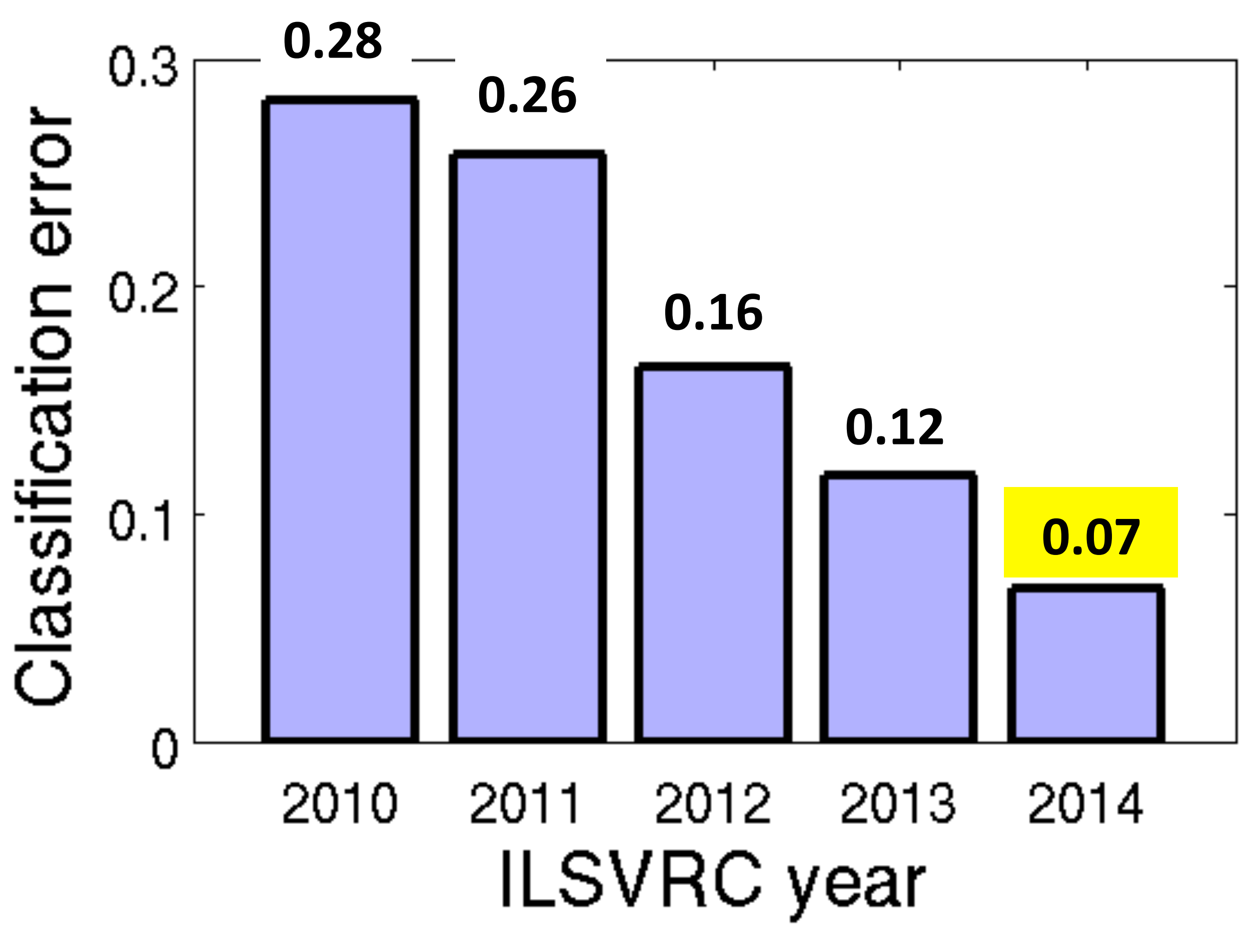
✔

| **Output:** |
| --- |
| Scale |
| T-shirt |
| Giant panda |
| Drumstick |
| Mud turtle |

✗

$$\text{Error} = \frac{1}{100{,}000} \sum_{\substack{100{,}000 \\ \text{images}}} 1[\text{incorrect on image i}]$$

# Progress



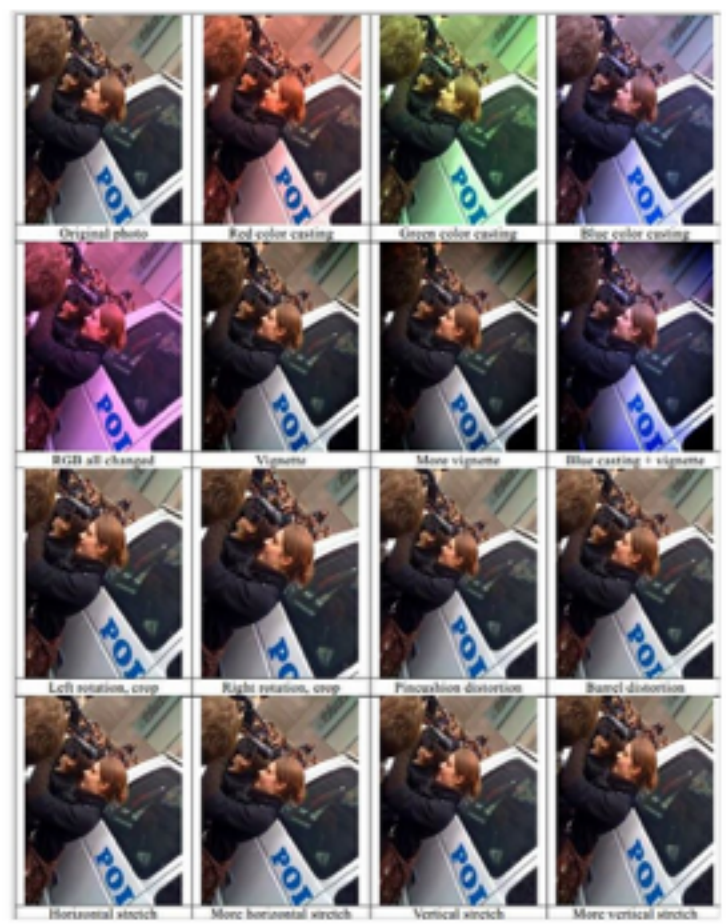

1.7x reduction in classification error since last year

4.2x reduction in classification error since 2010

# And the war goes on..

**Andrew Ng**
52 mins ·

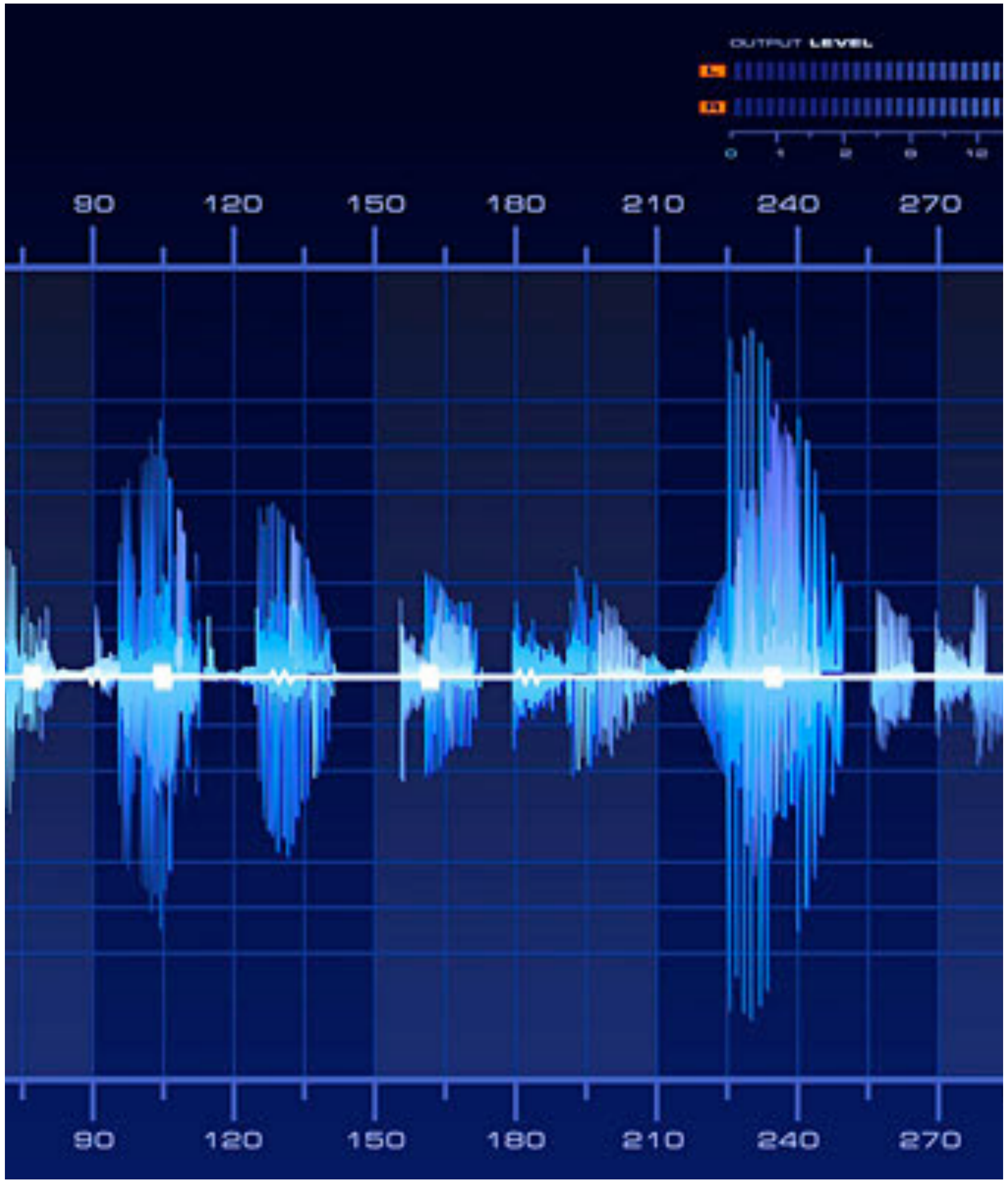
Baidu Research just attained the best computer vision ImageNet classification result 5.98% error (vs. GoogLeNet's 6.66%). The key to this was our multi-GPU deep learning infrastructure, which by using a mix of model-parallelism and data-parallelism, allows us to train our model 24.7x faster than using only a single GPU. This scale also allows us to use higher-resolution images, and absorb more (synthetic) training data. Paper here: bit.ly/deepimage




Unlike · Comment · Share

# Speech recognition

- Google
- Apple
- Baidu
- Achievements:
  - Error rates constantly drop since 2009, halved or so..
  - “Speech 2.0”



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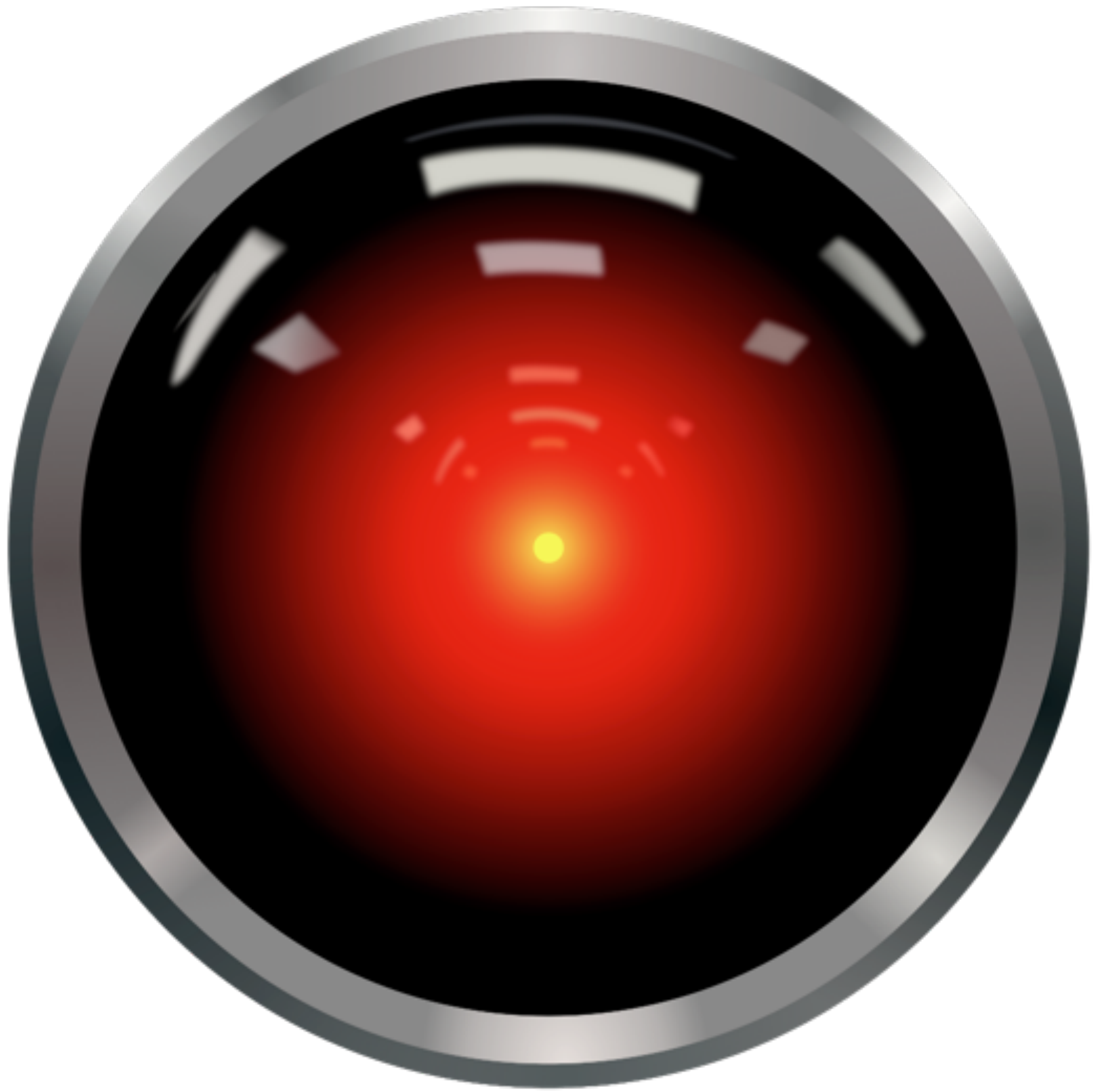

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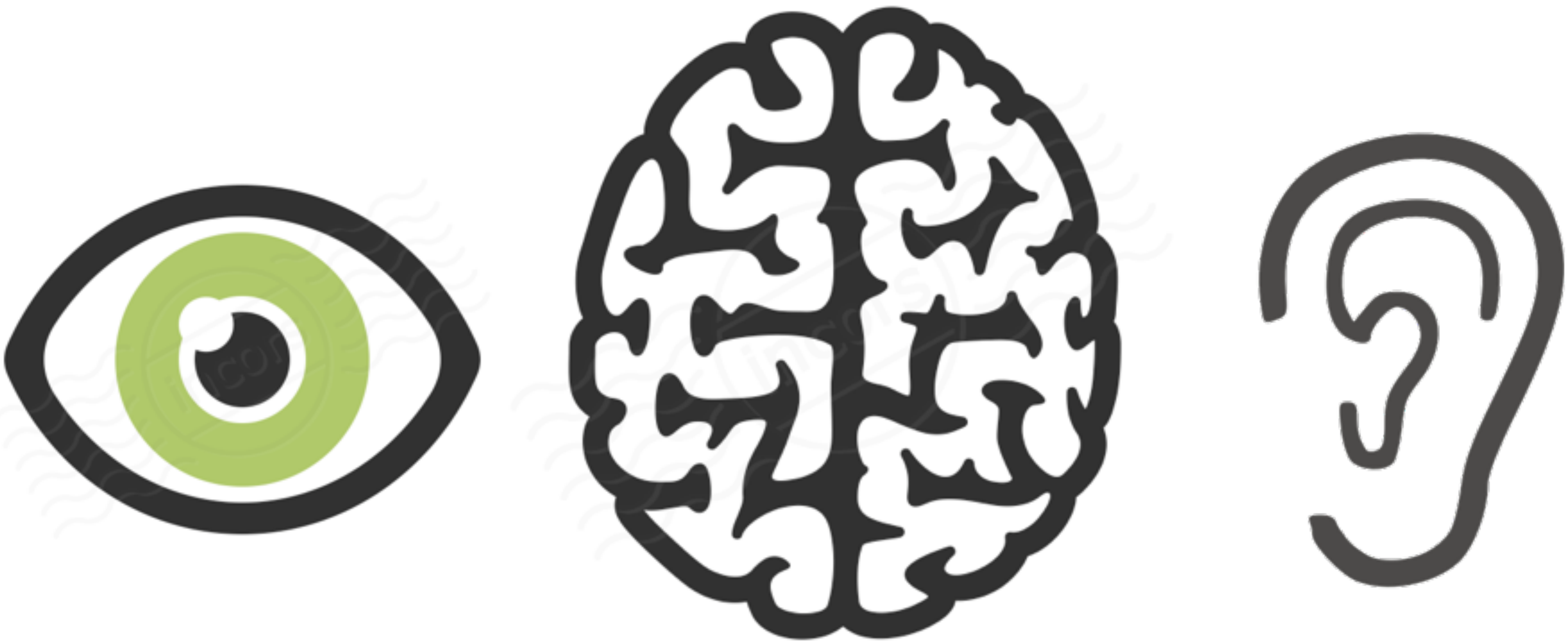
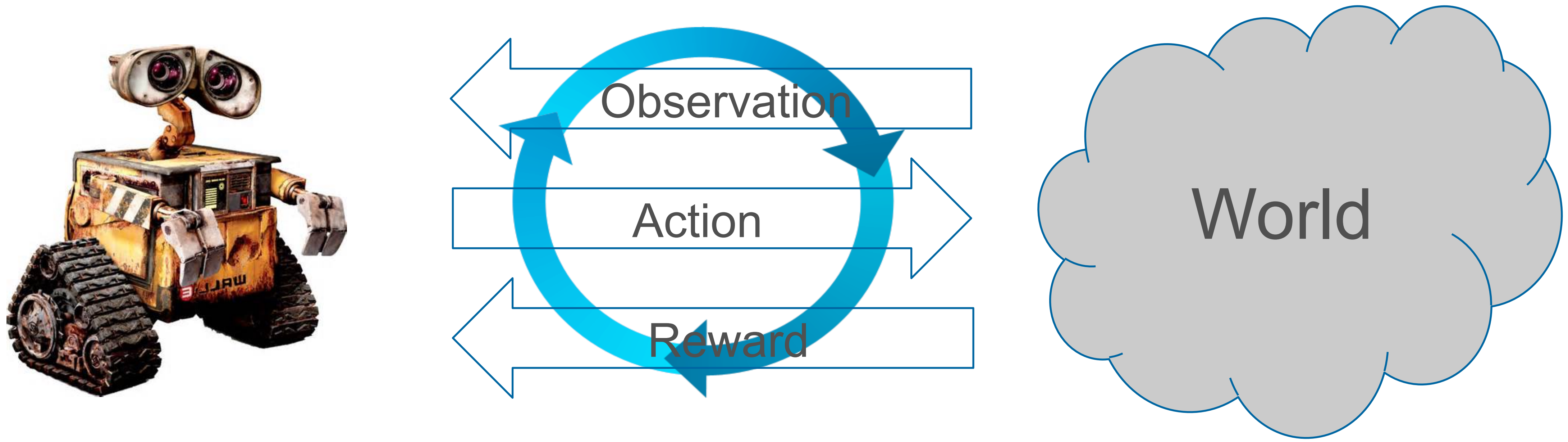


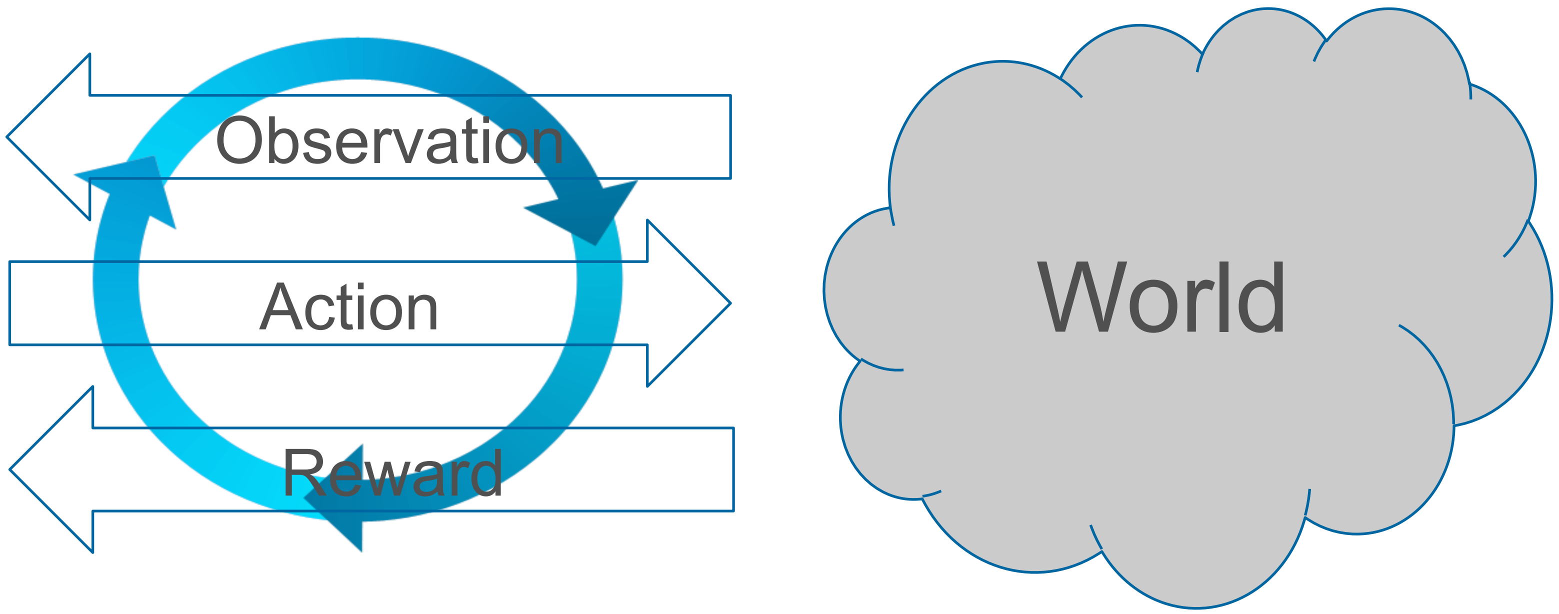
# Are we done?

Need to make decisions!

# RL to the Rescue

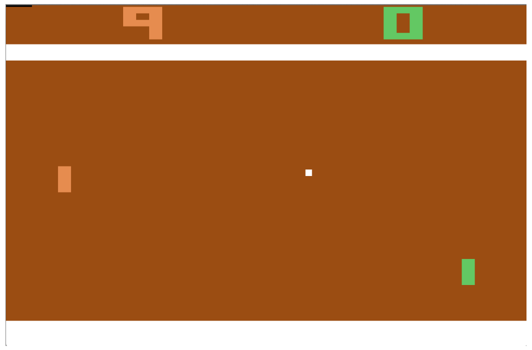


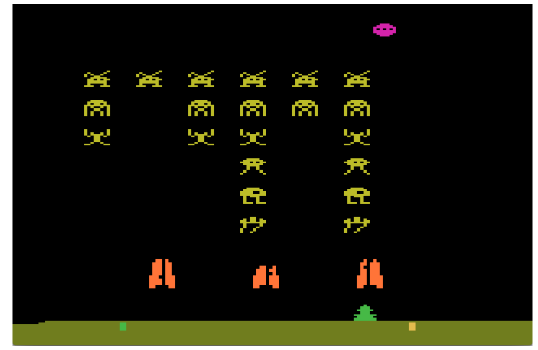
# RL to the Rescue




Goal: Maximize the total reward collected

# Google DeepMind: RL meets Deep Learning and Big Data







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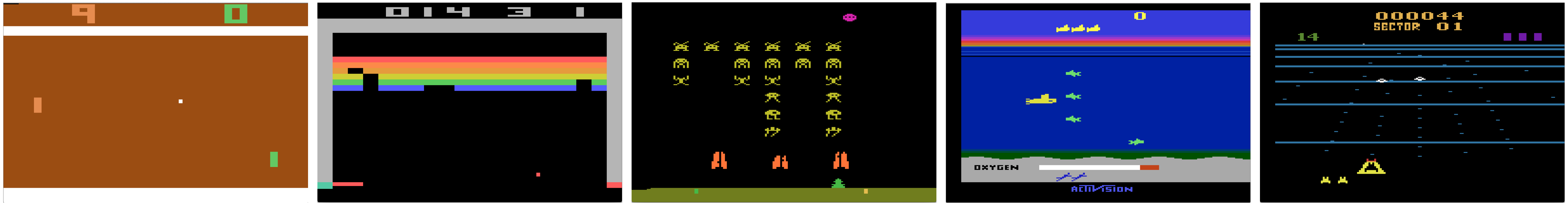



| | B. Rider | Breakout | Enduro | Pong | Q*bert | Seaquest | S. Invaders |
|---|---|---|---|---|---|---|---|
| **Random** | 354 | 1.2 | 0 | −20.4 | 157 | 110 | 179 |
| **Sarsa** [3] | 996 | 5.2 | 129 | −19 | 614 | 665 | 271 |
| **Contingency** [4] | 1743 | 6 | 159 | −17 | 960 | 723 | 268 |
| **DQN** | **4092** | **168** | **470** | **20** | **1952** | **1705** | **581** |
| **Human** | 7456 | 31 | 368 | −3 | 18900 | 28010 | 3690 |
| **HNeat Best** [8] | 3616 | 52 | 106 | 19 | 1800 | 920 | **1720** |
| **HNeat Pixel** [8] | 1332 | 4 | 91 | −16 | 1325 | 800 | 1145 |
| **DQN Best** | **5184** | **225** | **661** | **21** | **4500** | **1740** | 1075 |

Table 1: The upper table compares average total reward for various learning methods by running an $\epsilon$-greedy policy with $\epsilon = 0.05$ for a fixed number of steps. The lower table reports results of the single best performing episode for HNeat and DQN. HNeat produces deterministic policies that always get the same score while DQN used an $\epsilon$-greedy policy with $\epsilon = 0.05$.

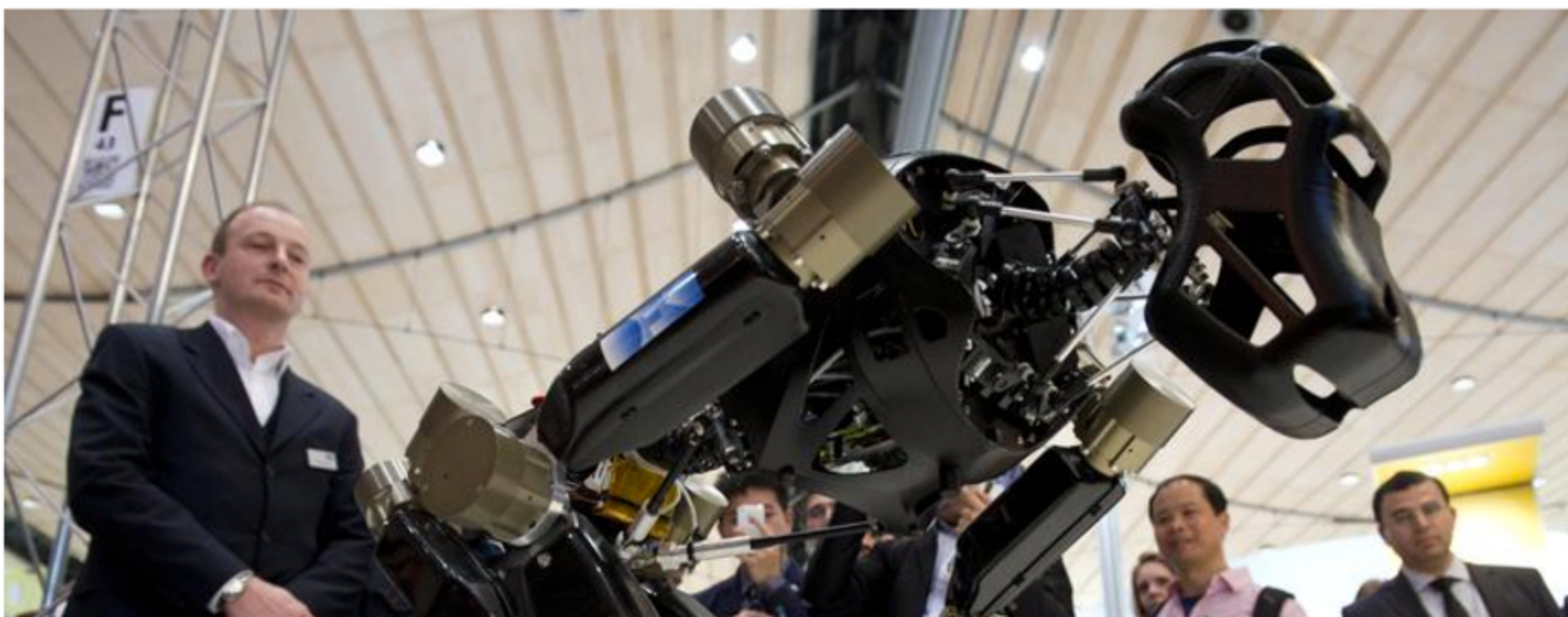
# Google DeepMind:
# RL meets Deep Learning and Big Data

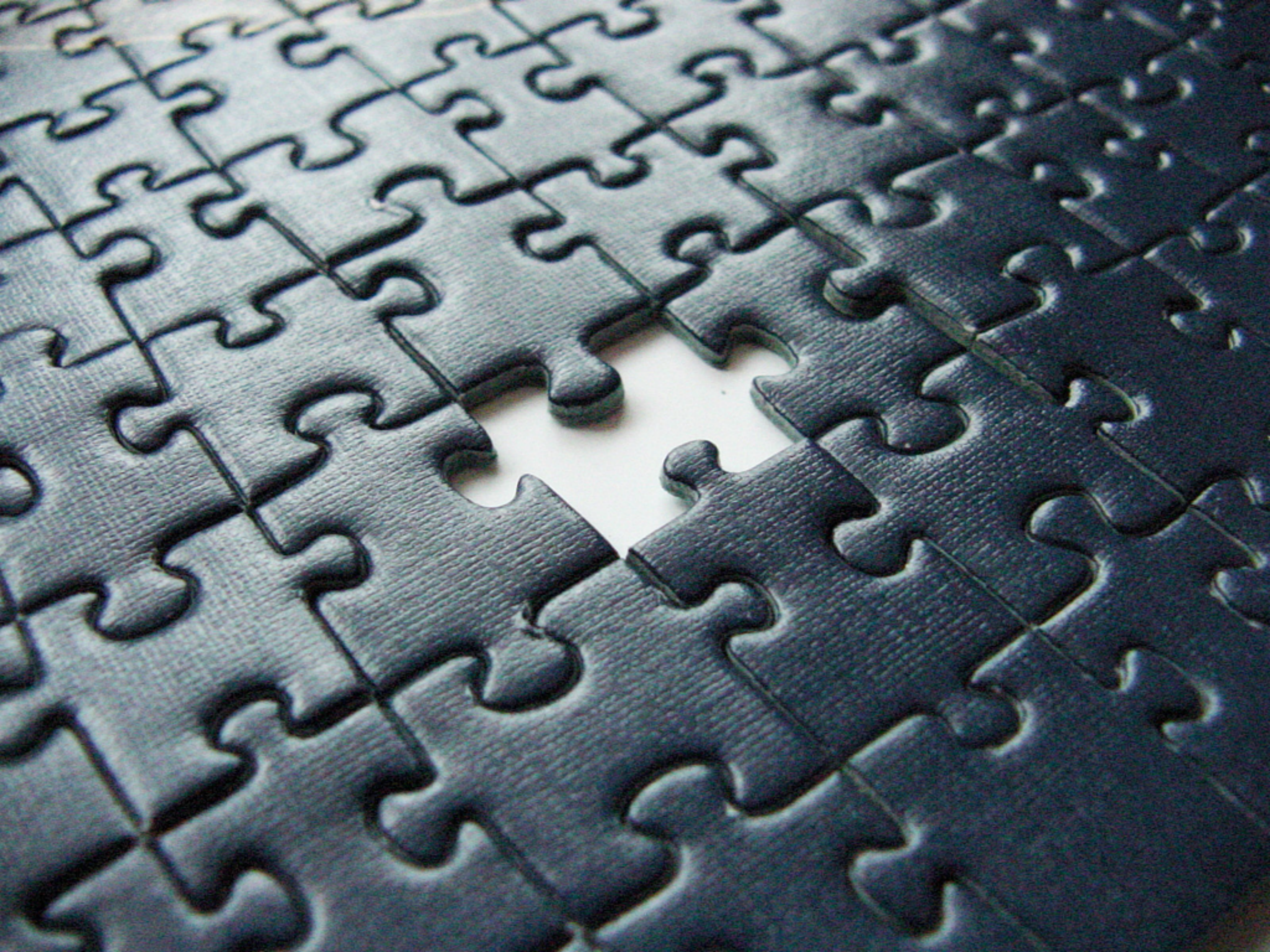
# Artificial intelligence experts sign open letter to protect mankind from machines

The Future of Life Institute wants humanity to tread lightly while developing really smart machines.

by Nick Statt @nickstatt / 12 January 2015 12:10 am GMT

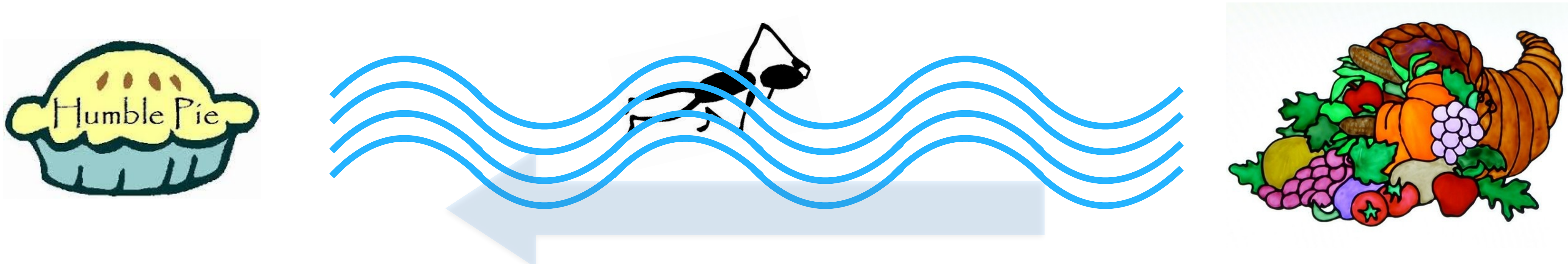
73 / 1.2K / 1.1K / 249 / / more +





# On Data Collection

# A Swimming Lesson




Slide graphics courtesy of Ben van Roy.
Problem due to [Strehl-Littman,'08]

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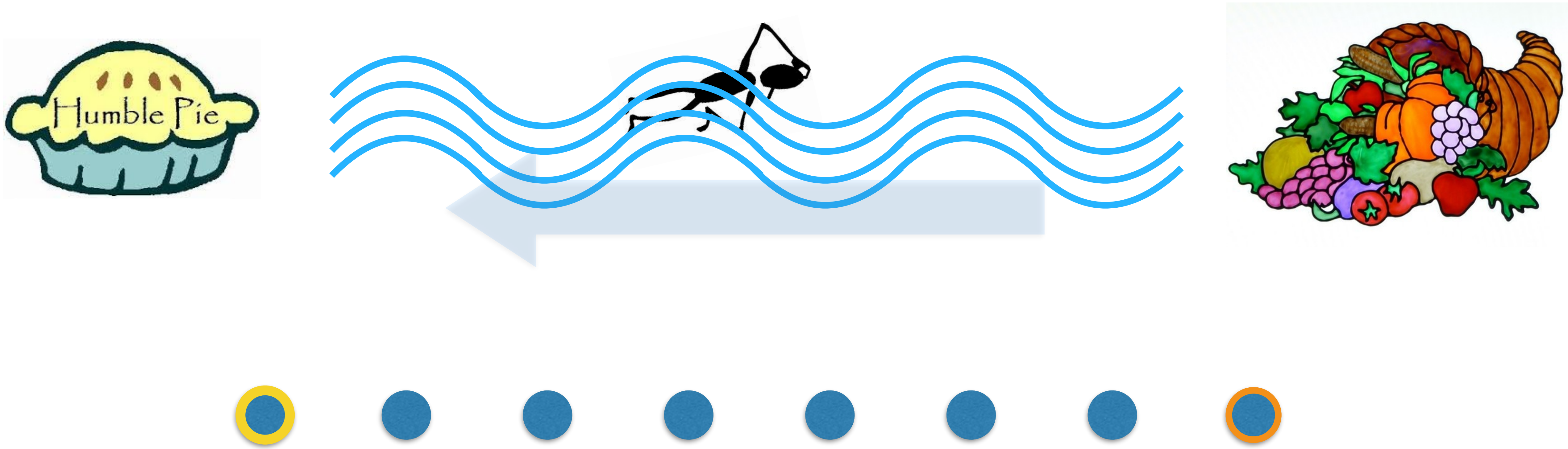


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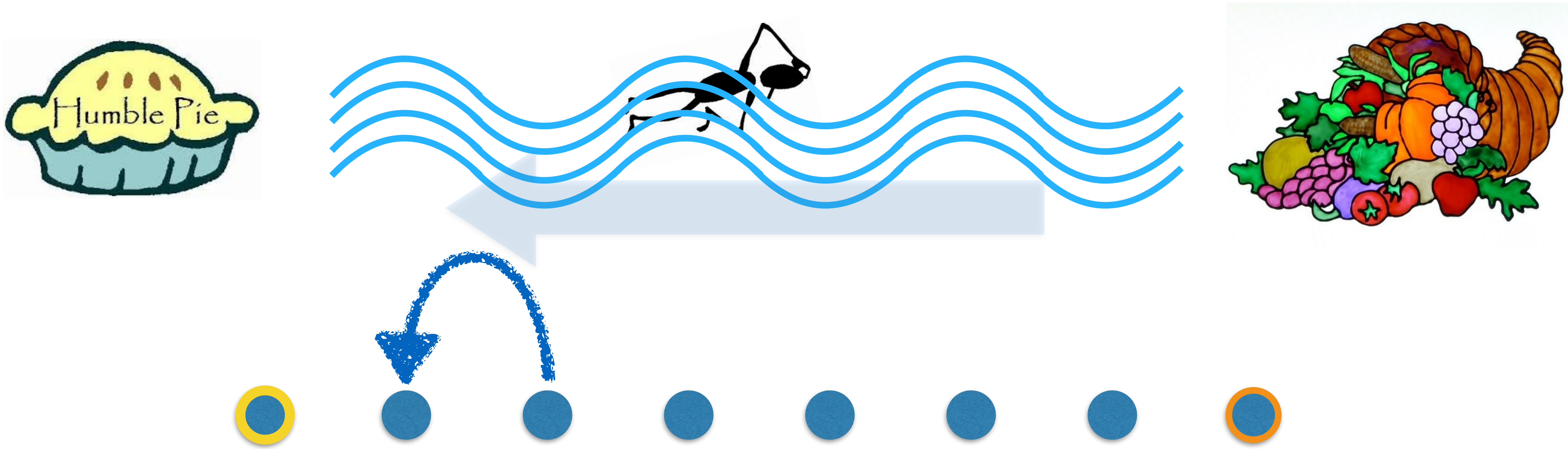


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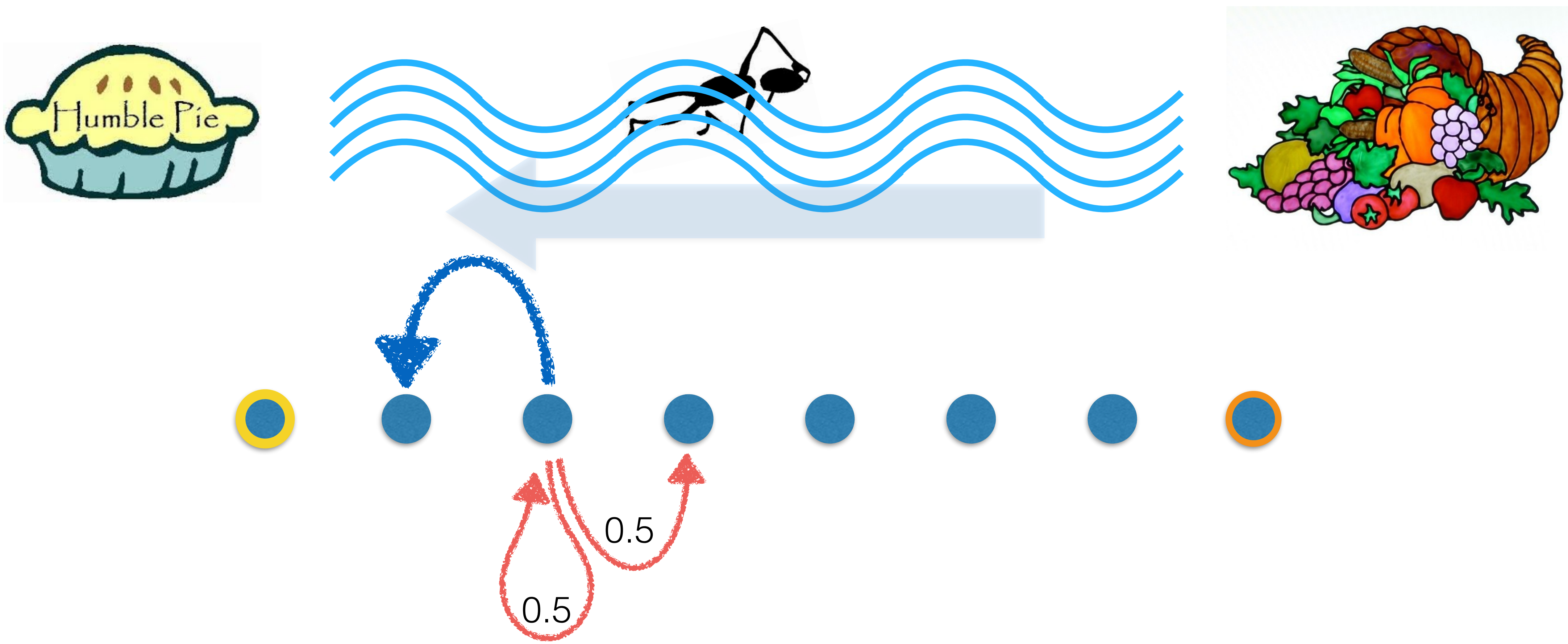


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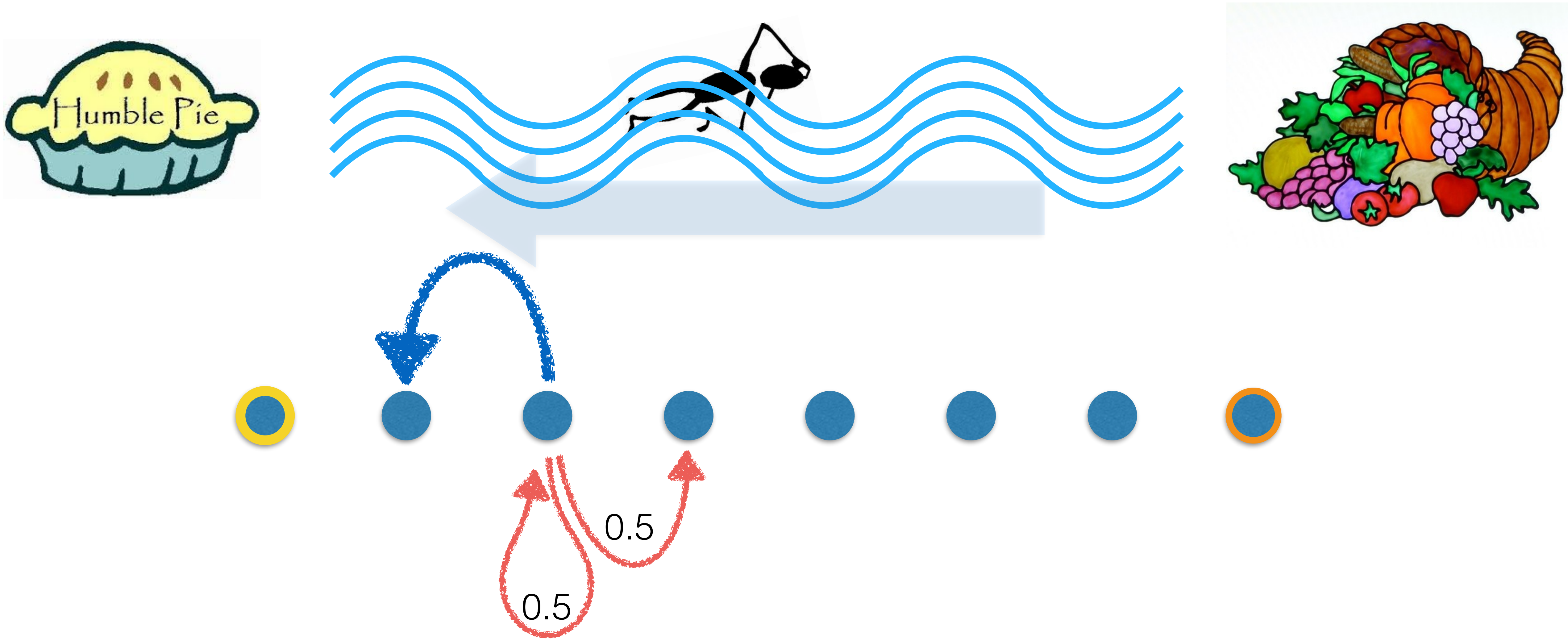


- **Reckless** data collection: Choose the actions ***uniformly at random***!

Slide graphics courtesy of Ben van Roy.
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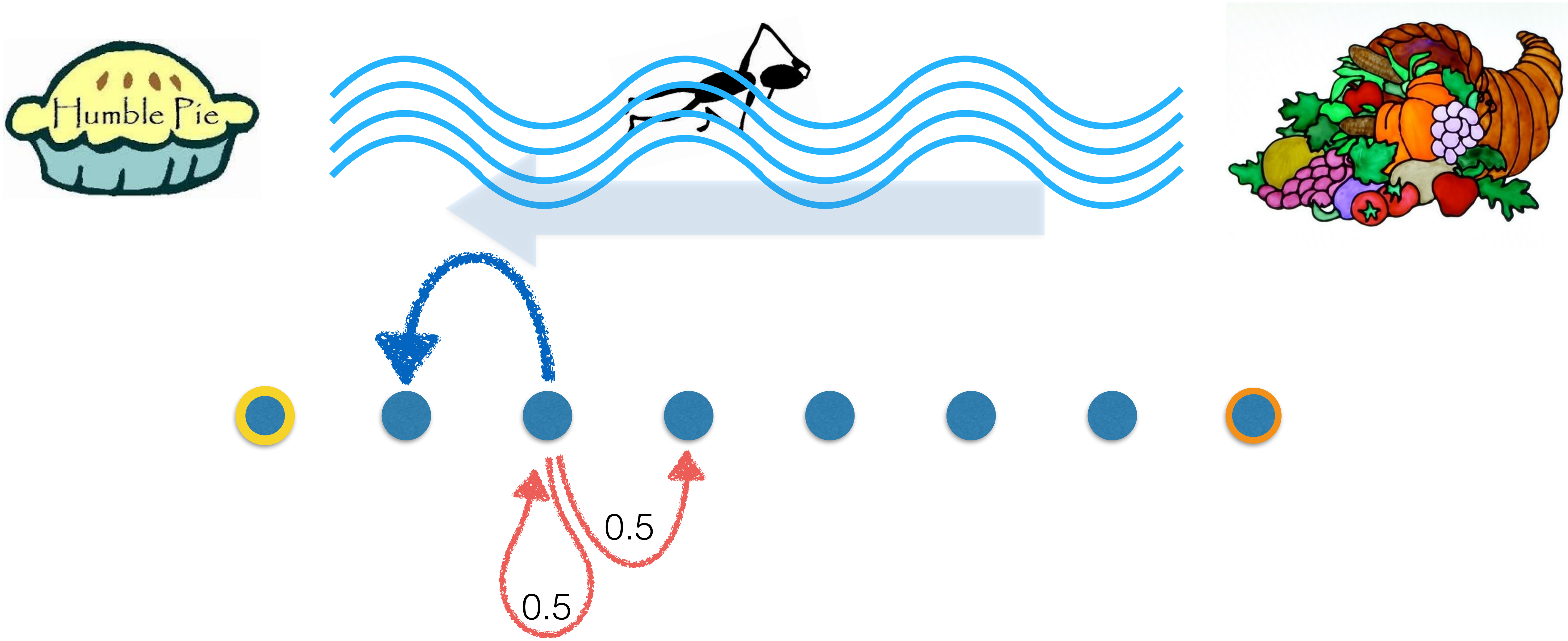


- **Reckless** data collection: Choose the actions ***uniformly at random***!
- **How much data** do we need to collect before we see the bounty for the first time, starting from the middle?

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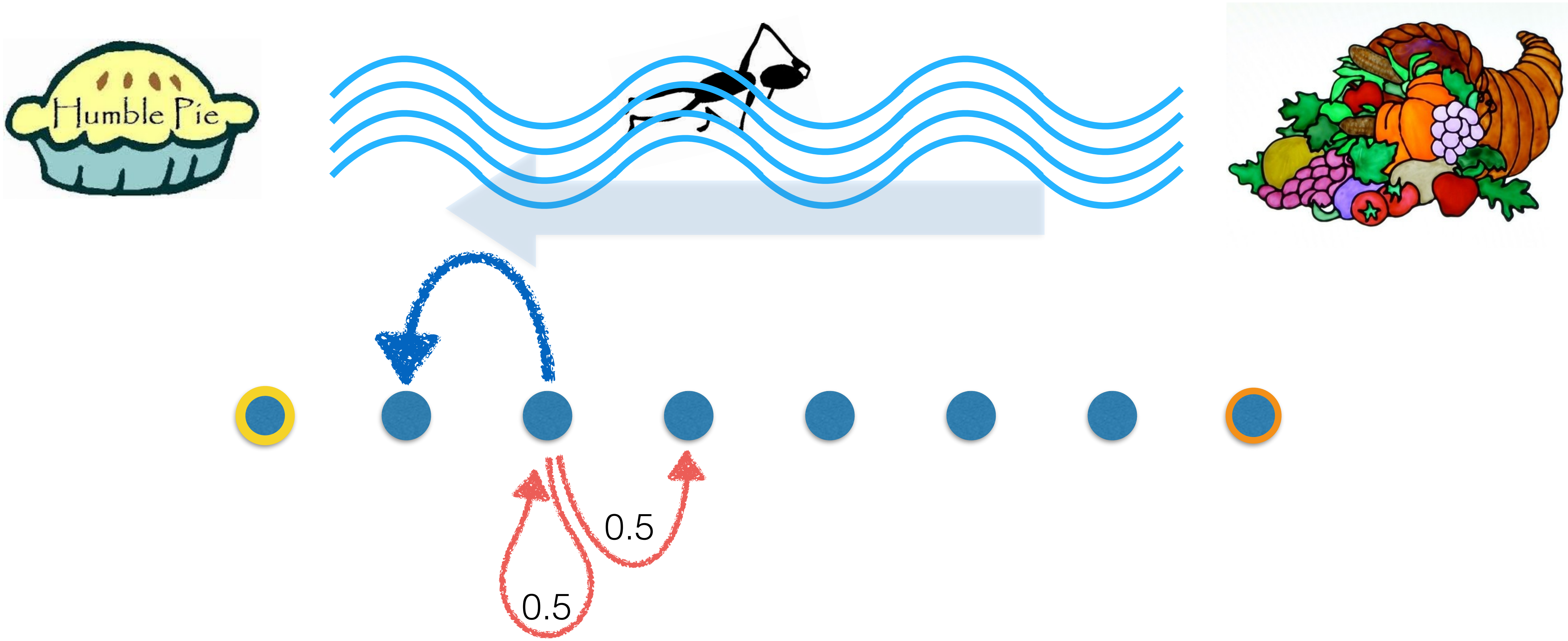
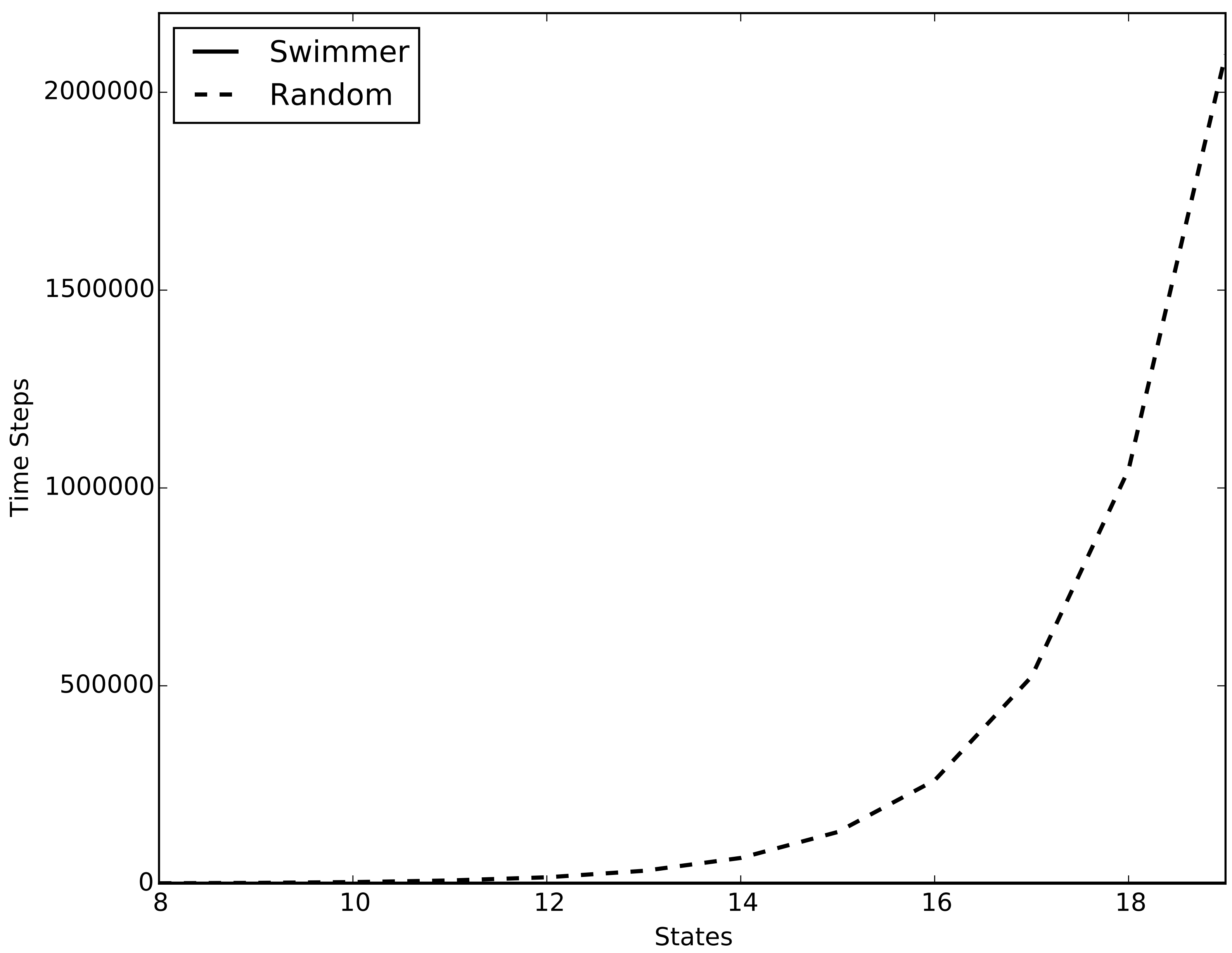
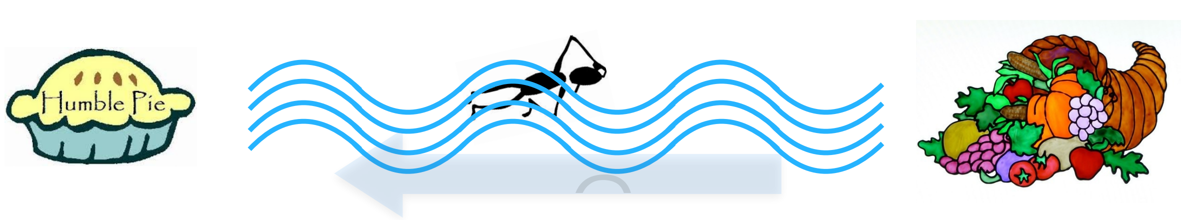



- **Reckless** data collection: Choose the actions ***uniformly at random***!
- **How much data** do we need to collect before we see the bounty for the first time, starting from the middle?
- How does this depend on the number of states?

Slide graphics courtesy of Ben van Roy.
Problem due to [Strehl-Littman,'08]

# Time before bounty is found



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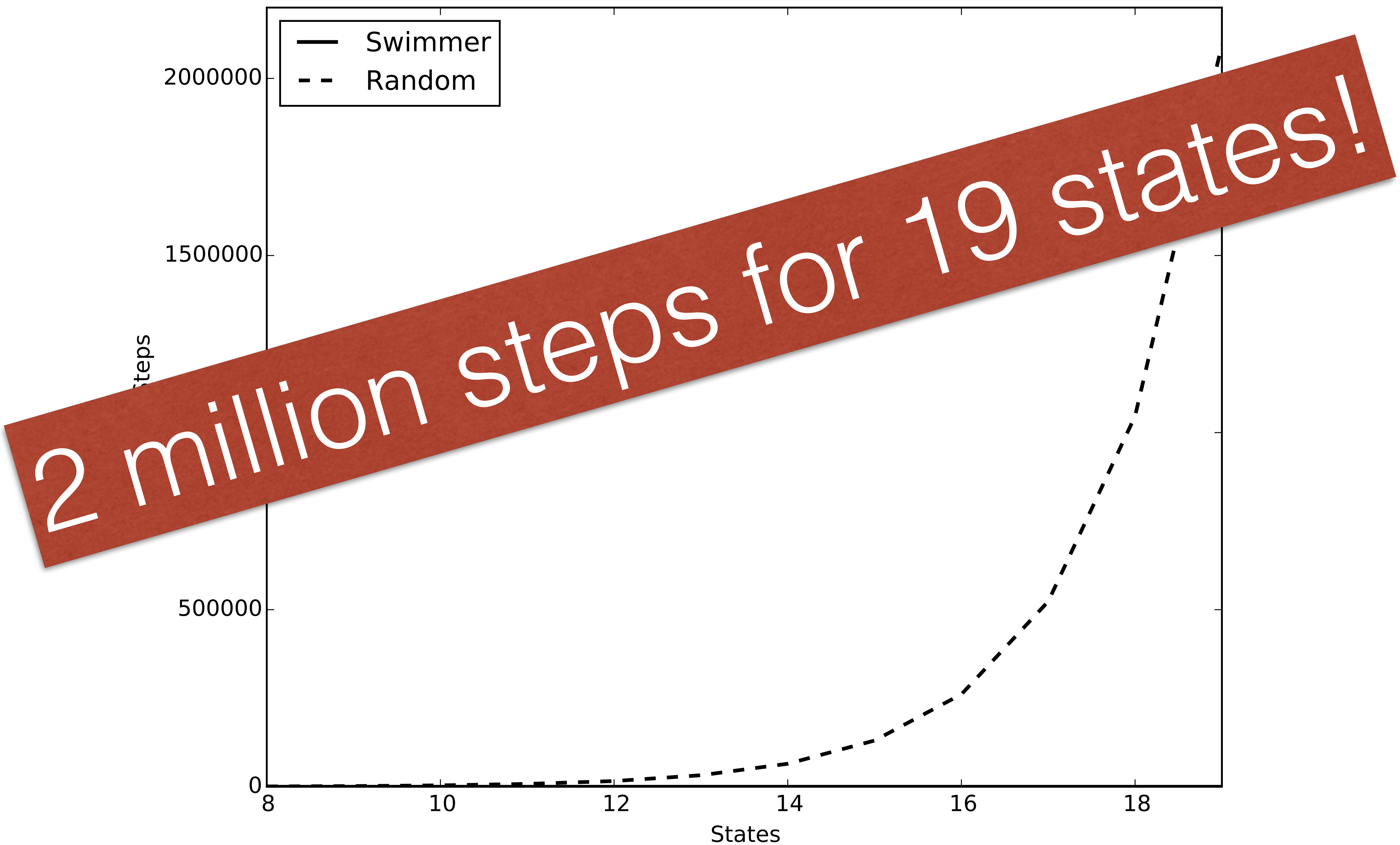


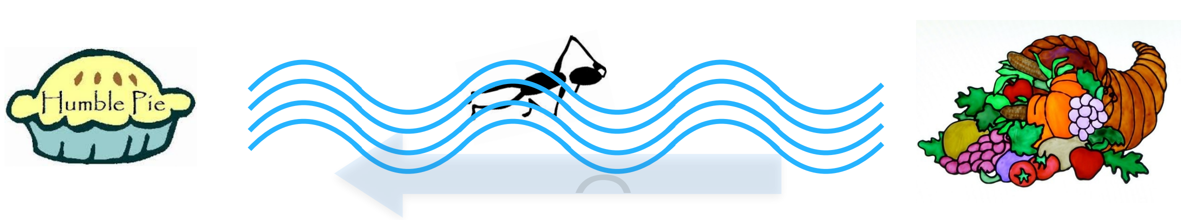

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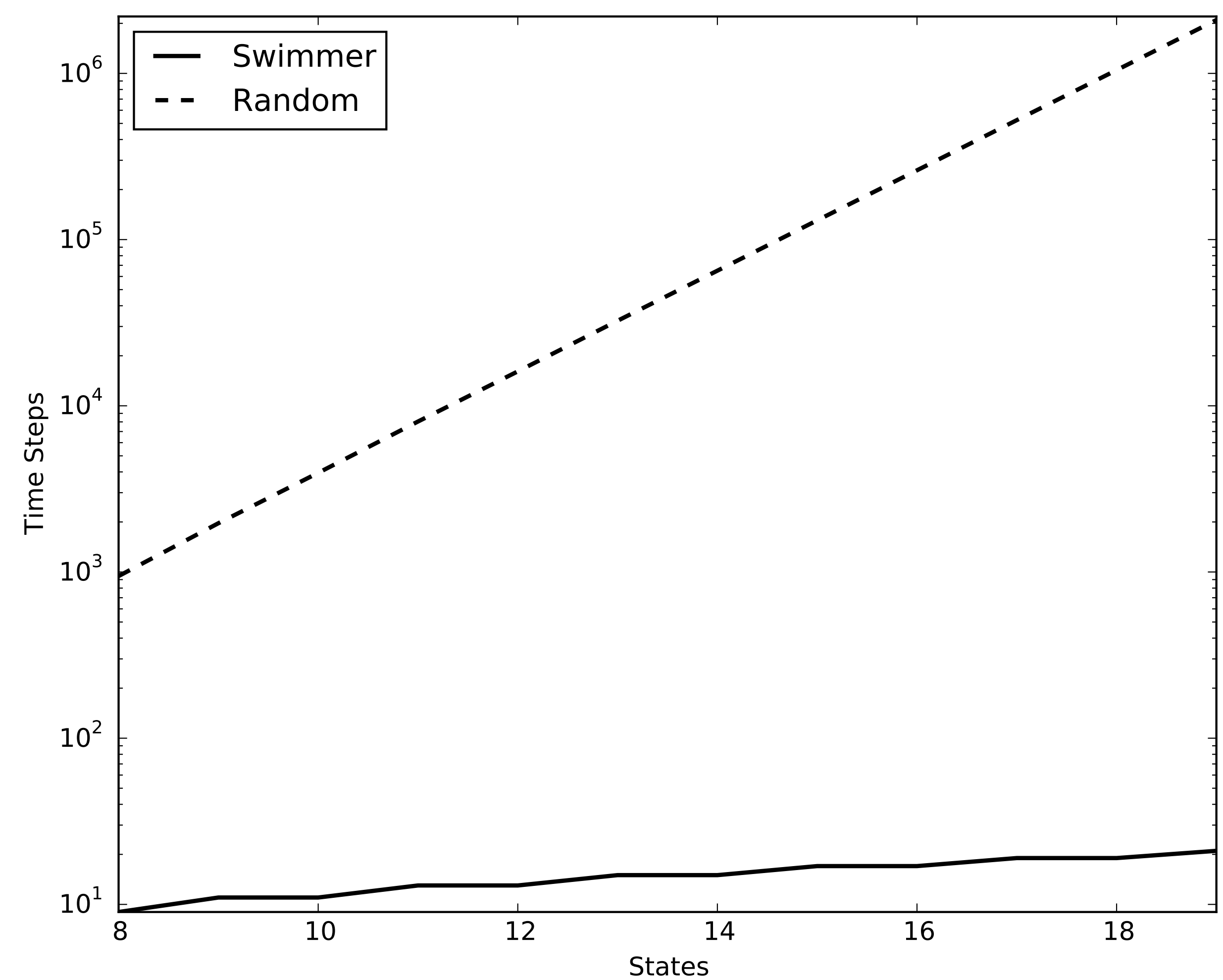
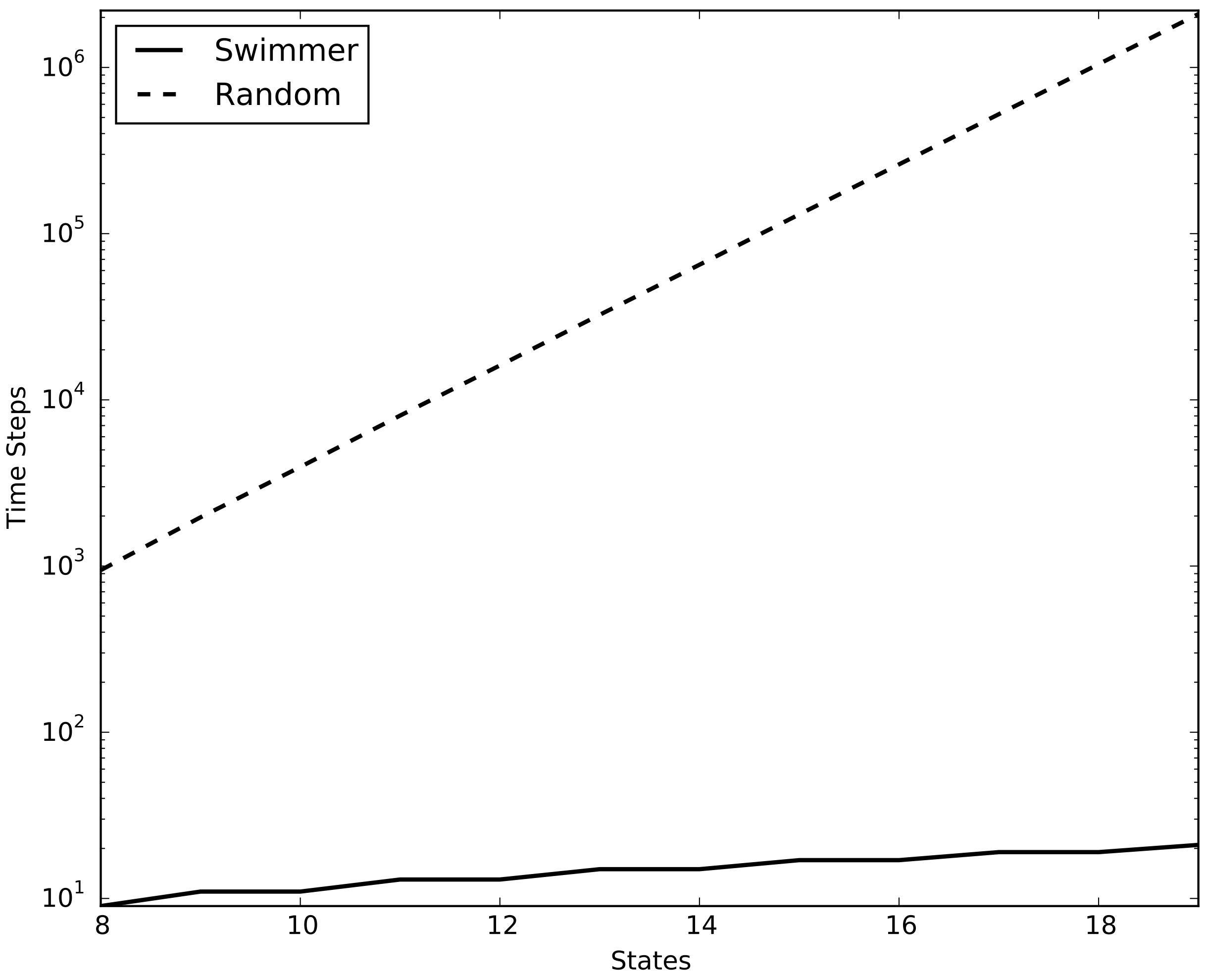


# Time before bounty is found

- Hitting time for random policy:

$$\Theta(2^n)$$



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- Hitting time for "swimming policy":

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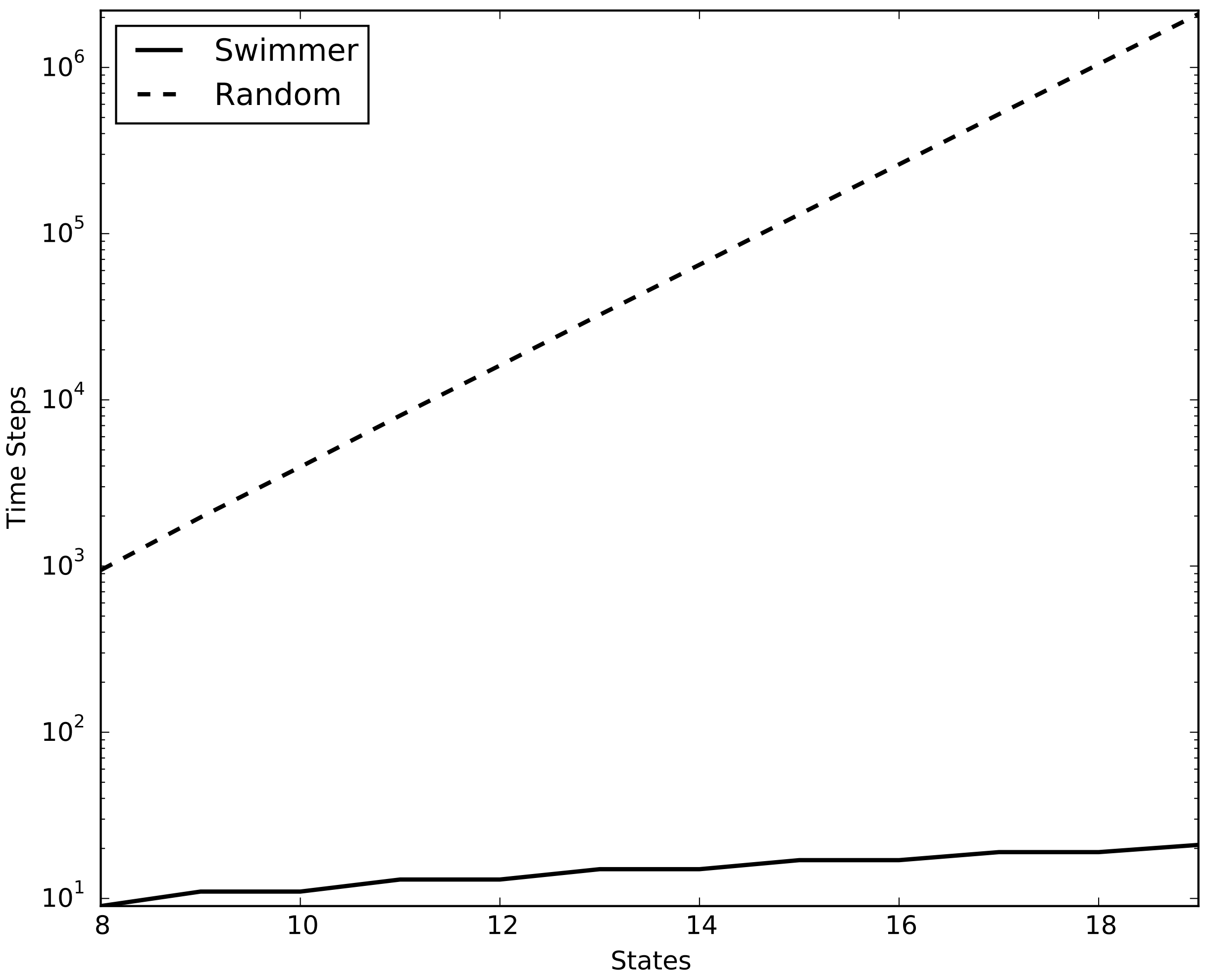

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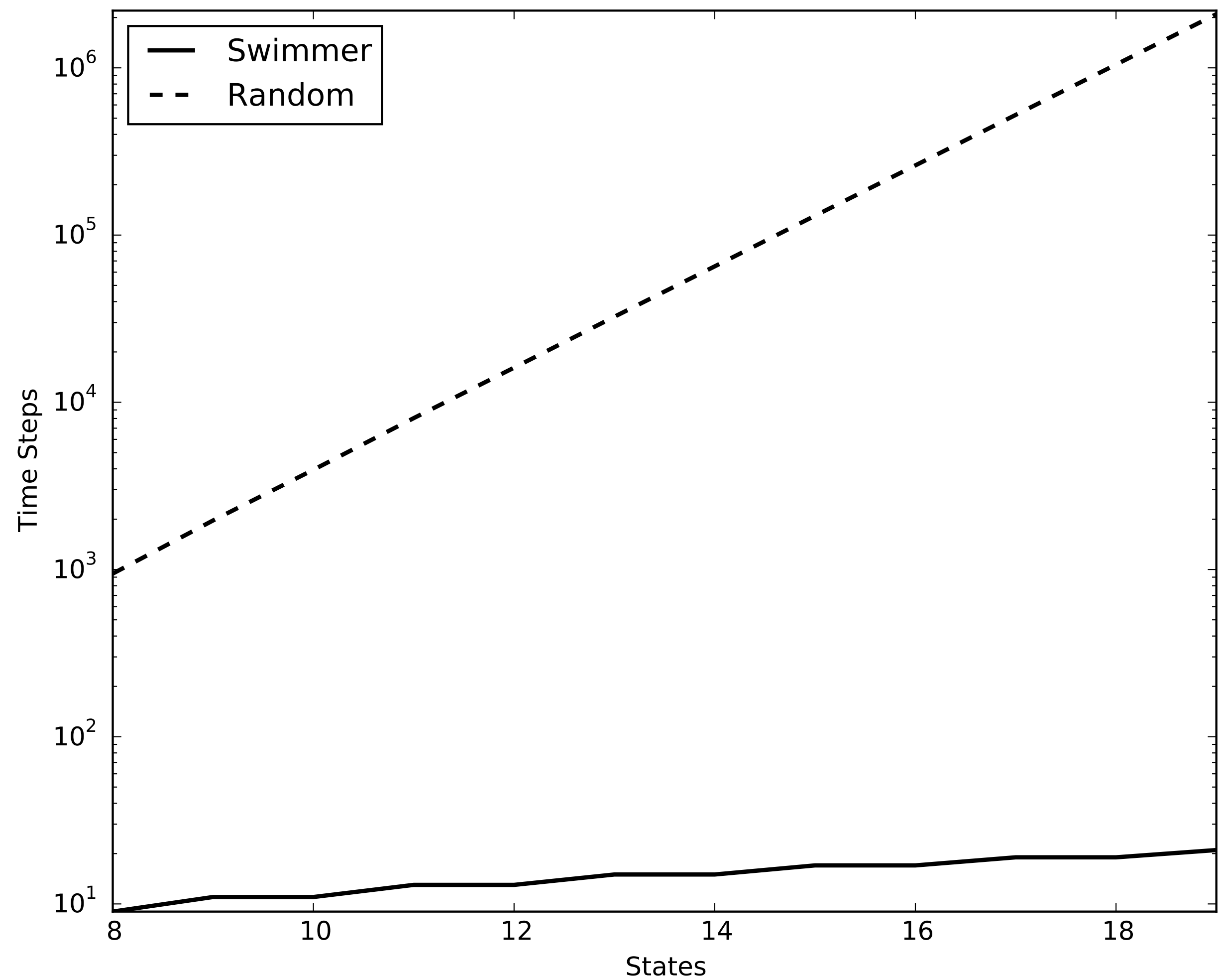


- Exponential gap on a very simple example!
..could be ***much*** worse on a real problem

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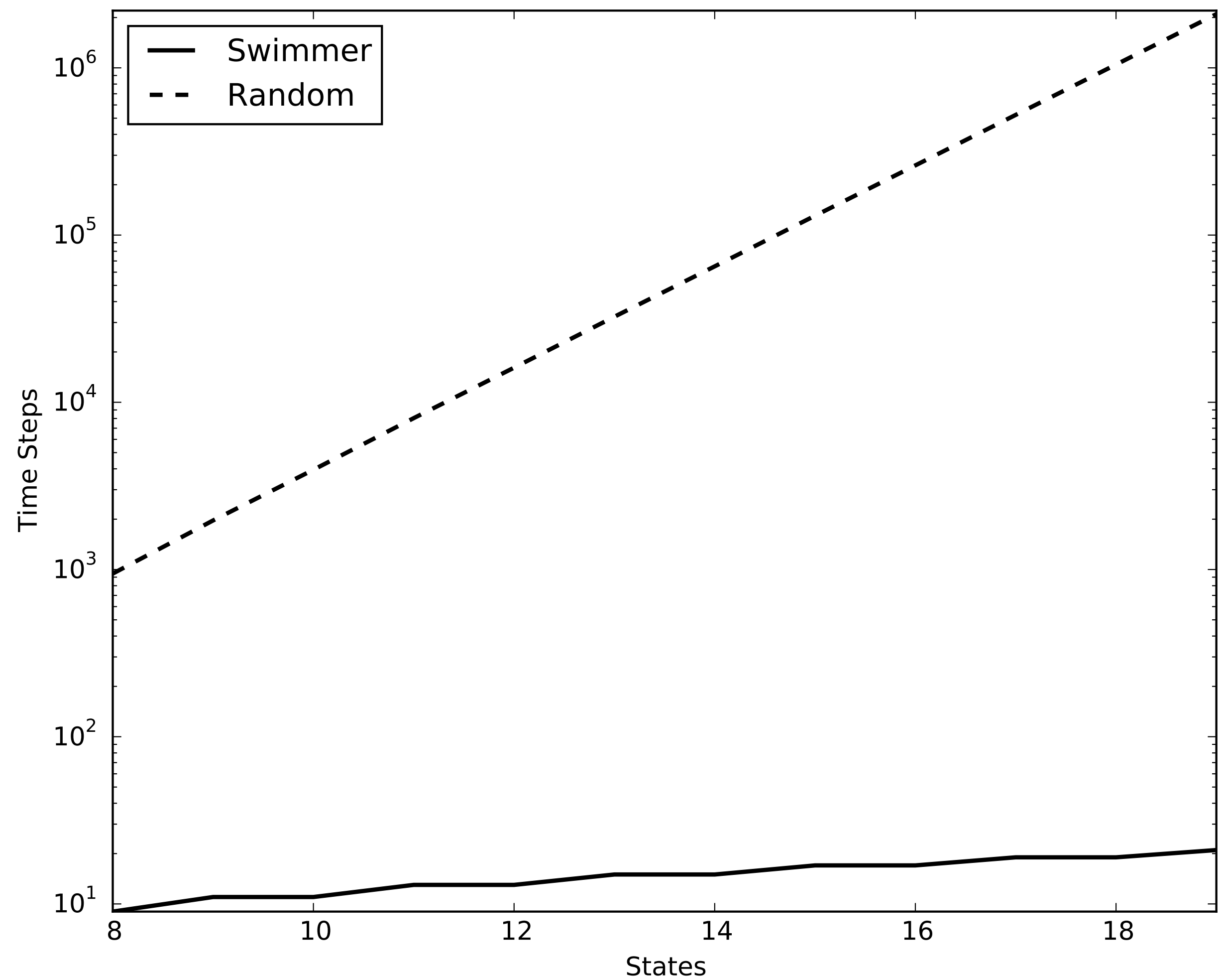


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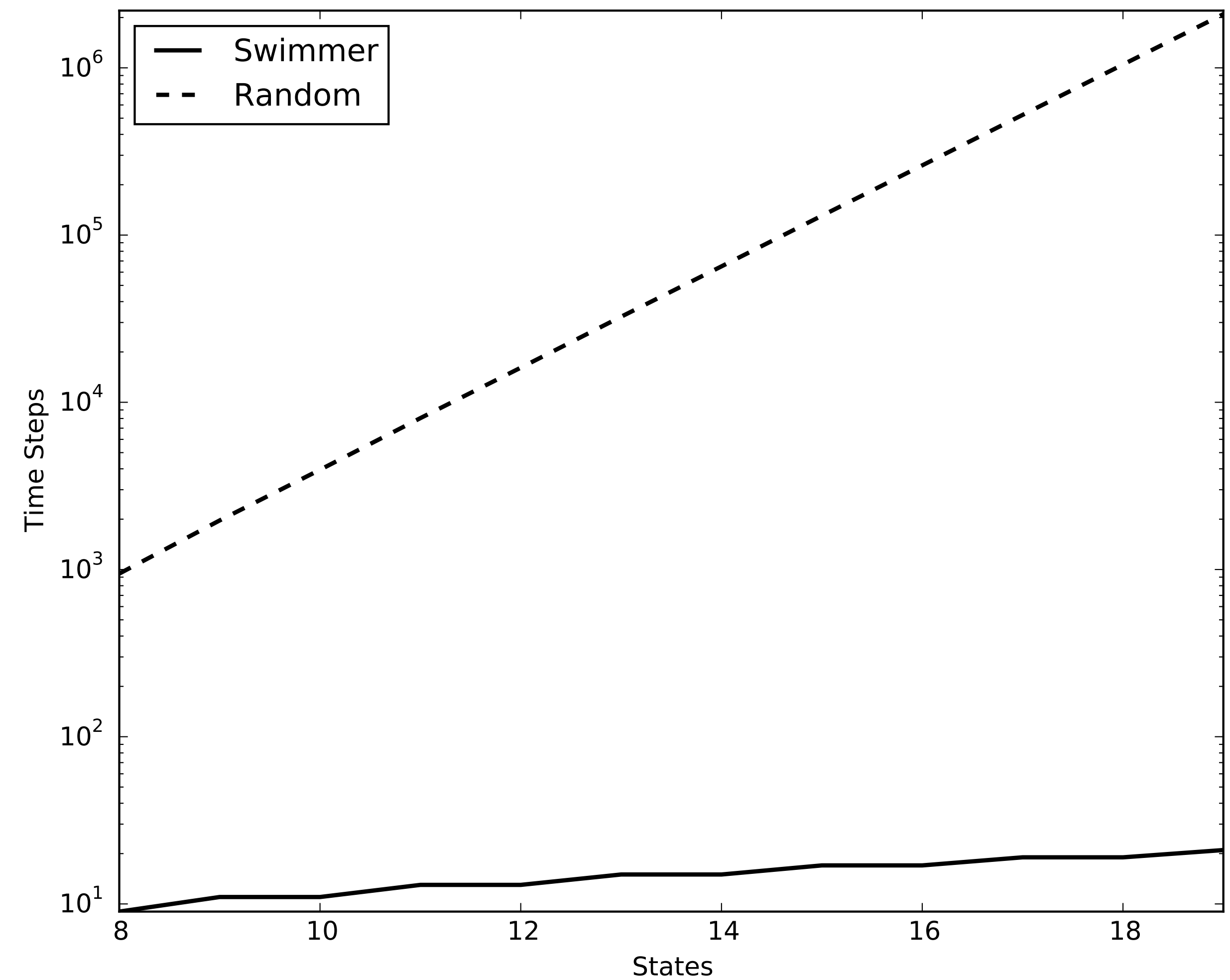


- Exponential gap on a very simple example! ..could be ***much*** worse on a real problem
- How "big" is big enough?
- Will we ever have enough data? Can we do better?

# Changing the game...

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- Allow data to be collected by a policy we select

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- Can we design more efficient data collection policies?

# Standard RL Approach

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- Repeat:

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# Standard RL Approach

- Repeat:
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- “epsilon-greedy”, “Boltzmann exploration”
- “Dithering”

# What happens with dithering in RiverSwim?

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What is the policy learned initially?
How long do we need to wait until the bounty is first collected?

# What happens with dithering in RiverSwim?

Dithering is NOT sufficient
Need smart exploration methods

What is the policy learned initially?
How long do we need to wait until the bounty is first collected?

# How do we evaluate a data collection strategy?

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# How do we evaluate a data collection strategy?

- How much data is needed to find a good policy?
  ..reward collected/lost during data collection does not matter: **"pure exploration" problem**

- How much reward is incurred during data collection? **"exploitation" problem**
  Must optimize *while* learning. Explore or exploit?
  Metric: **Regret**.

# The Exploitation Problem

# Optimism in the Face of Uncertainty

Lai and Robbins (1985), Burnetas and Katehakis (1996),
Auer, Cesa-Bianchi and Fischer UCB1 (2002), and many others

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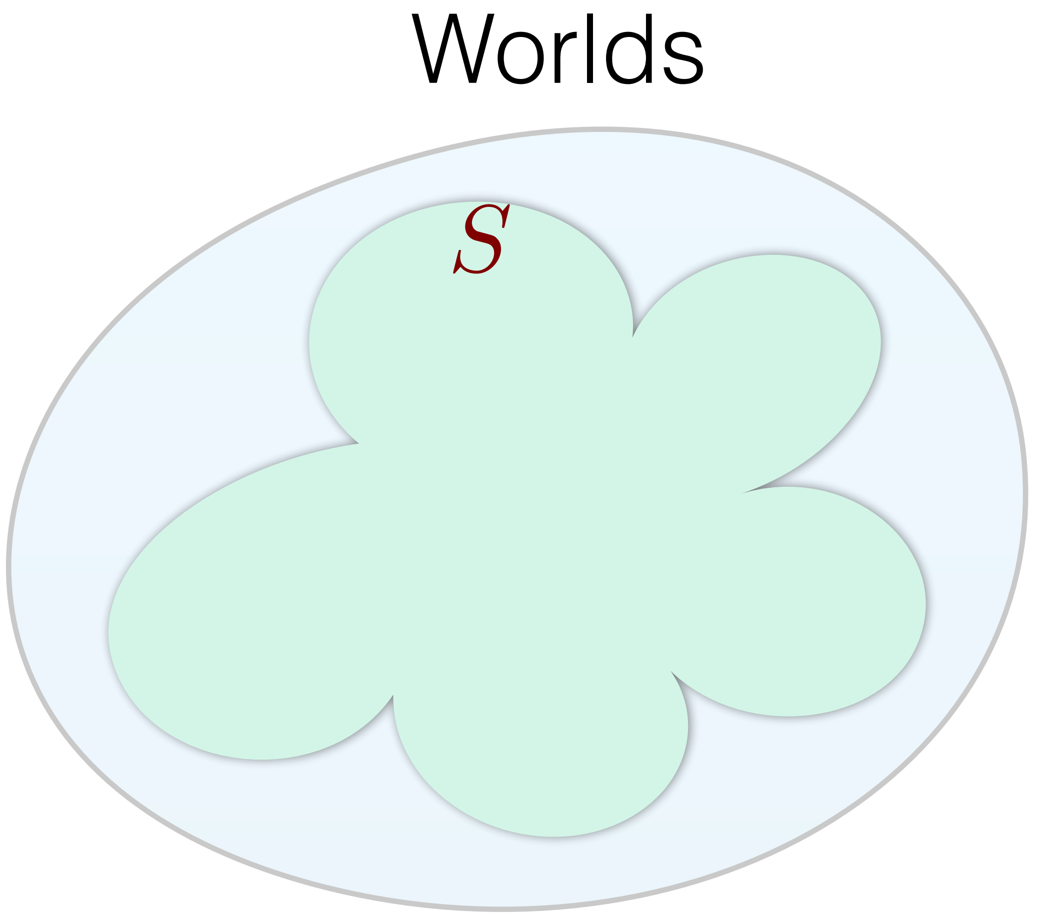


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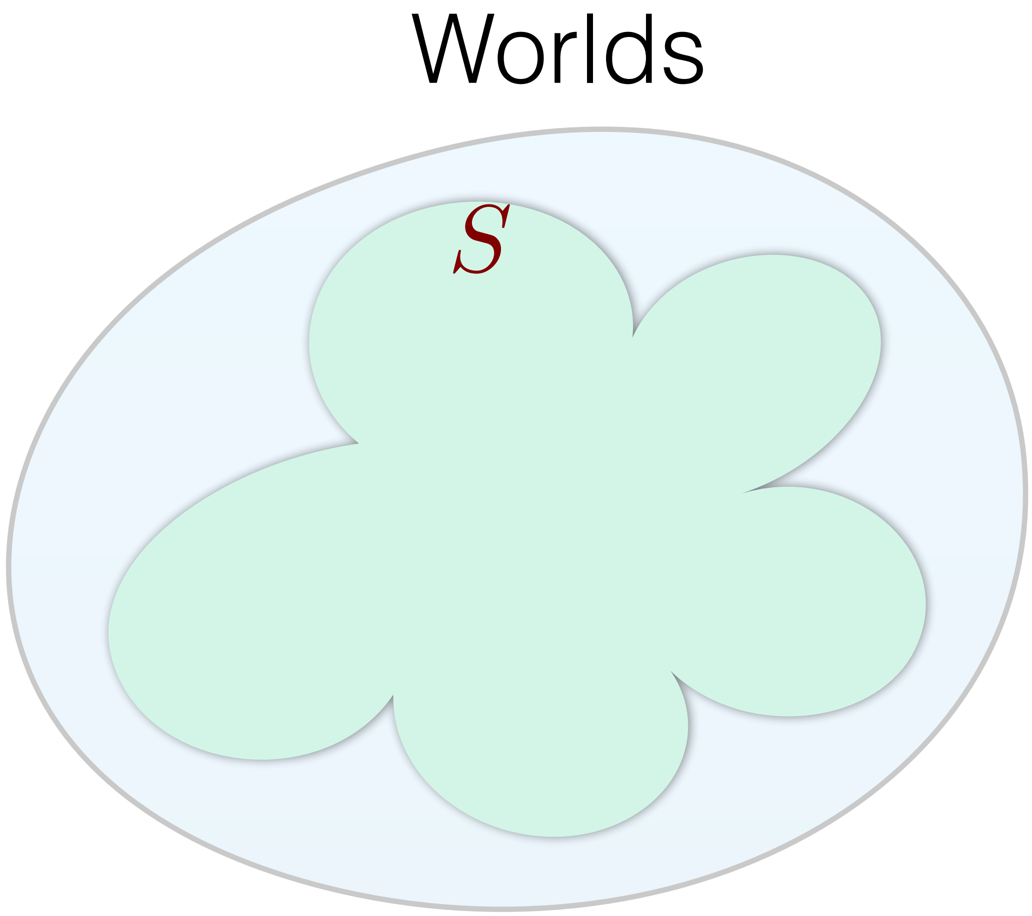
$$W = \operatorname*{argmax}_{W \in S} J(W)$$

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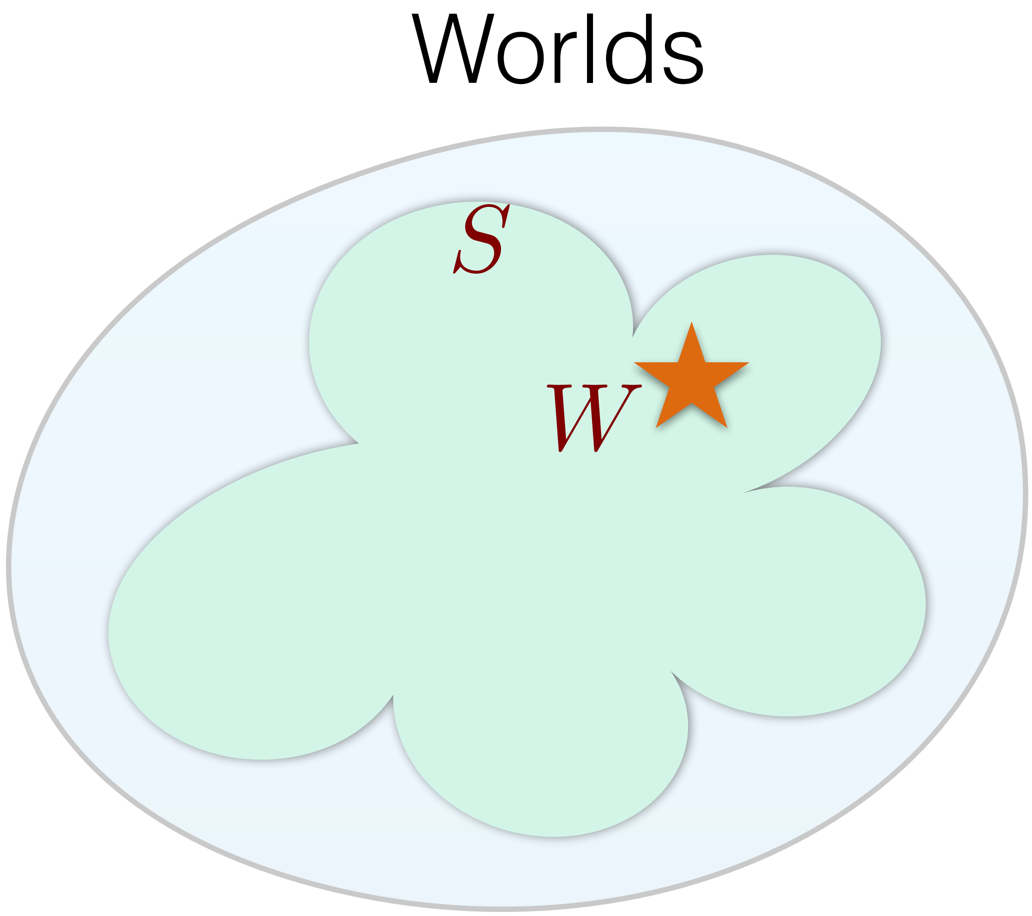


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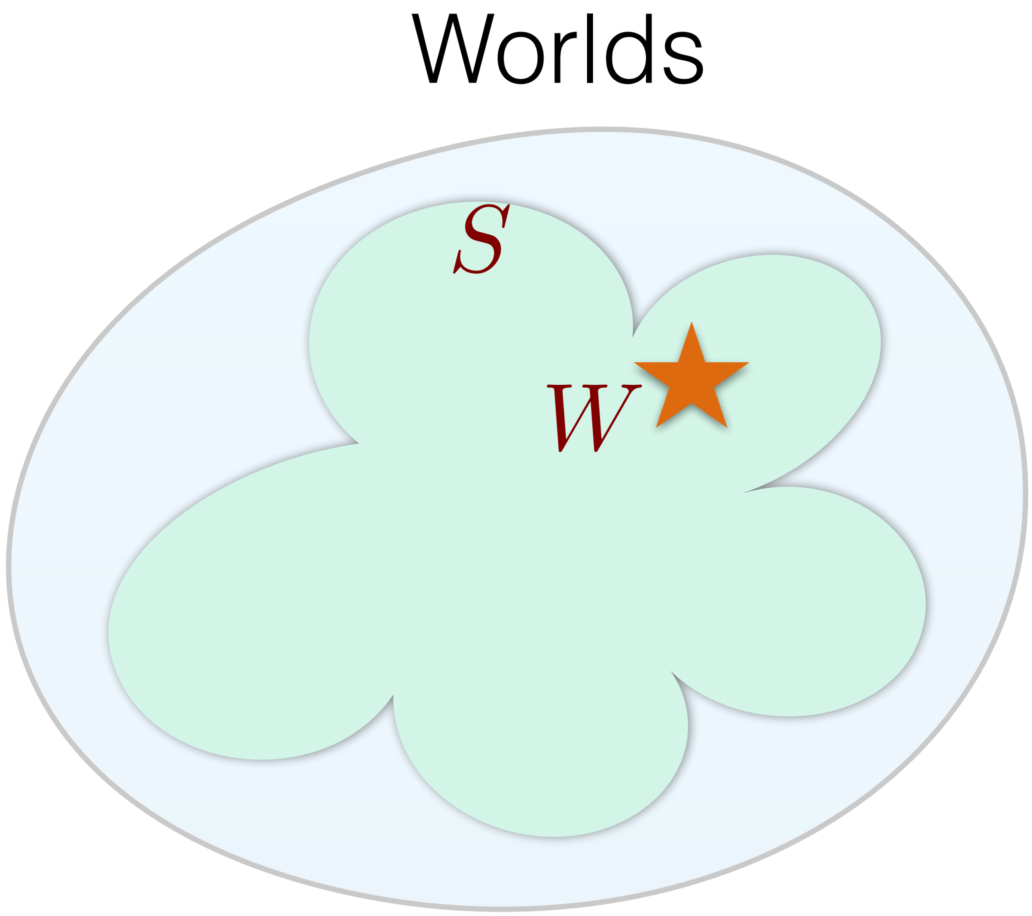


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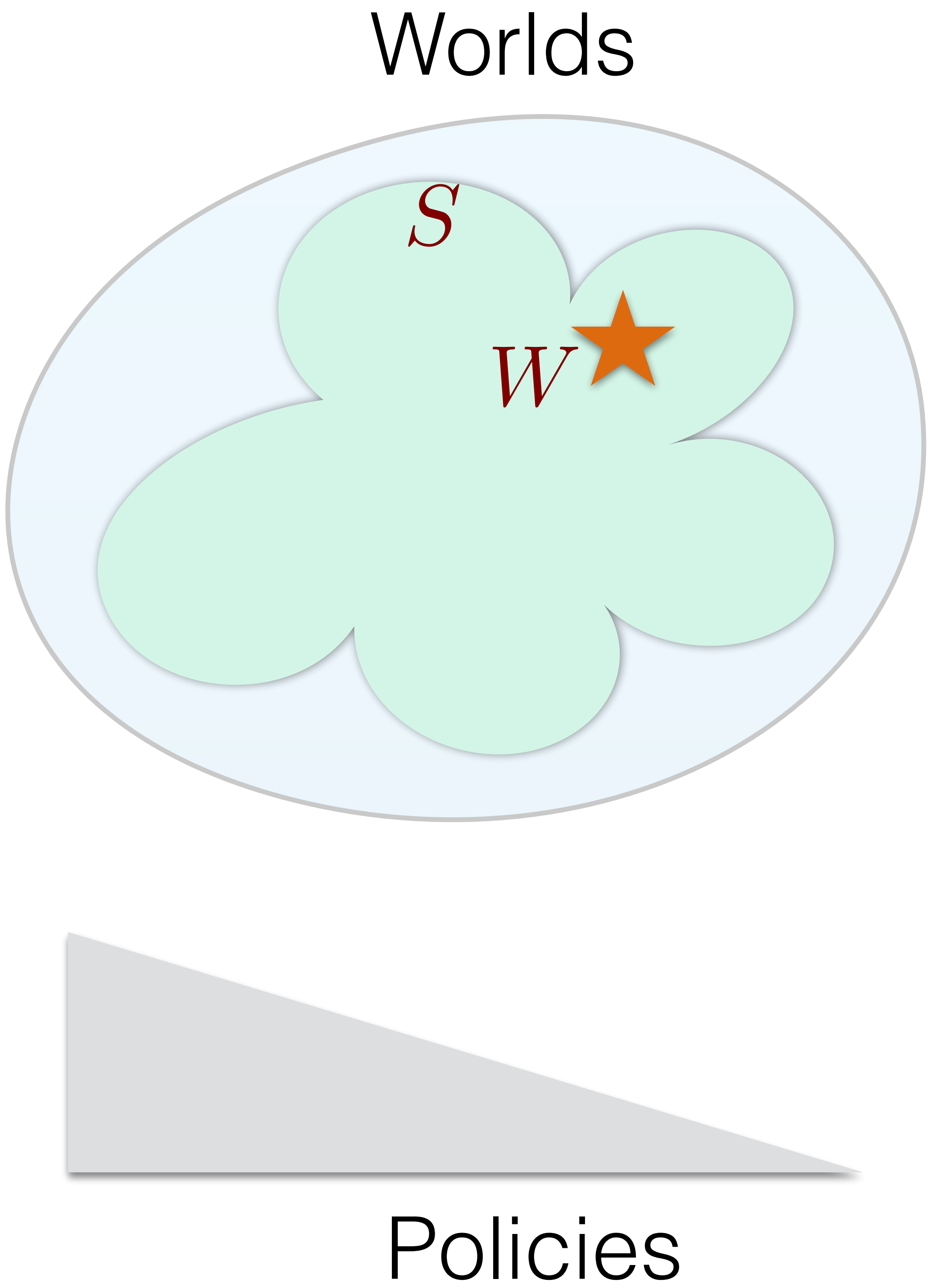


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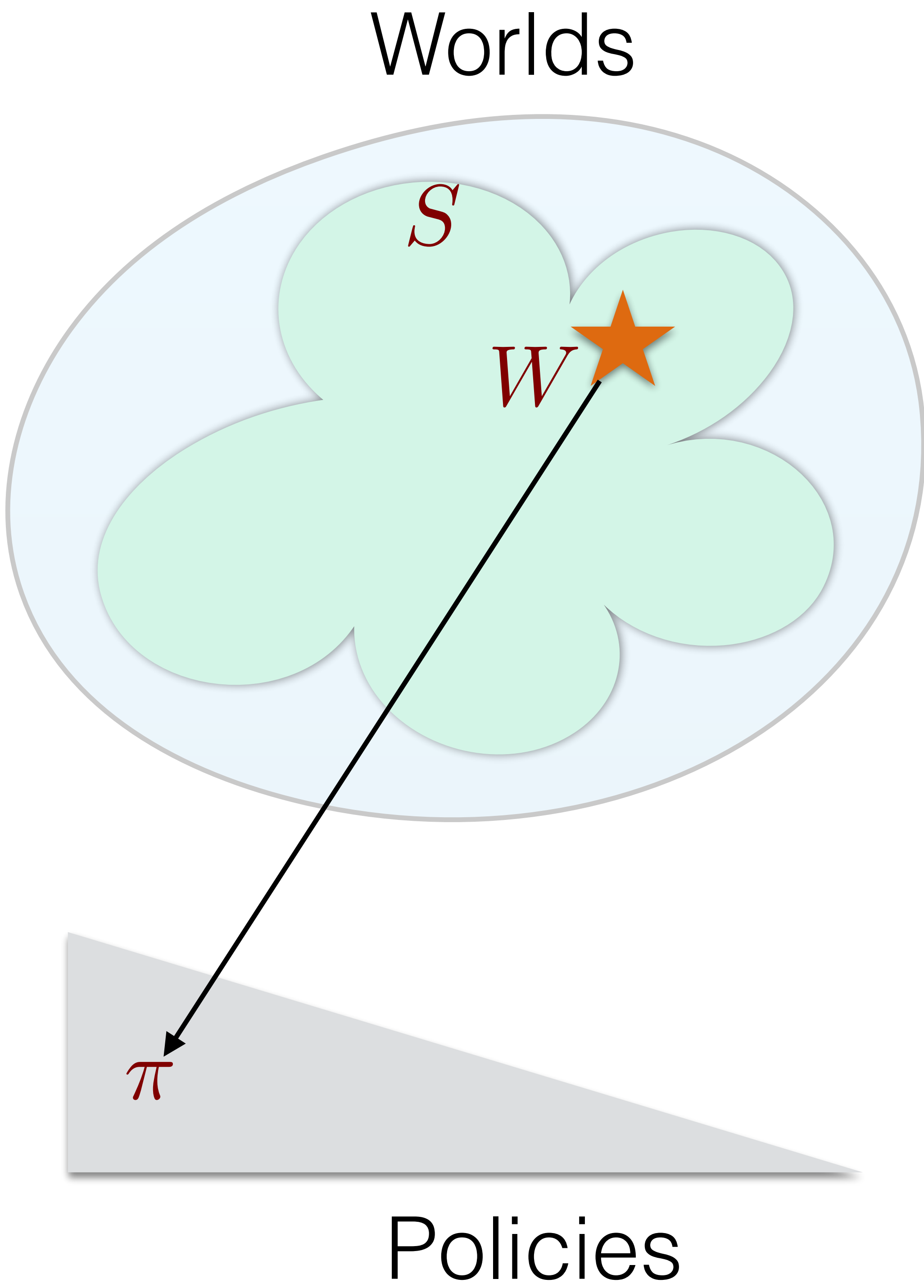


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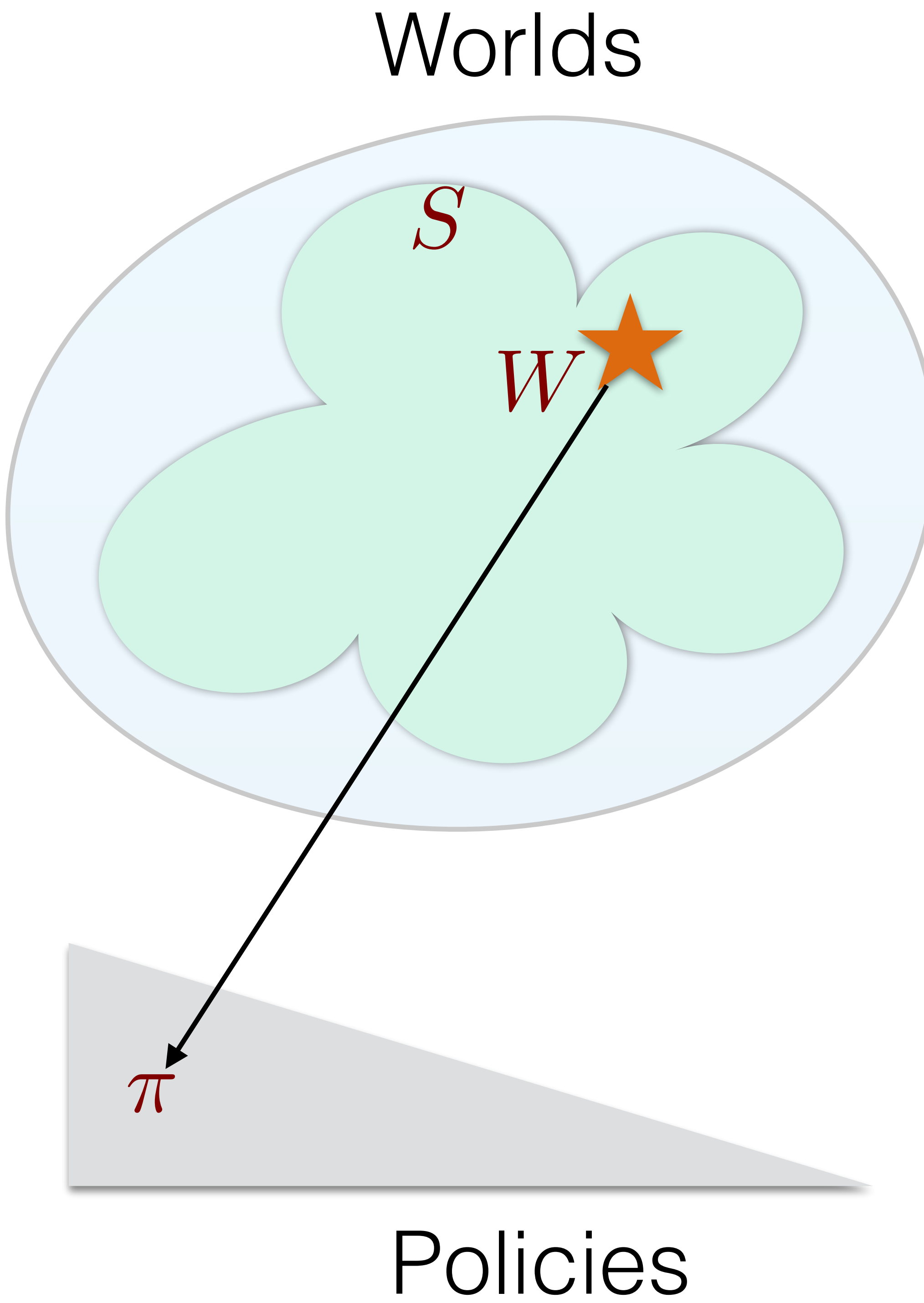


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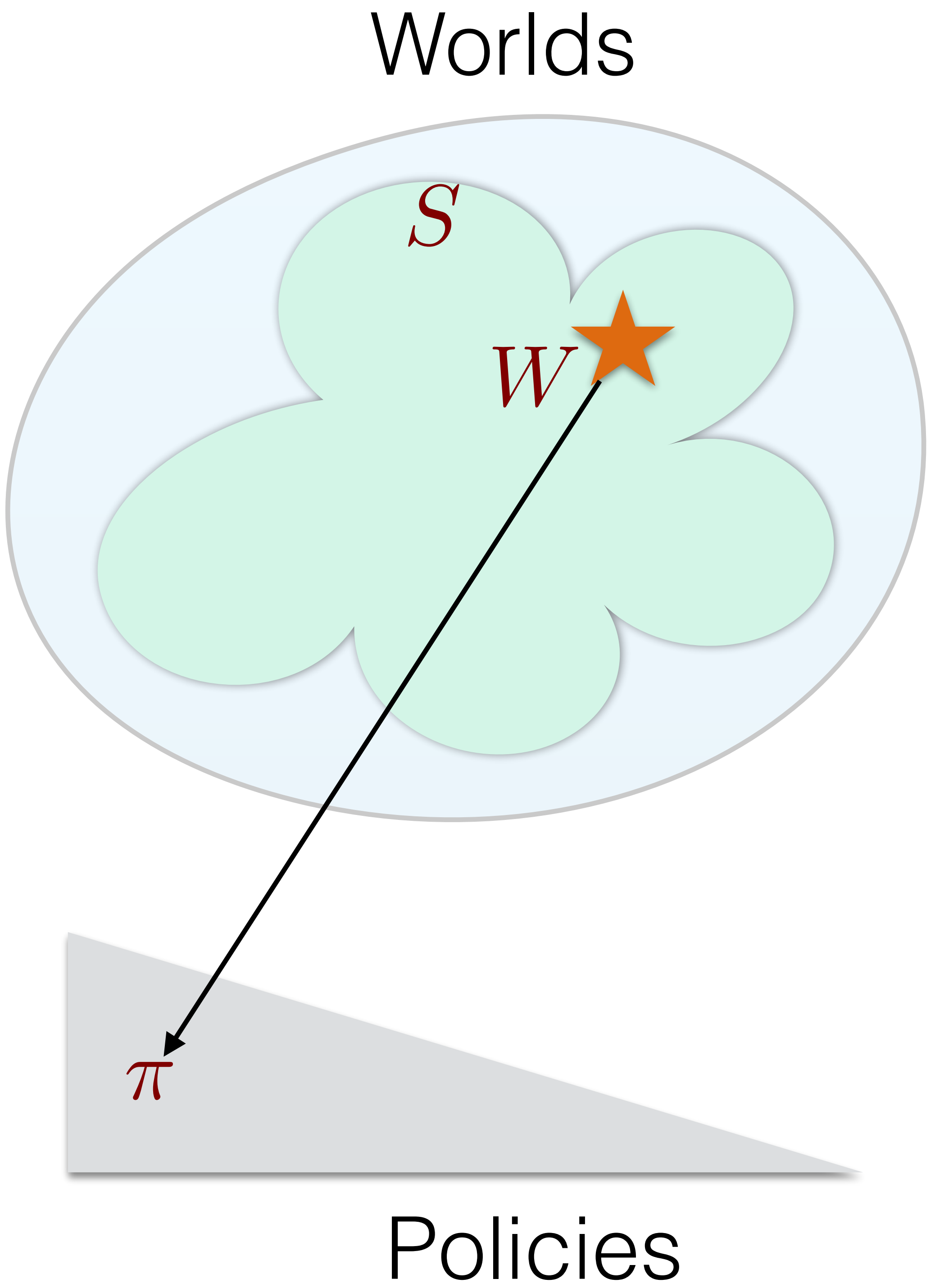
4. Use this policy until ***S*** significantly shrinks

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[Jaksch-Ortner-Auer,’10]

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***S*** states, ***A*** actions, rewards in [0,1].

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OFU for finite problems: UCRL2

[Jaksch-Ortner-Auer,'10]

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**<u>Definition:</u>** Diameter := maximum of best travel times between pairs of states. River swim: ***D = S***

OFU for finite problems: UCRL2

[Jaksch-Ortner-Auer,'10]

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- <u>**Theorem:**</u> For any algorithm,

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# Posterior Sampling Reinforcement Learning

[Thompson, 1933(!), Strens '00]

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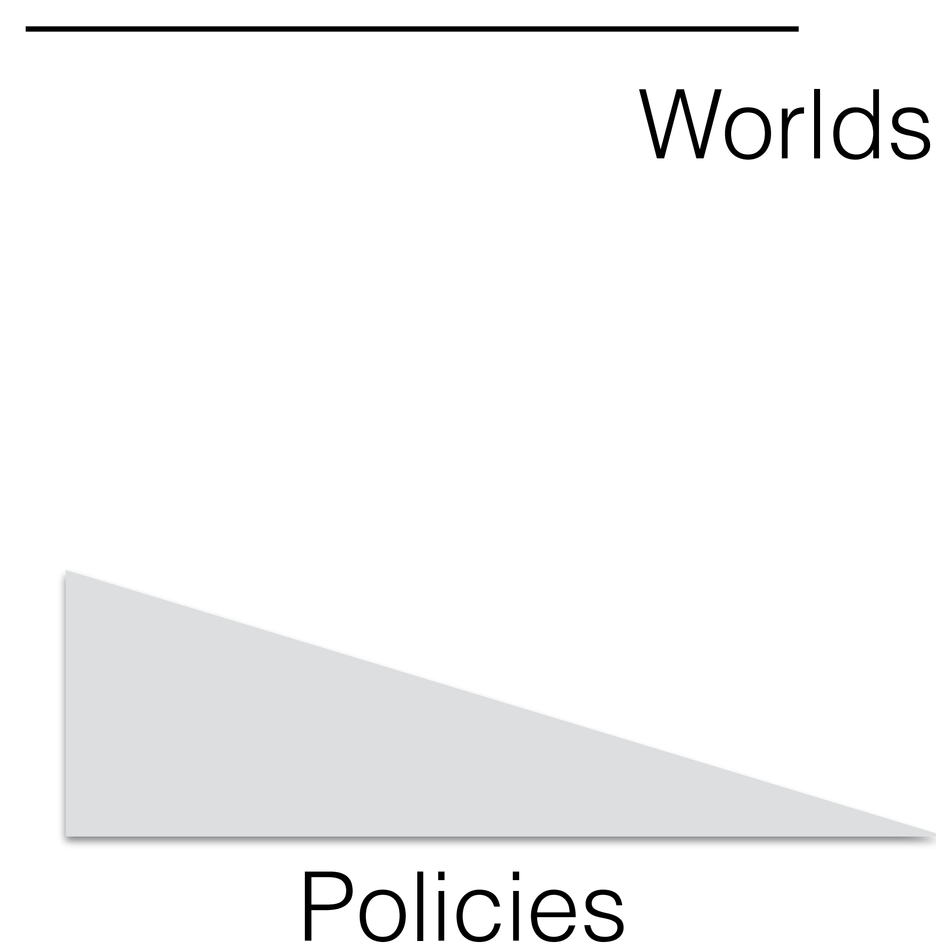


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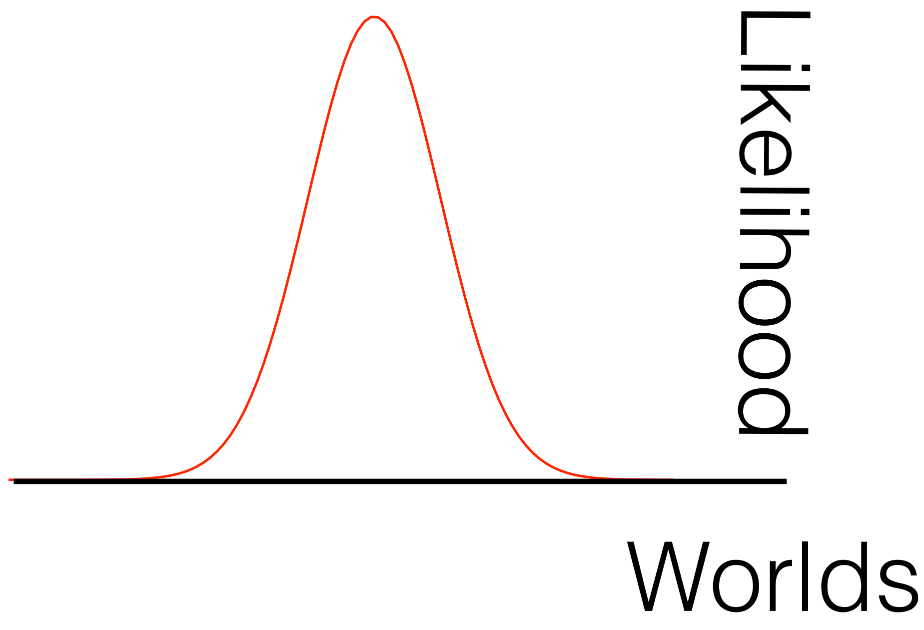



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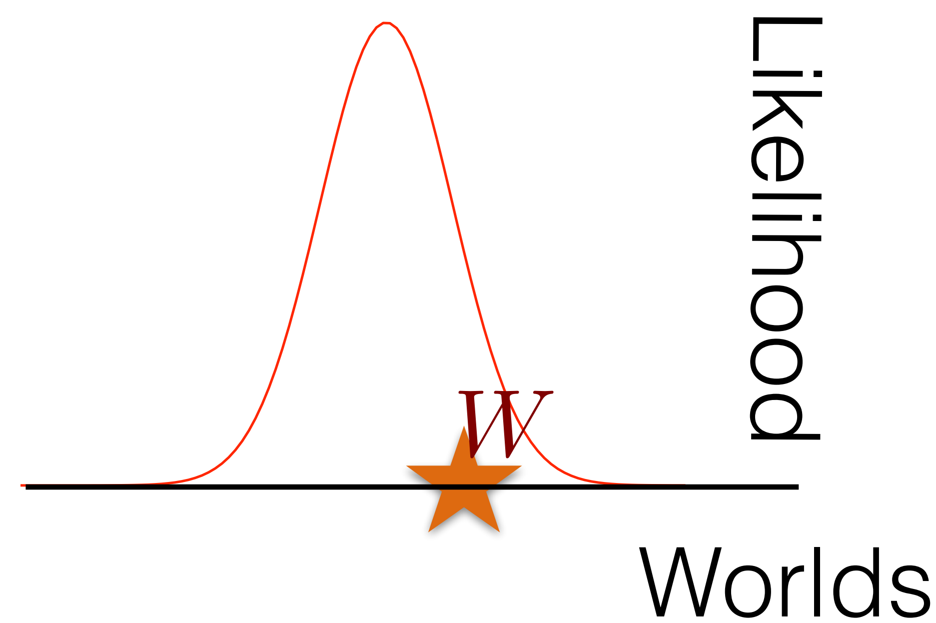



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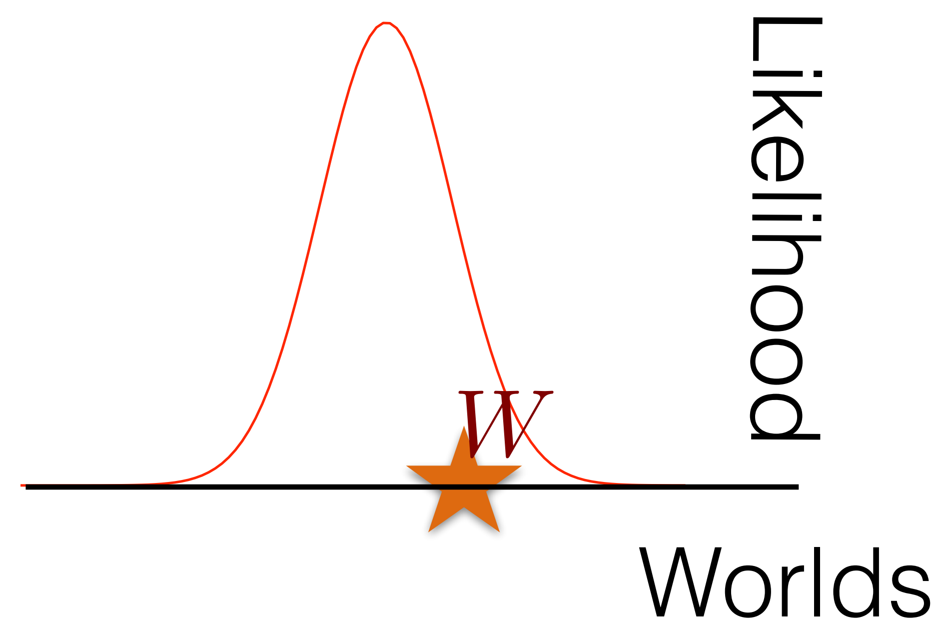



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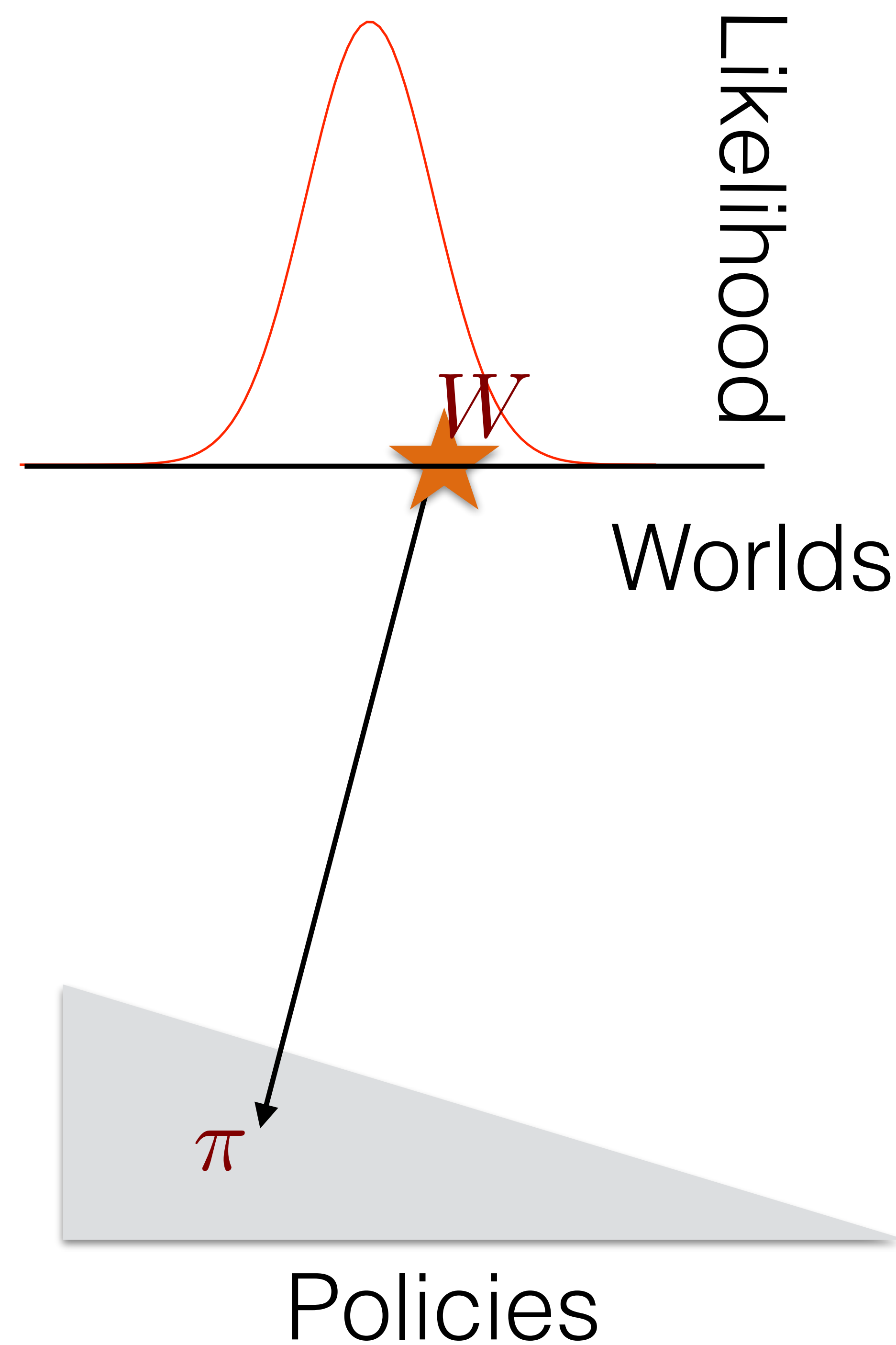


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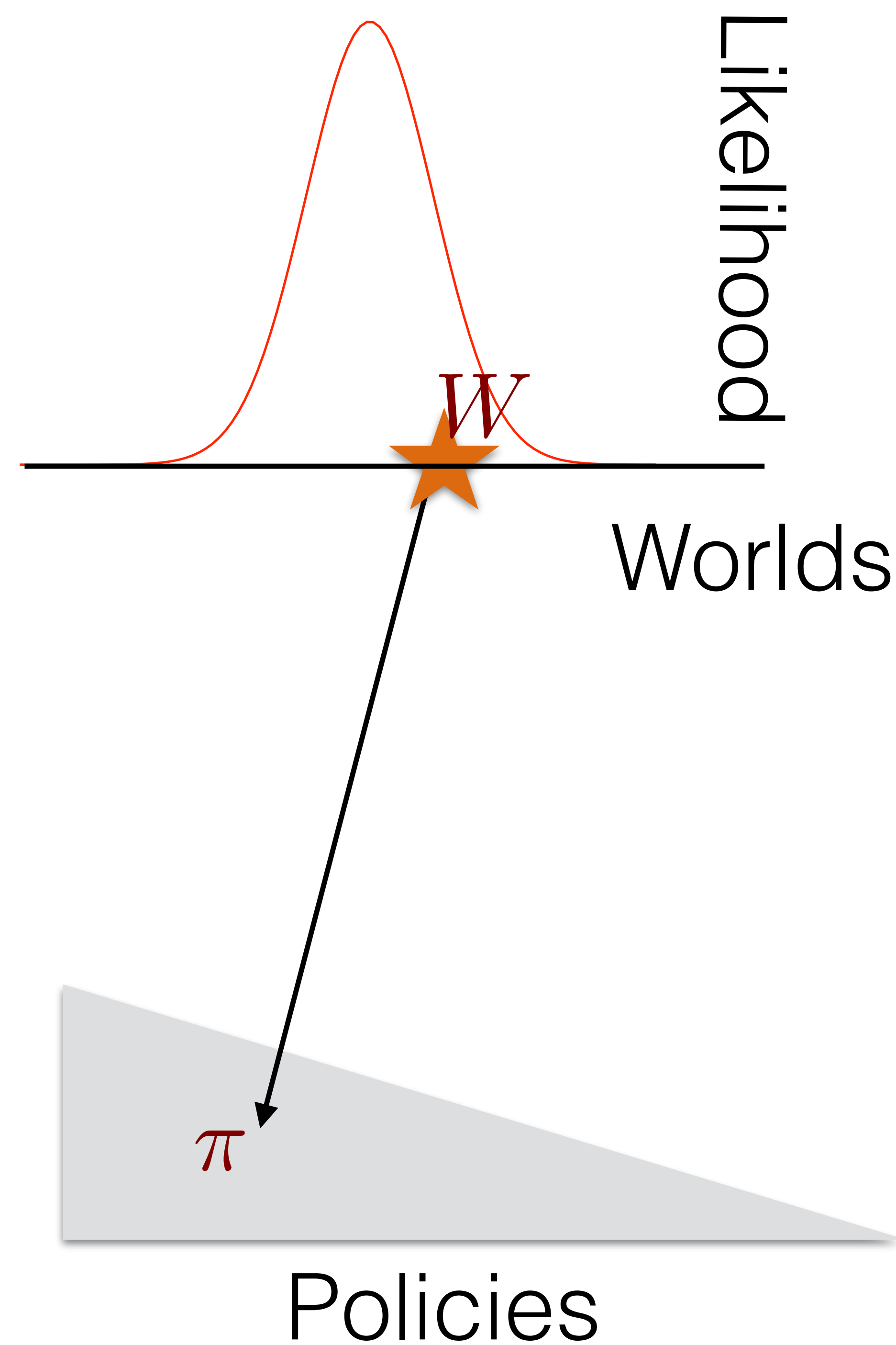


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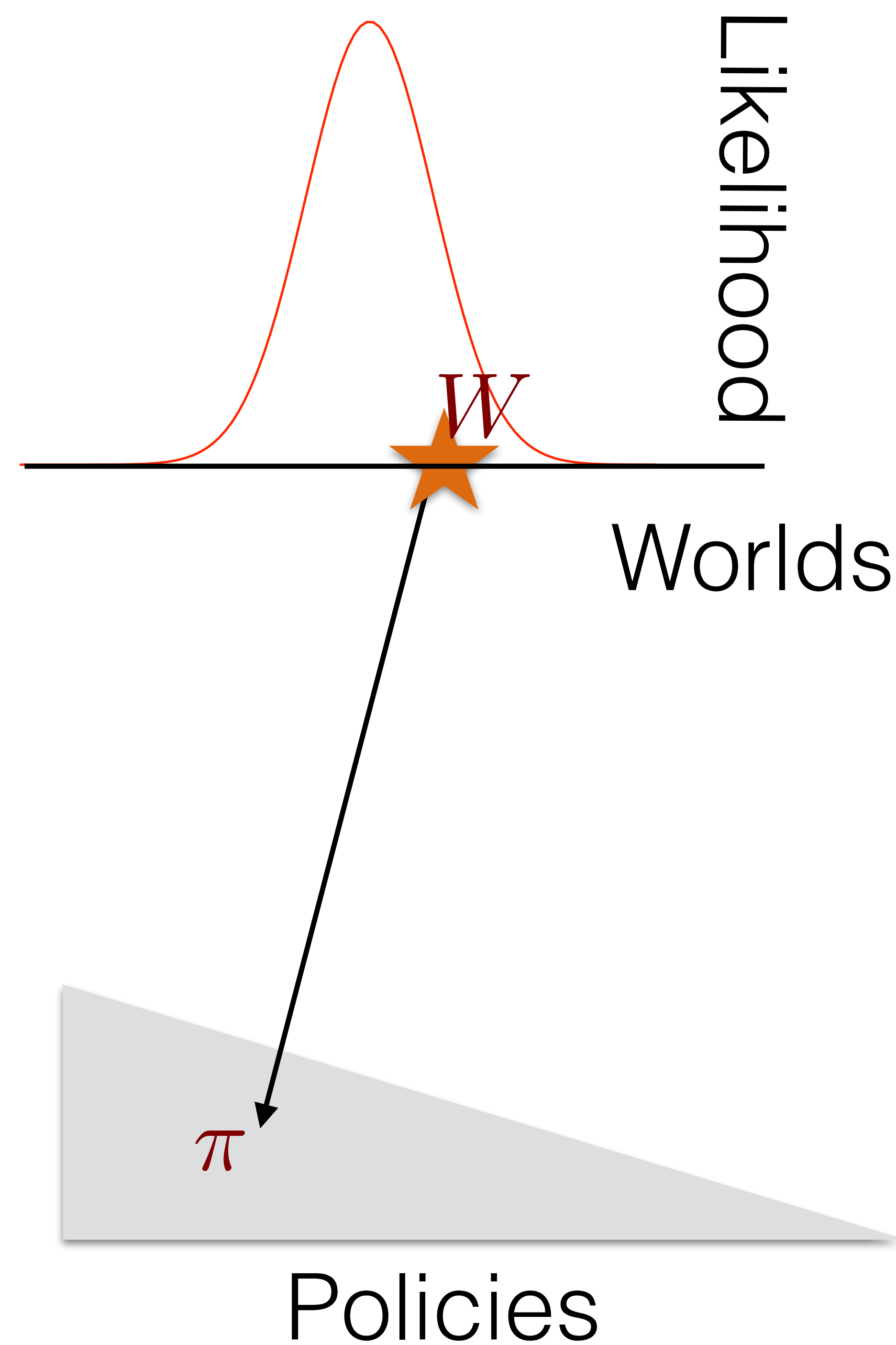
3. Use this policy a “little while”

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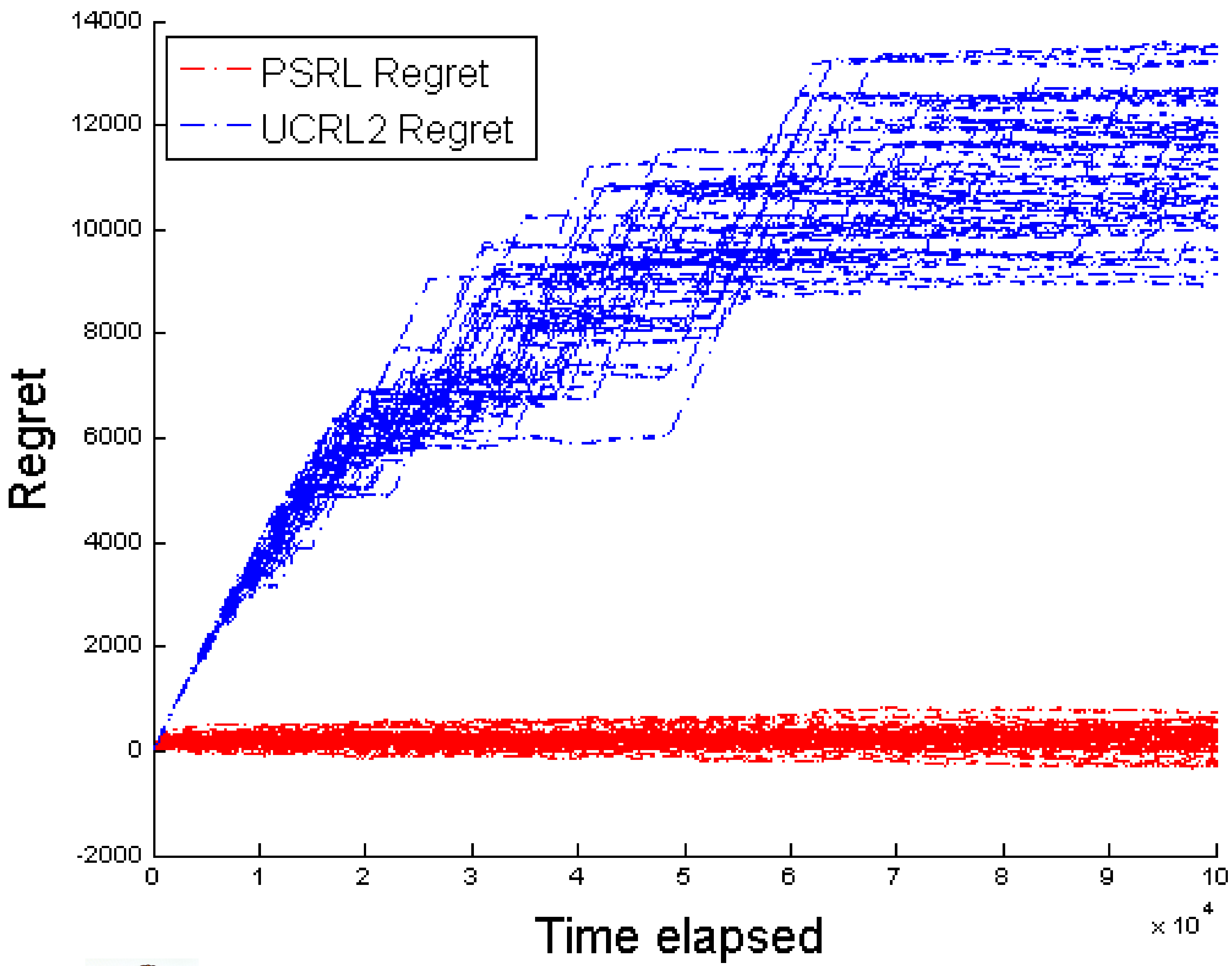
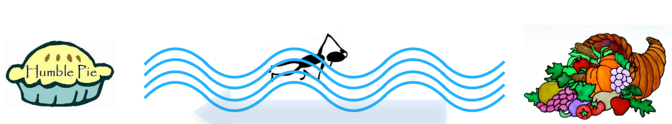
[Osband,Van Roy, Russo '13]

# Beating a near-optimal algorithm



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- Key idea: Estimate the unknown parameter using l2 regularized least-squares, develop tight confidence sets

# Nonlinear systems?

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- Smoothness:

$$y = f(x, a, \theta, z), y' = f(x, a, \theta', z)$$
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$$\mathbb{E}\left[\|y - y'\|\right] \leq \|\theta - \theta'\|_{M(x,a)}$$

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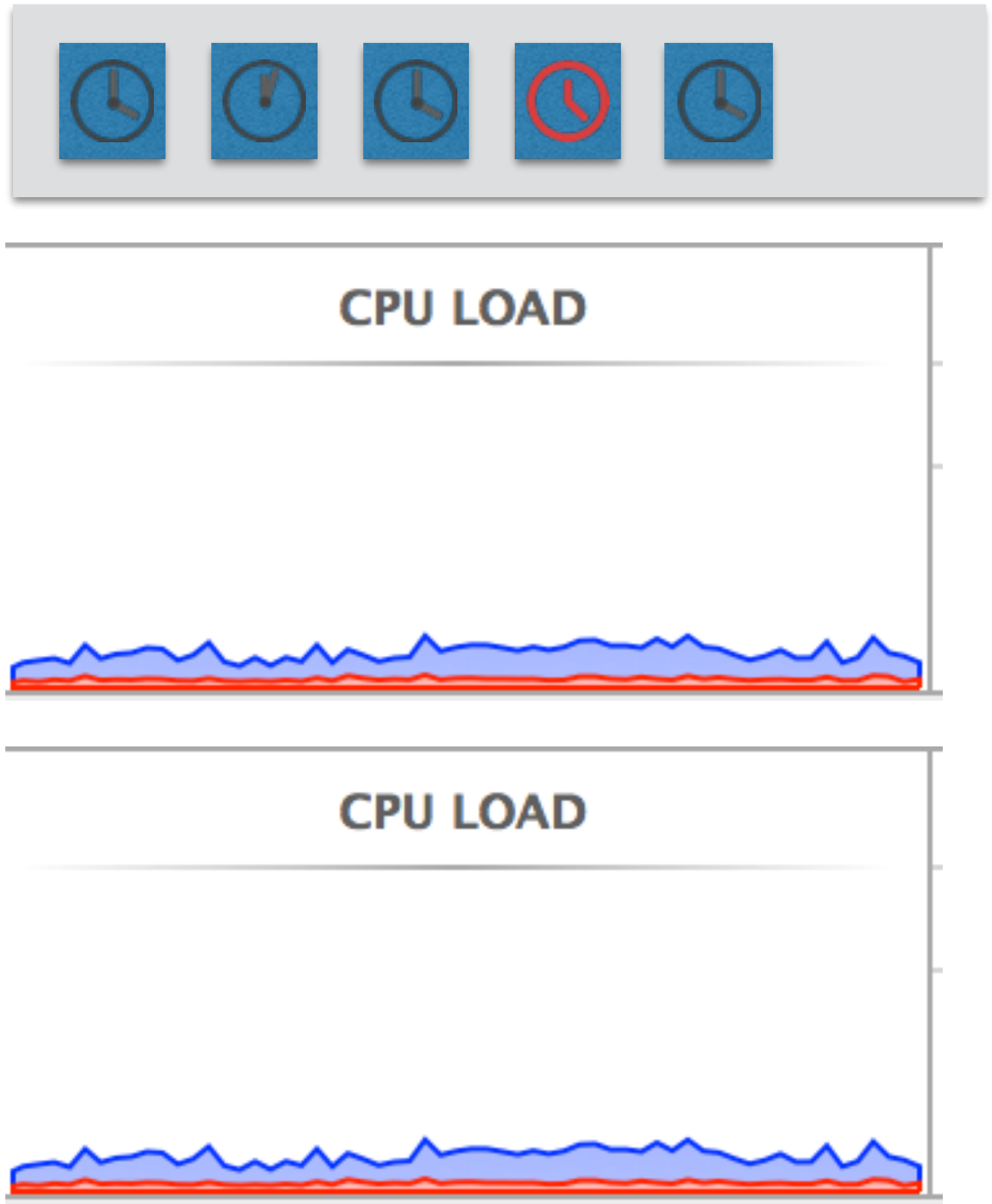
- Key idea: Use $M(x, a)$ to measure information.

# Web Server Control



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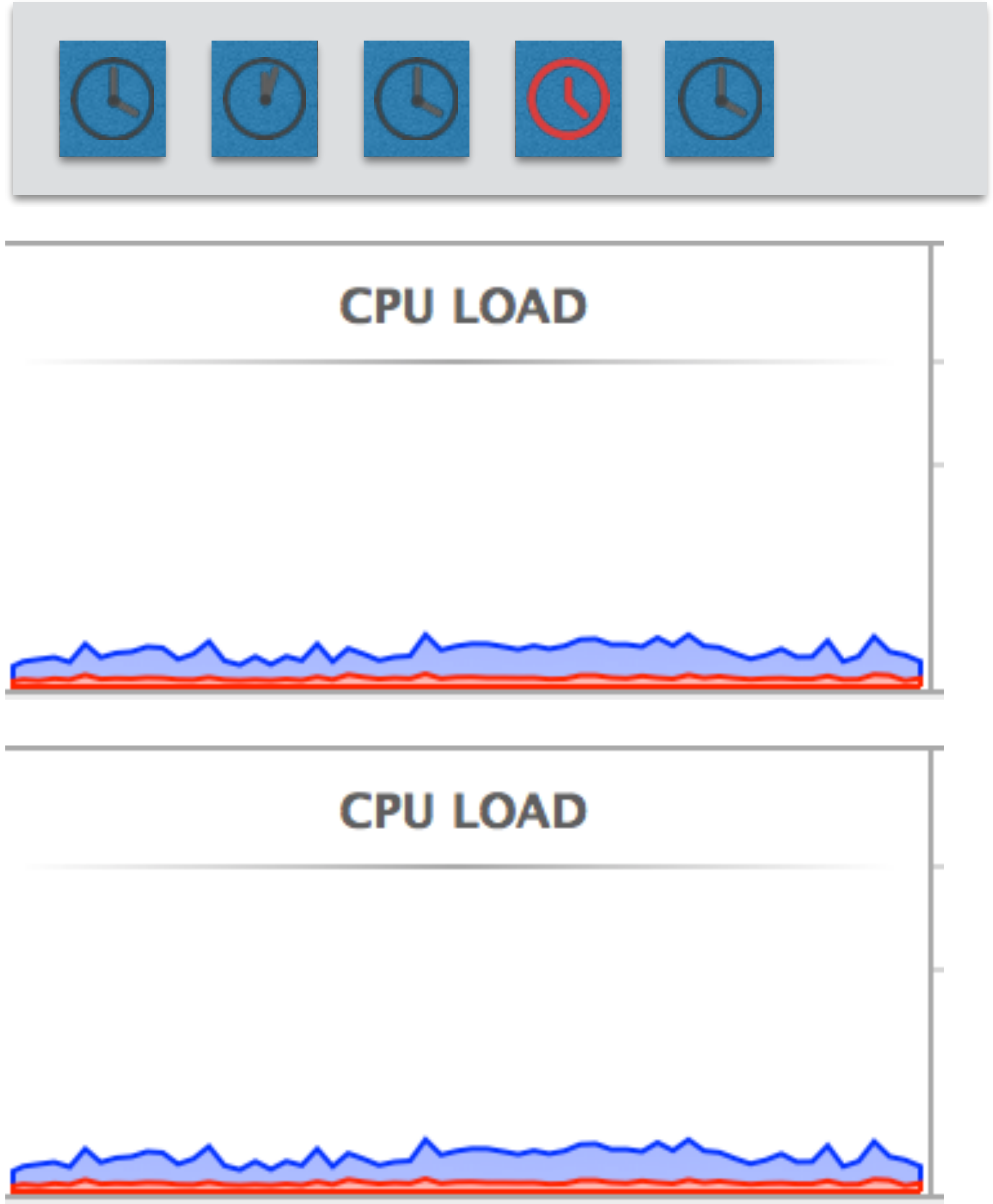

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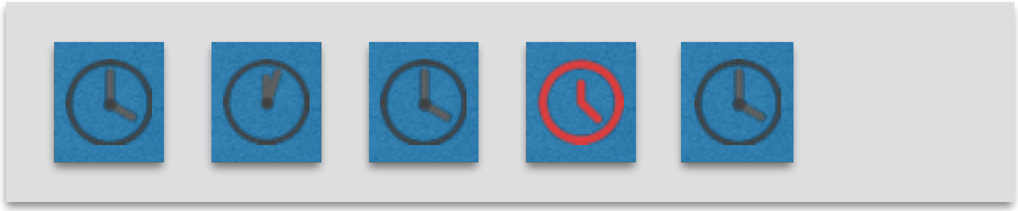

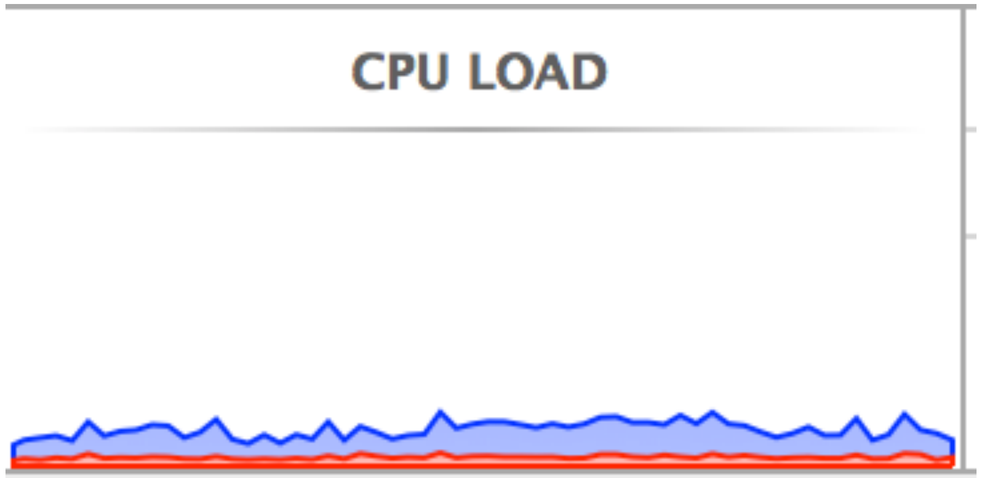


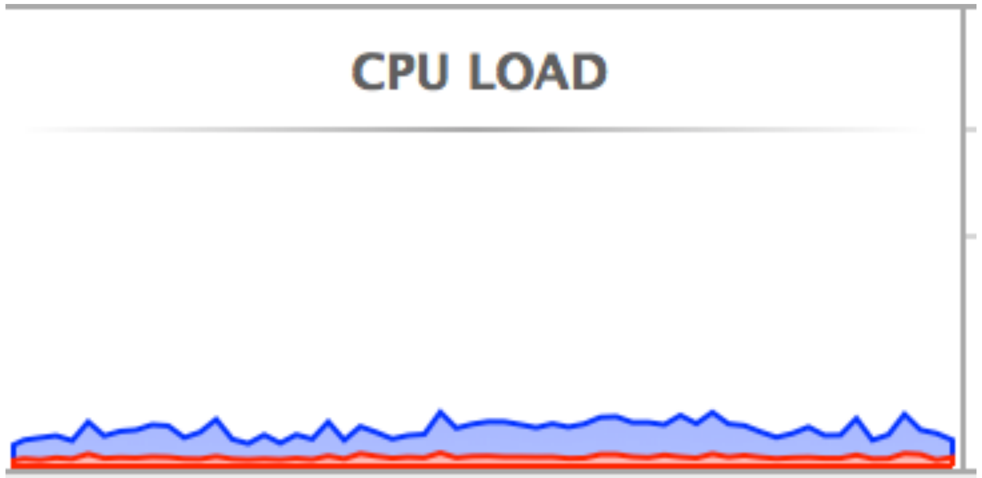

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- Control variables:
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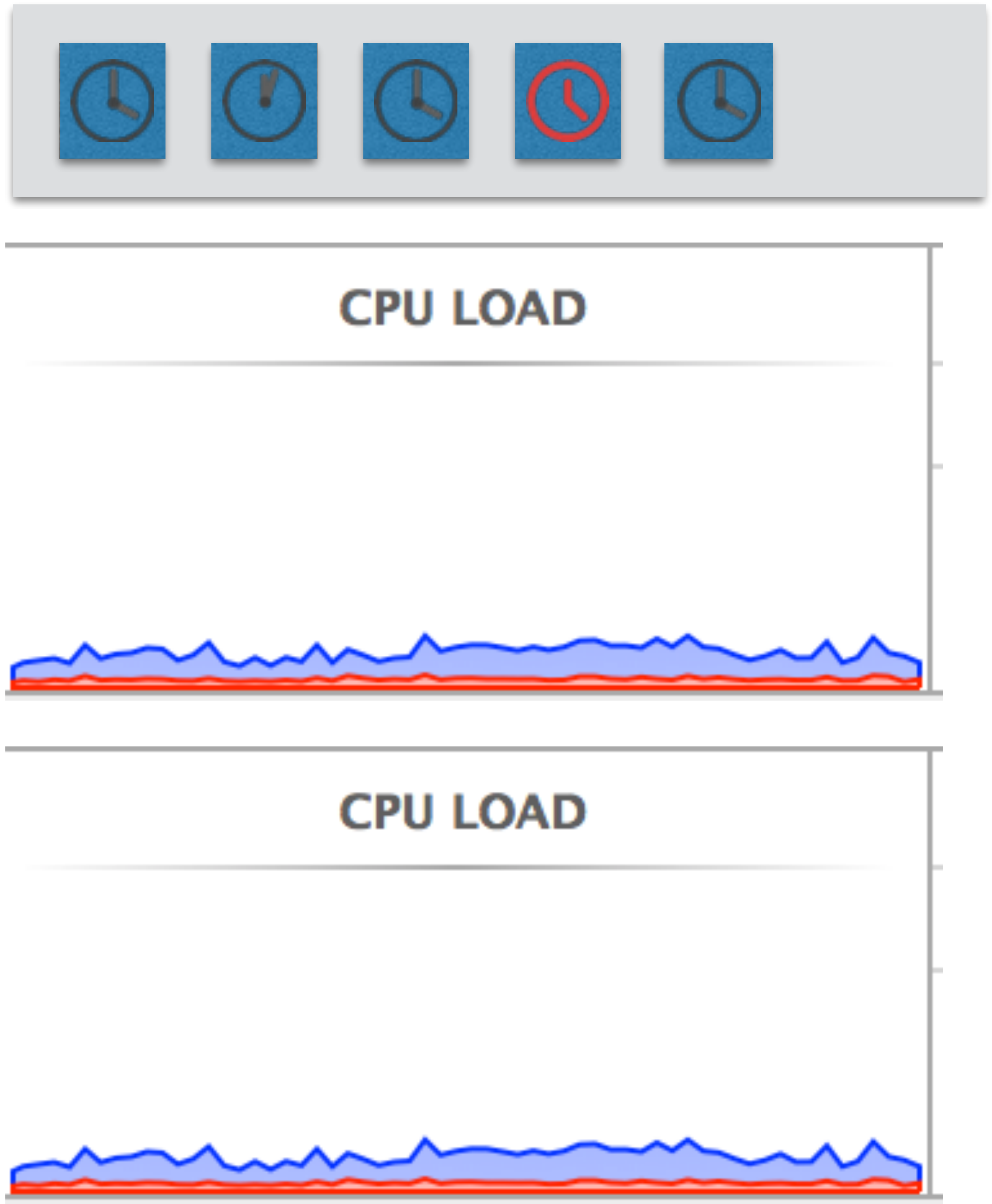

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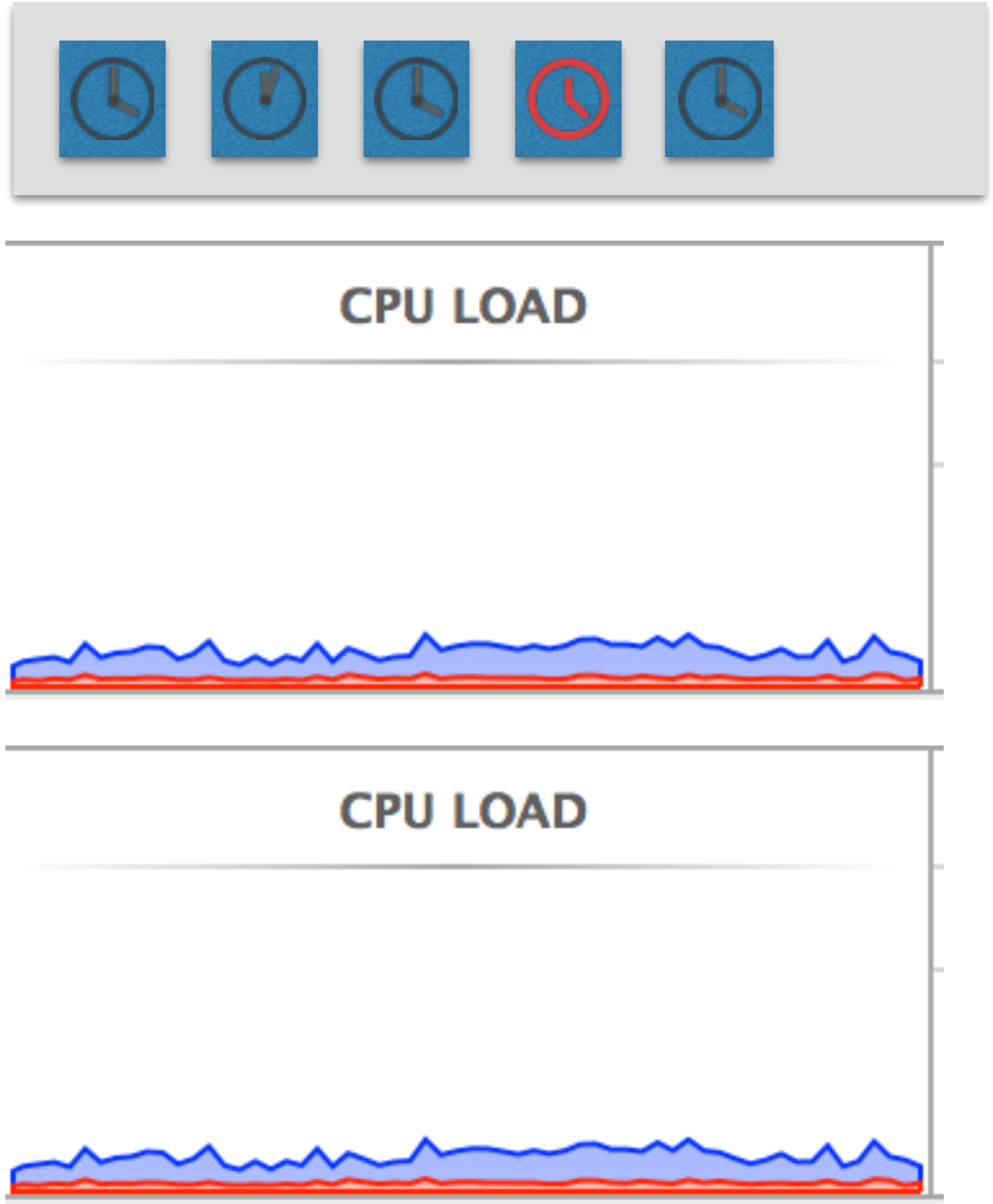

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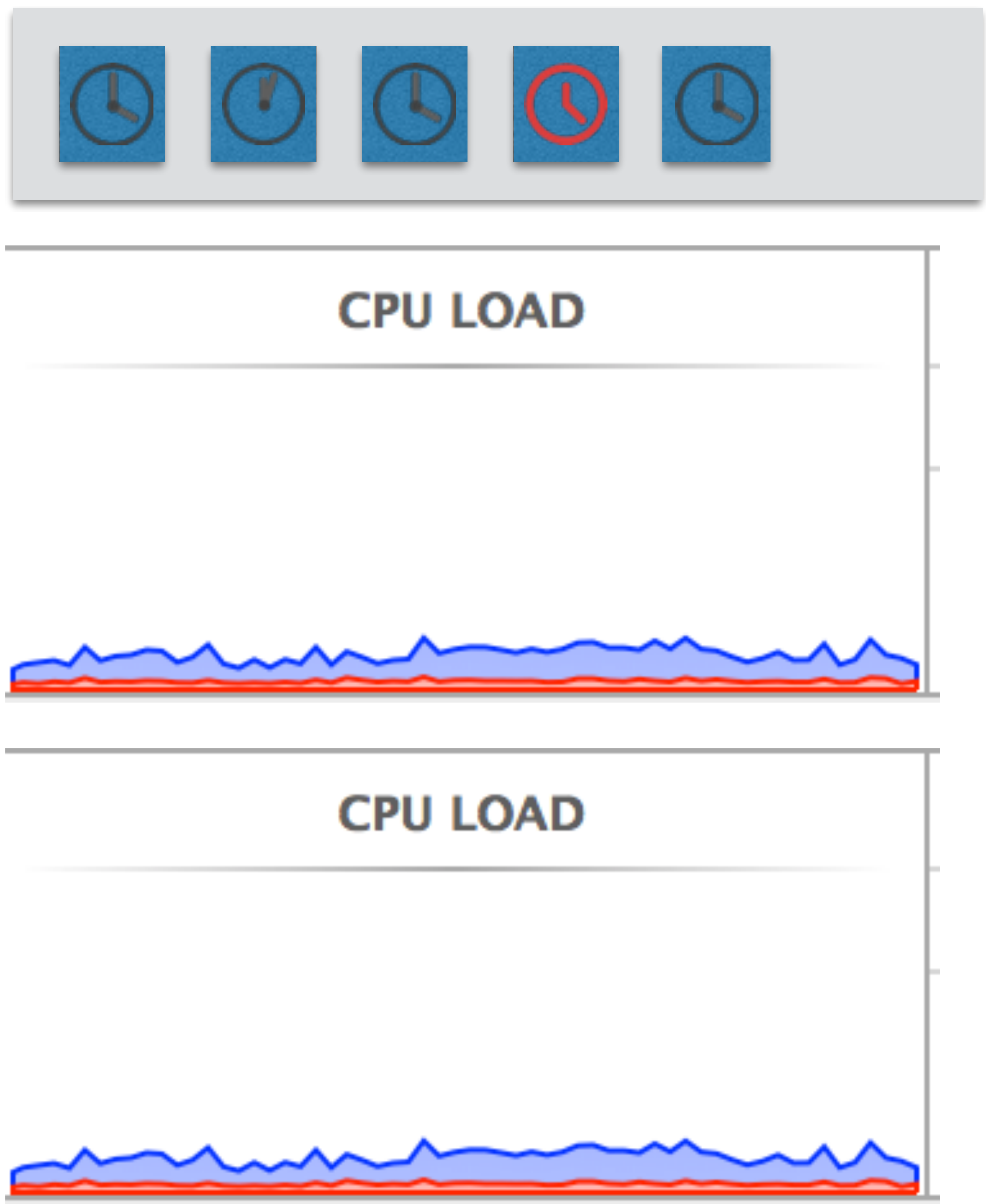

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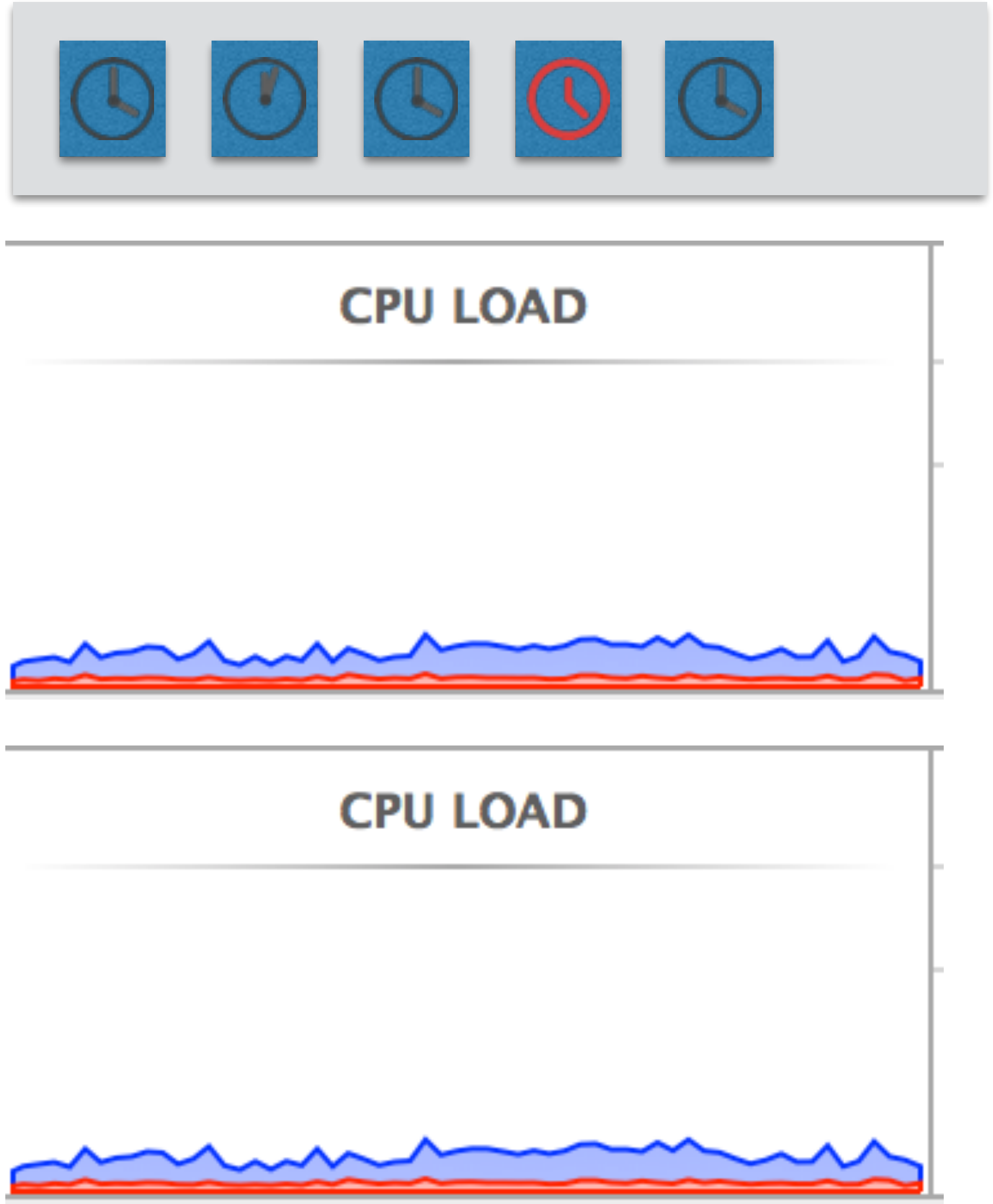
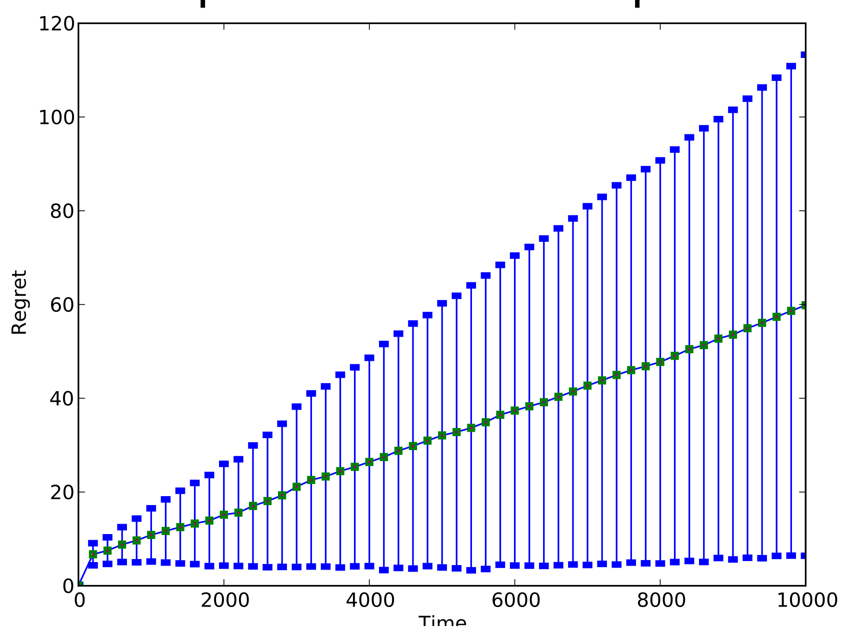
  - Memory usage relative to ideal memory usage

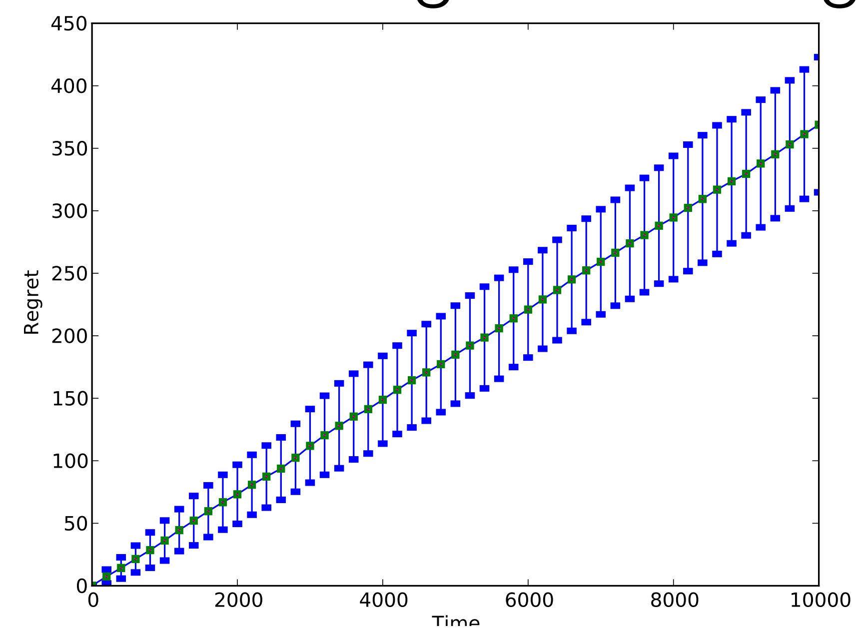


# Results

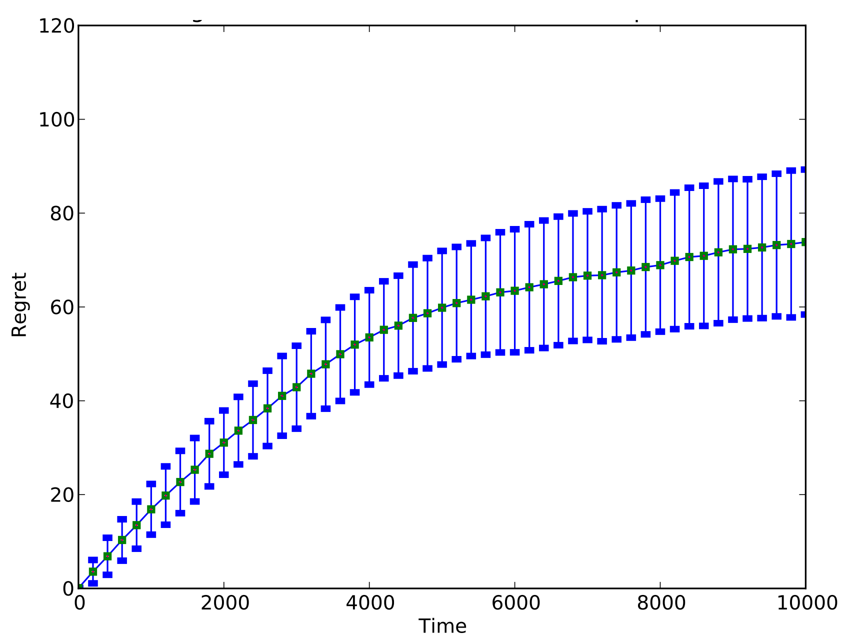
Explore then exploit



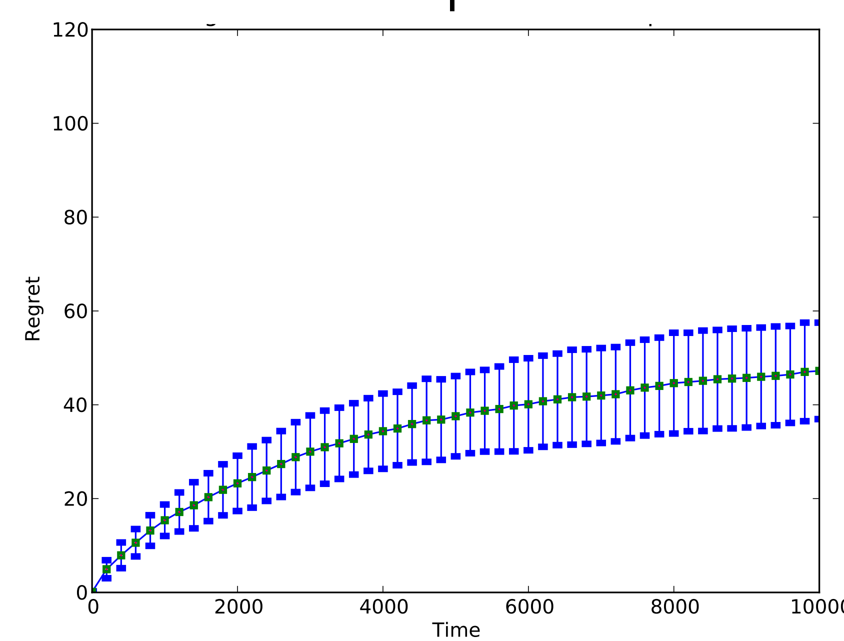

Q-learning w. dithering




OFULQ

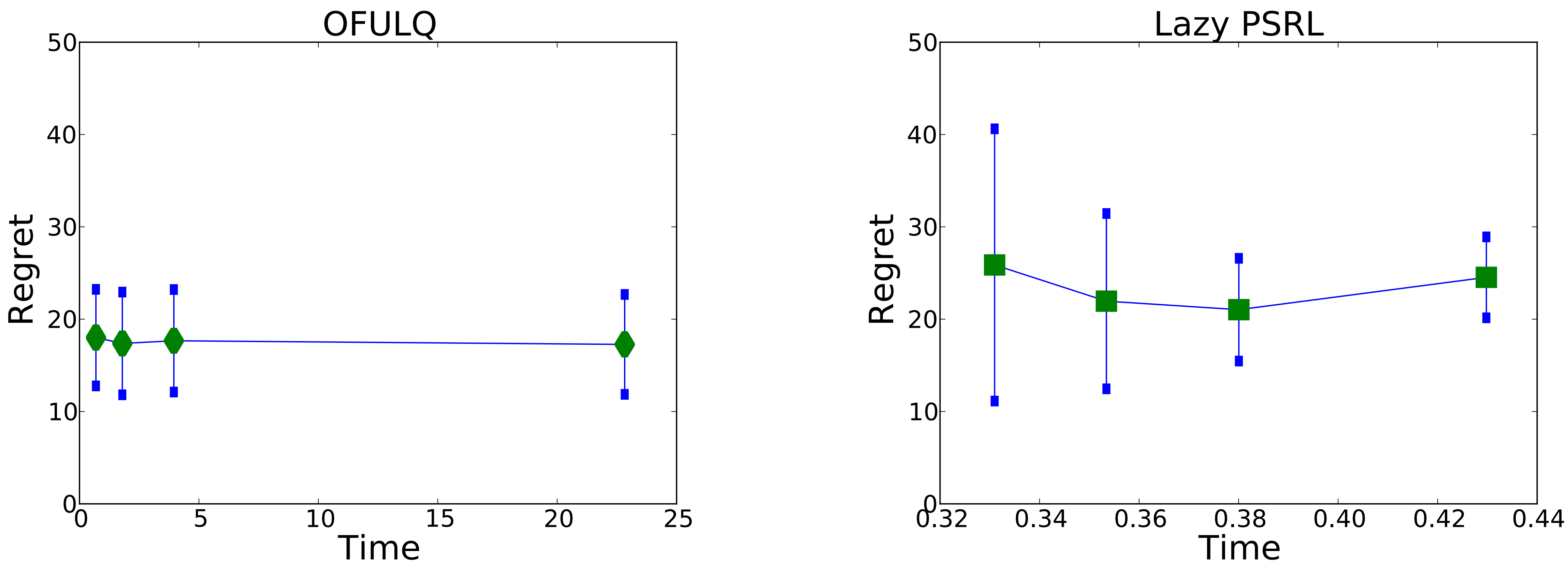



OFULQ prefetch



# OFULQ vs. PSRL

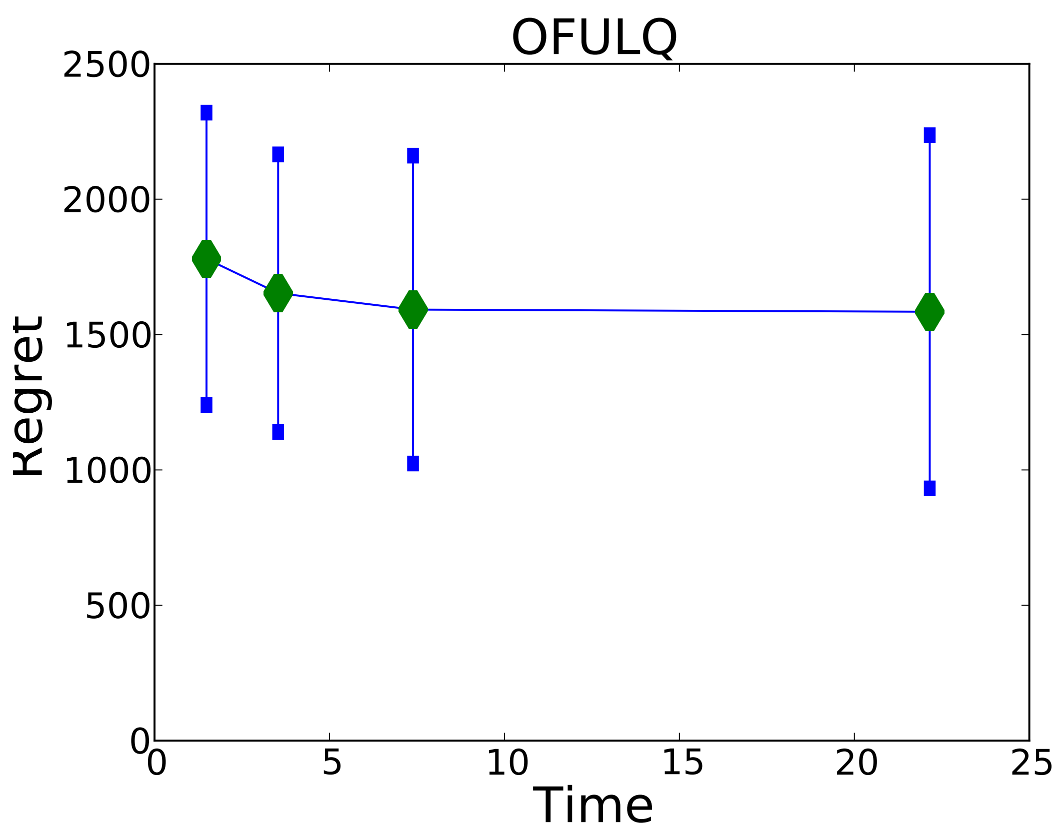
The frequency of policy switches is controlled by a parameter, which ultimate controls the computation time

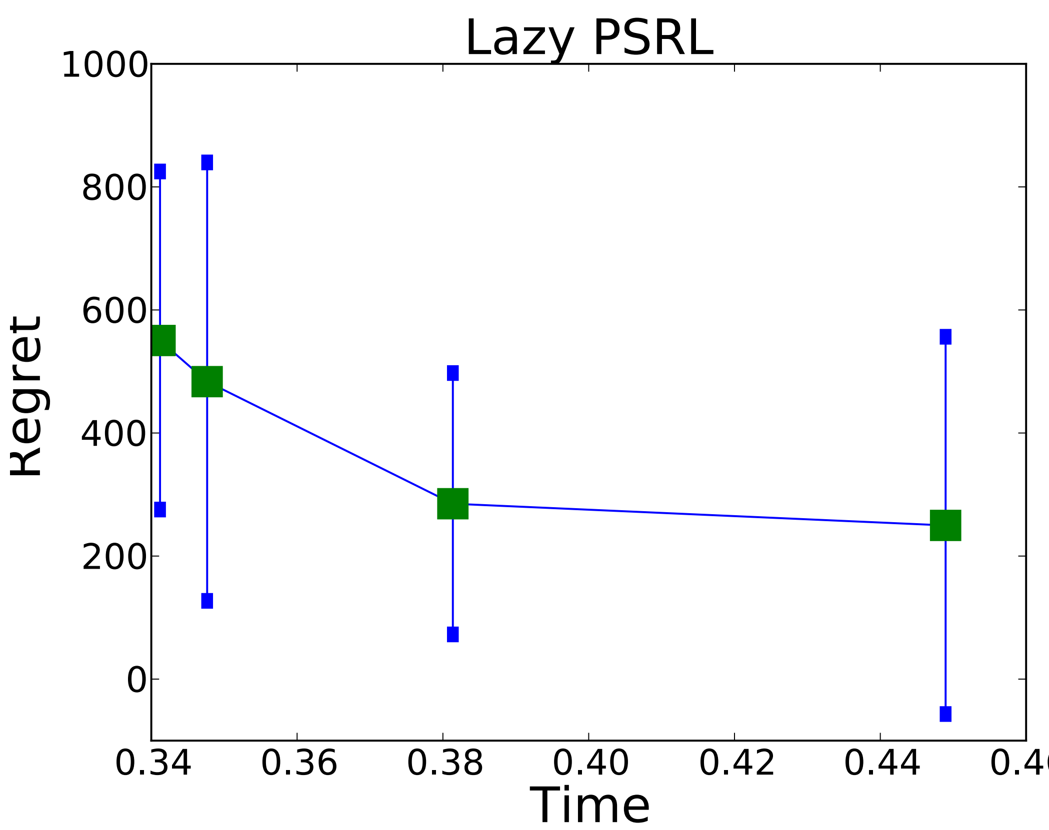



OFULQ = OFU on LQR

Lazy PSRL = PSRL that switches to new policy based on $M(x,a)$

# Higher noise

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# High dimensional bandits

# Bandit Problems


**Lever 1**
**Known payout**
**$0.25 bet**
**$0.30 win!**

**Lever 2**
**Unknown payout**
**$0.25 bet**
**$? win**

**EXPLOITATION**

**EXPLORATION**

Goal: maximize the total reward incurred

# Linear Bandits

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- Actions are elements of a vector space:

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- **<u>Theorem [Dani et al '08]:</u>** For subgaussian noise, OFU's regret for the L2 problem is

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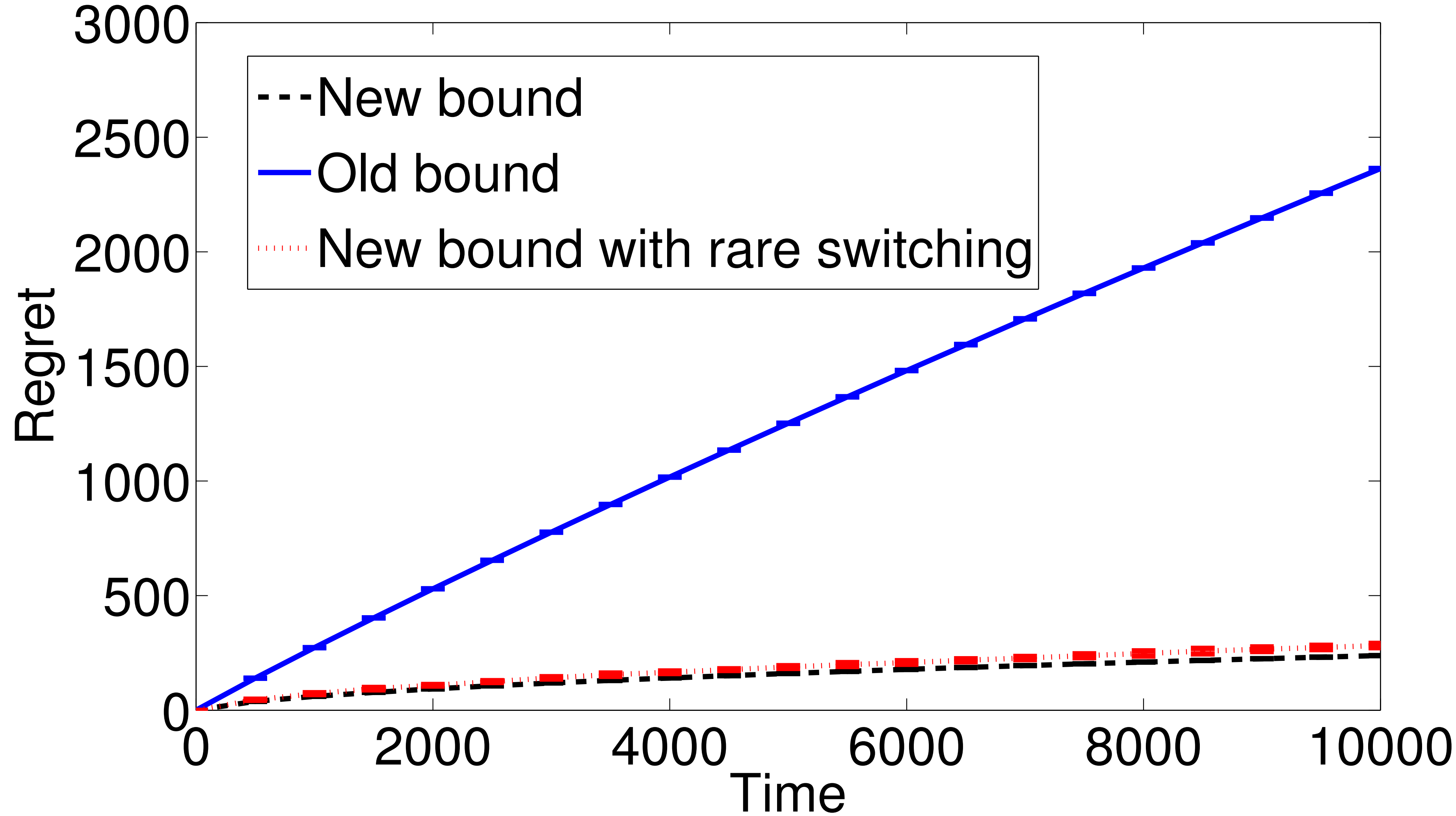
# Confidence sets matter




- “New bound”: Abbasi-Pal-Sz’11
- “Old bound”: Dani-Hayes-Kakade ‘08

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  such that $\mathbb{P}\left(\theta_* \in C_t\right) \ge 1 - \delta$

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- How to exploit sparsity of $\theta_*$?

# A general solution

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- If we have a good predictor for an adversarial linear regression problem with small regret, the predictions $\hat{R}_1, \ldots, \hat{R}_t$ and the regret bound $B_t$ should give us a honest, tight confidence set.

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- **<u>Theorem [Abbasi-Pal-Sz ’12]</u>**: With probability $1-\delta,\ \theta_* \in C_n$ holds for all $n \geq 1$, where

$$C_n = \left\{ \theta \in \mathbb{R}^d \ :\ \sum_{t=1}^{n} (\hat{R}_t - \langle A_t, \theta \rangle)^2 \right.$$

$$\left. \leq 1 + 2B_n + 32\gamma^2 \ln\left( \frac{\gamma\sqrt{8} + \sqrt{1+B_n}}{\delta} \right) \right\}$$

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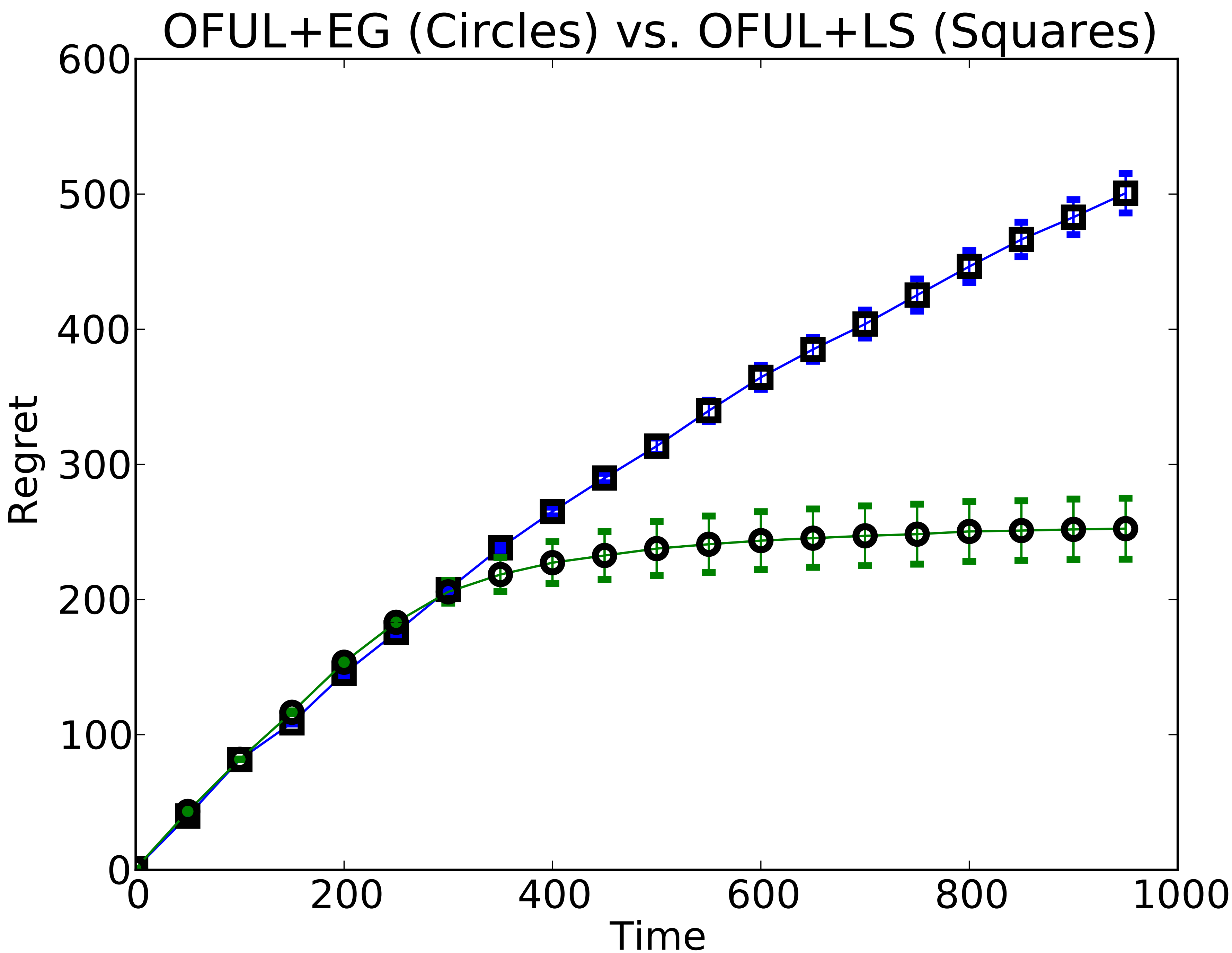
    - Good algorithms select good actions frequently ==> No RIP
    - Covariates are highly correlated

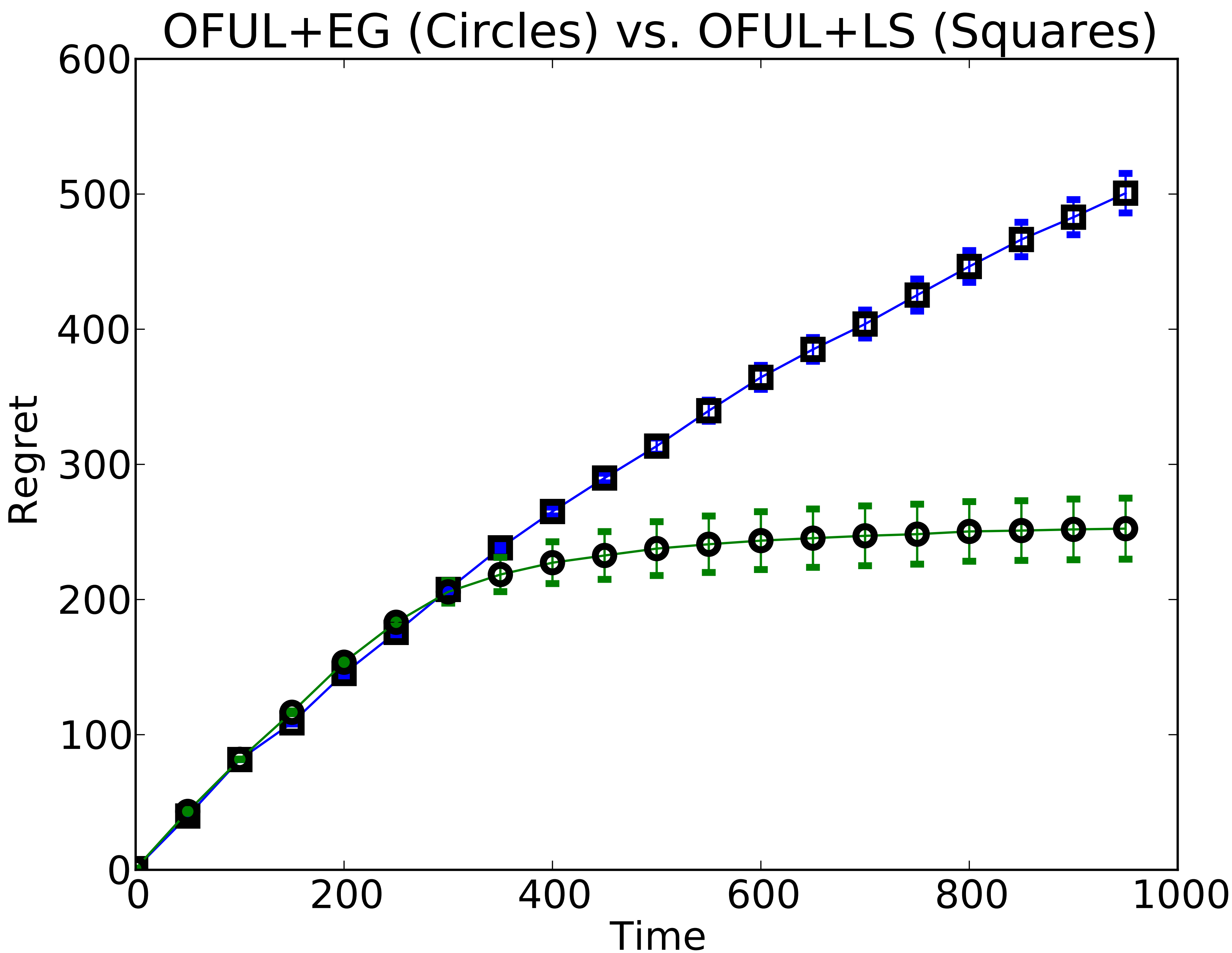
# Still.. does it work?




d = 100, p = 10

# Still.. does it work?

Yes, it does!

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Significant computational, algorithmic and statistical challenges remain. Much to be done!!

Thanks for being here!
Questions?