Block-oriented data-driven modeling starting from the best linear approximation

Maarten Schoukens
Vrije Universiteit Brussel

Overview

Best Linear Approximation

Block-oriented models

Structure detection

Data-driven modeling

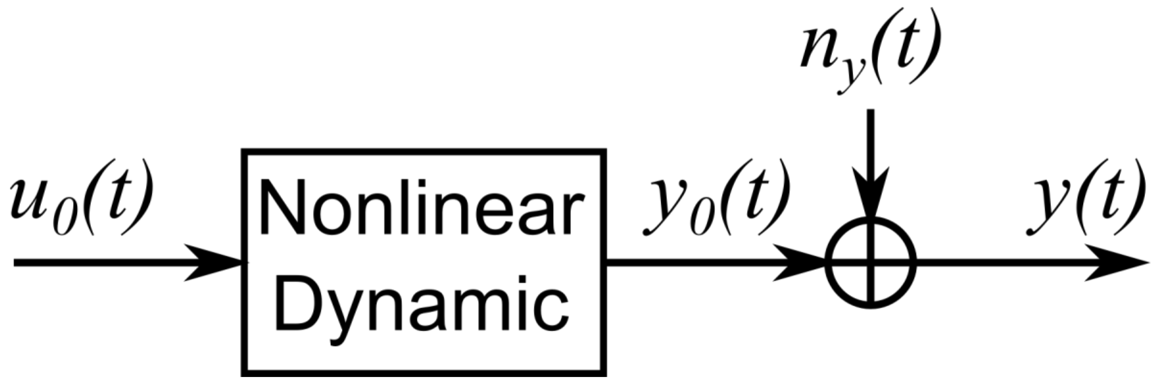
Future perspectives

Best Linear Approximation

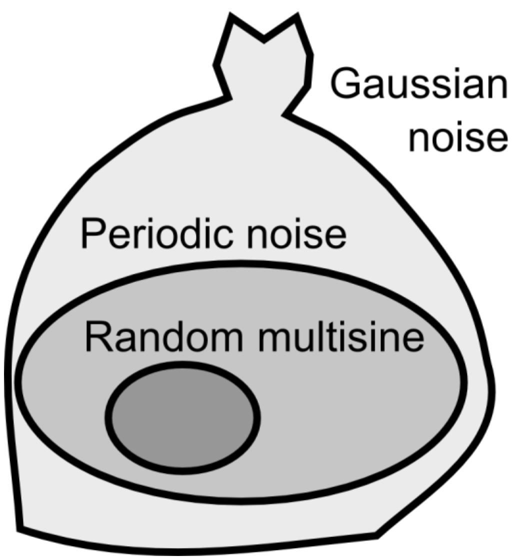
Best Linear Approximation

- Nonlinear framework
- BLA of a nonlinear system
- Coherent and non-coherent contributions
- Robust implementation
- Example

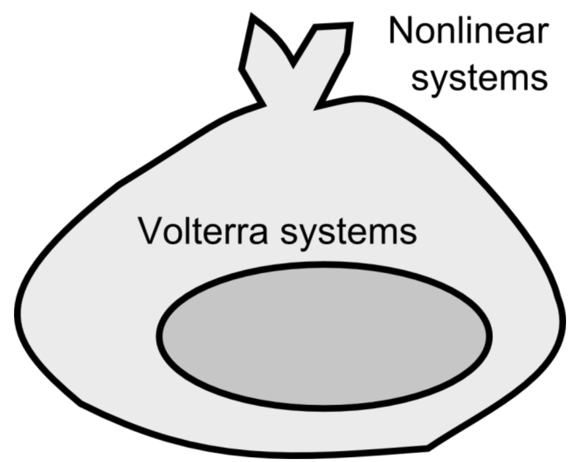
Best Linear Approximation



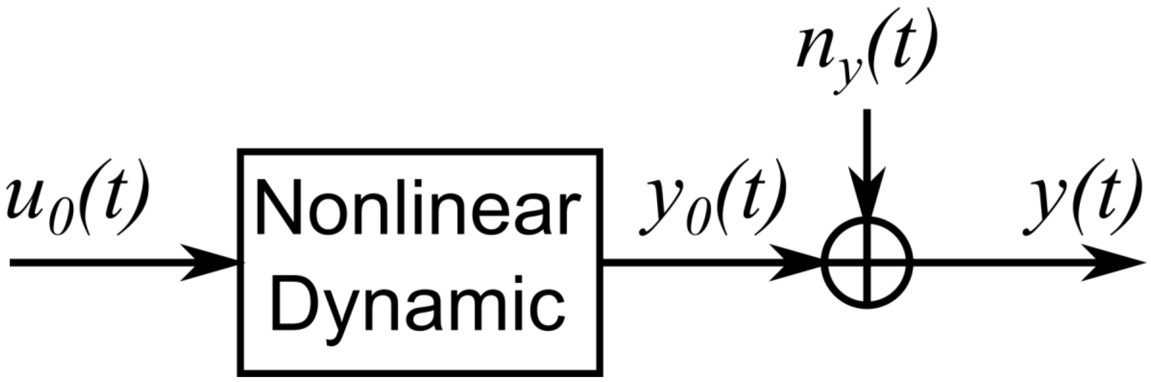
Input signal class

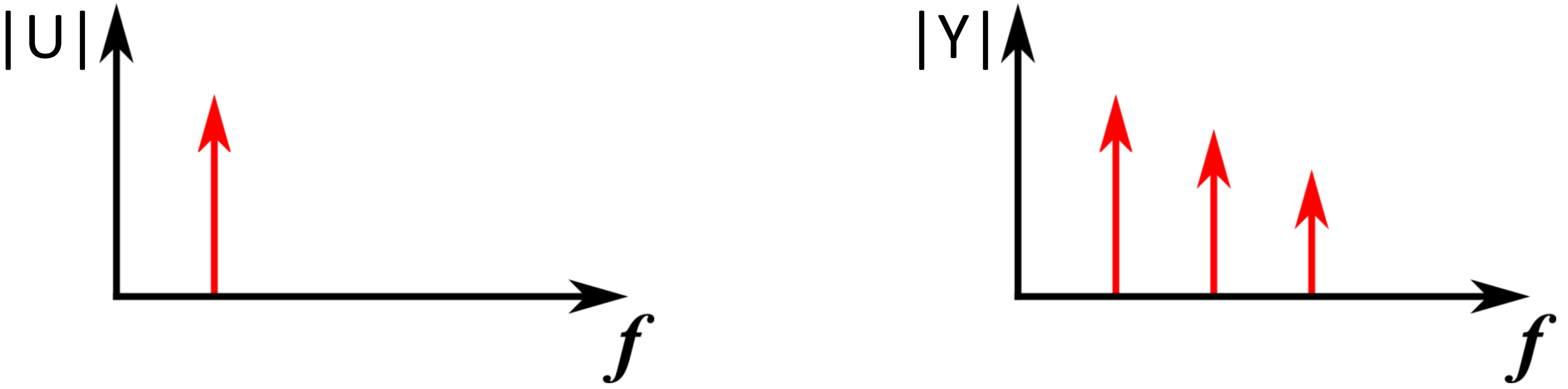


System class

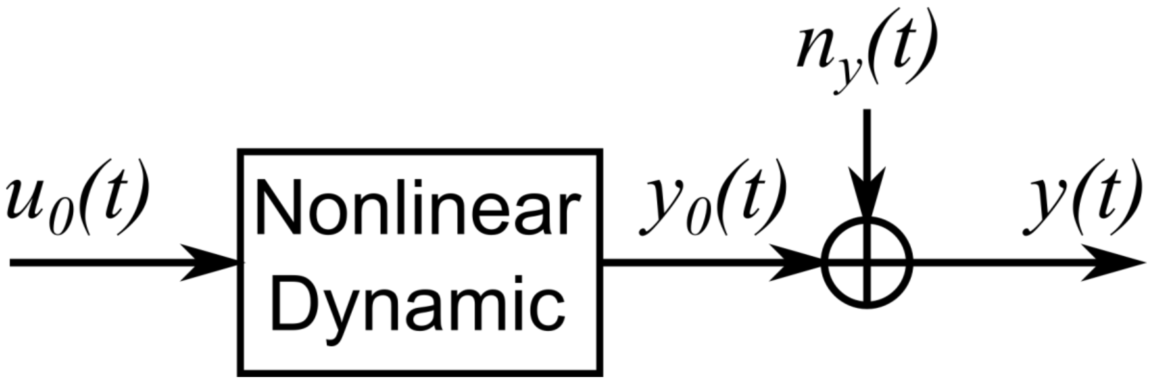


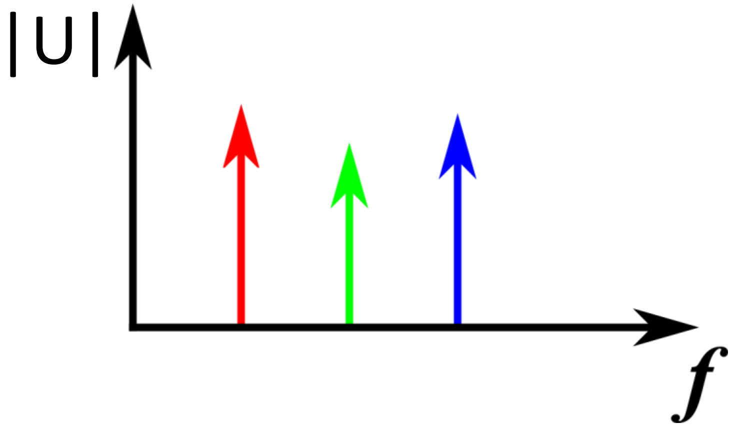
Nonlinear system output

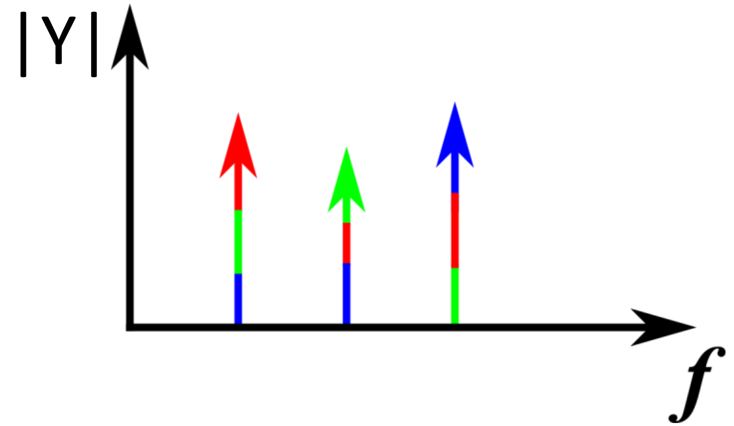




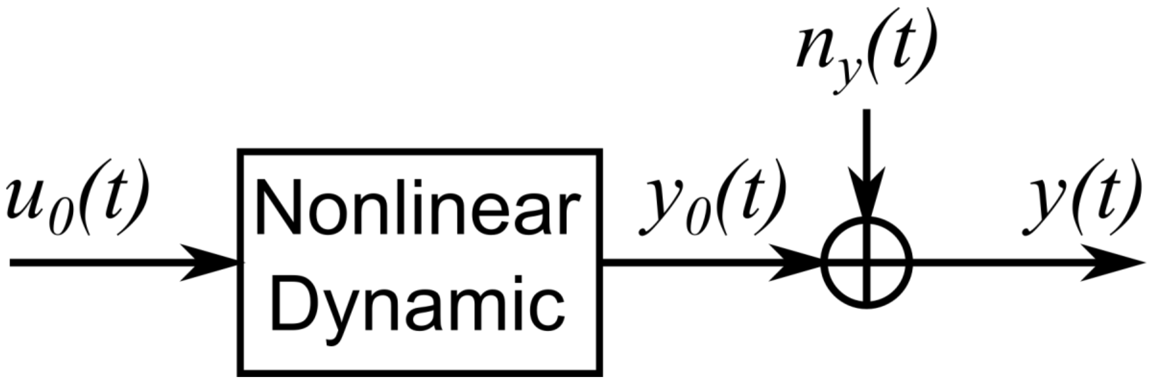
Nonlinear system output



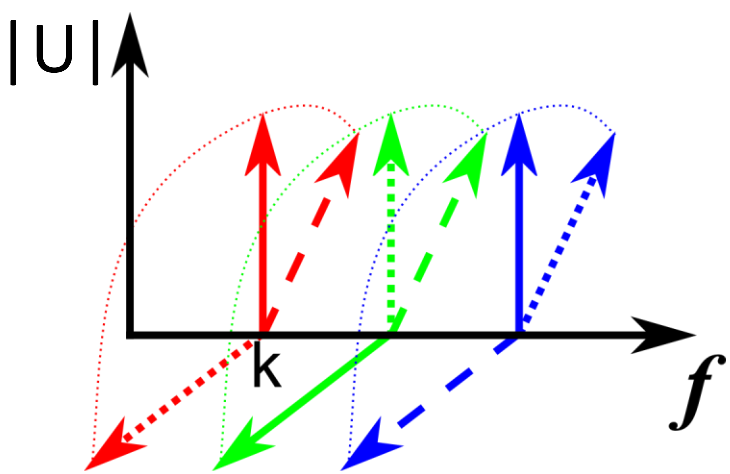


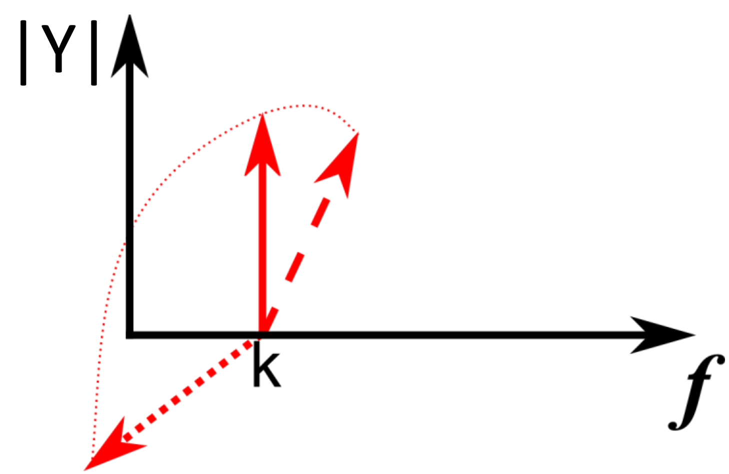


Coherent contributions

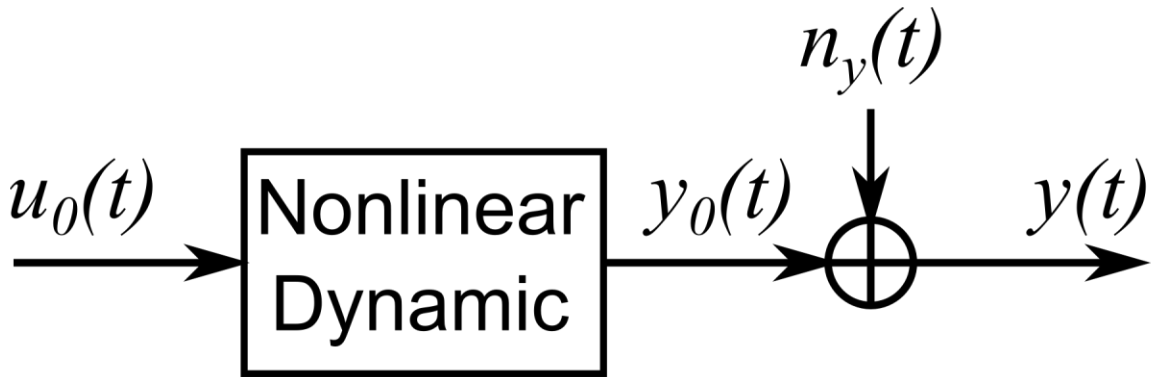


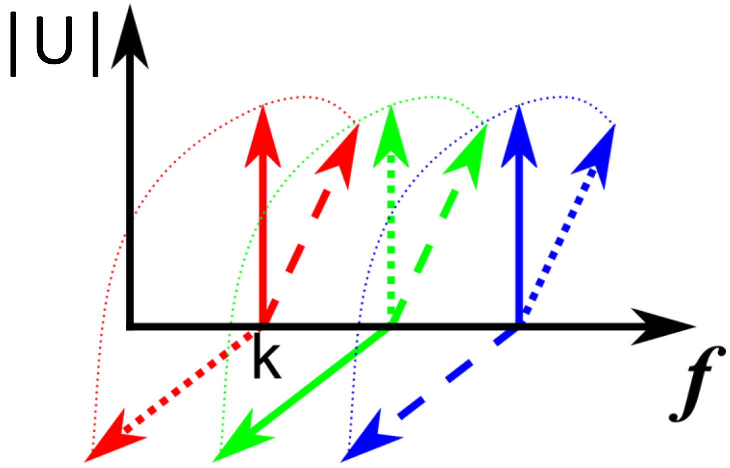
coherent $\angle Y(k) = \angle U(k) + \varphi_0(k)$

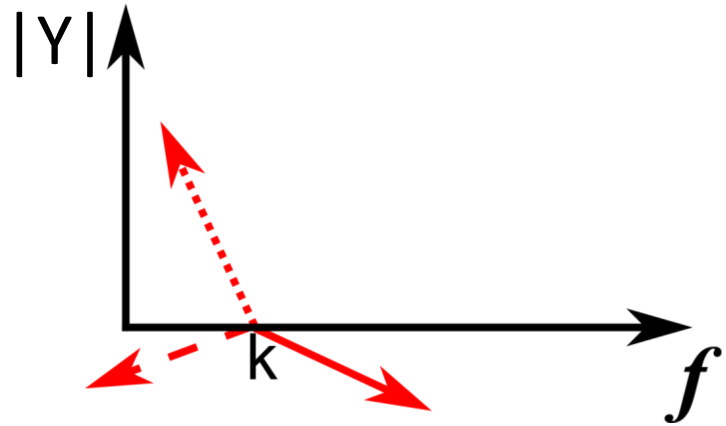




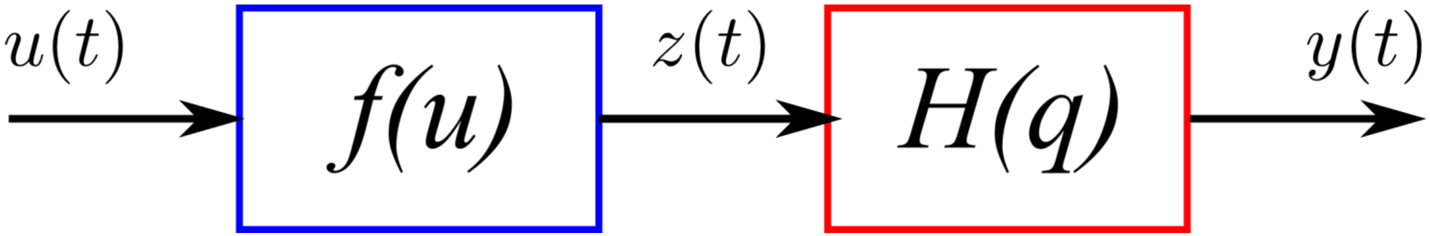
Non-coherent contributions







(non-)coherent contributions



3rd Degree nonlinearity

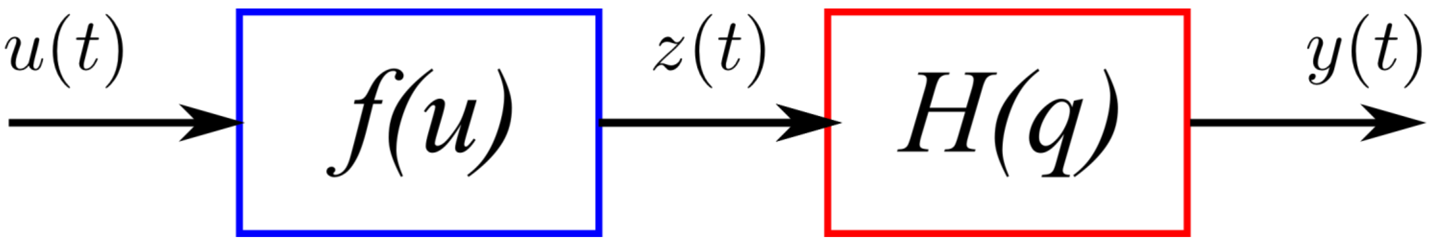
Y(k) = H(k)U(k)U(l)U(-l) = H(k)U(k)|U(l)|^2

→ coherent

Y(k) = H(k)U(k-2)U(1)U(1)

→ non-coherent

(non-)coherent contributions

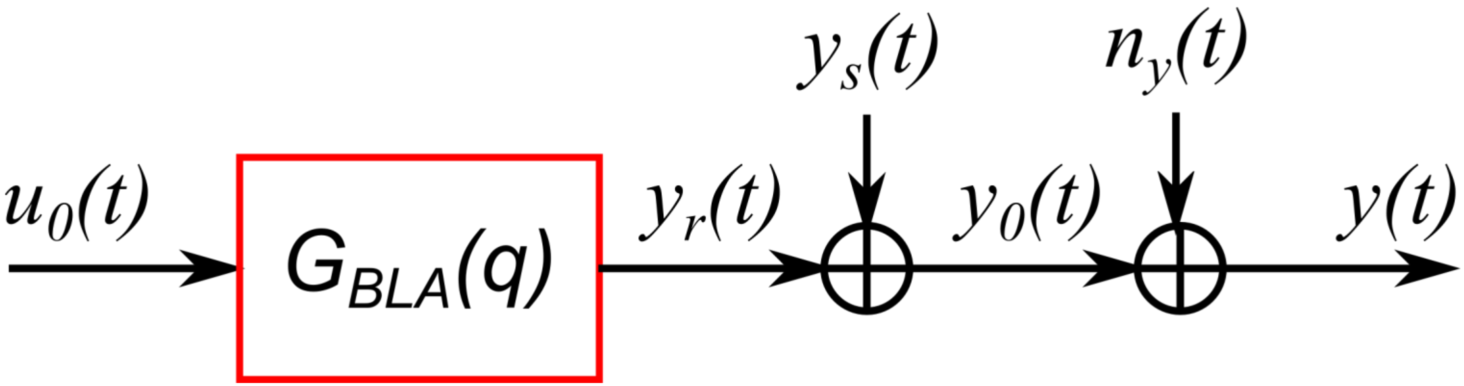


2^{nd} Degree nonlinearity

$Y(k) = H(k)U(k-l)U(l)$

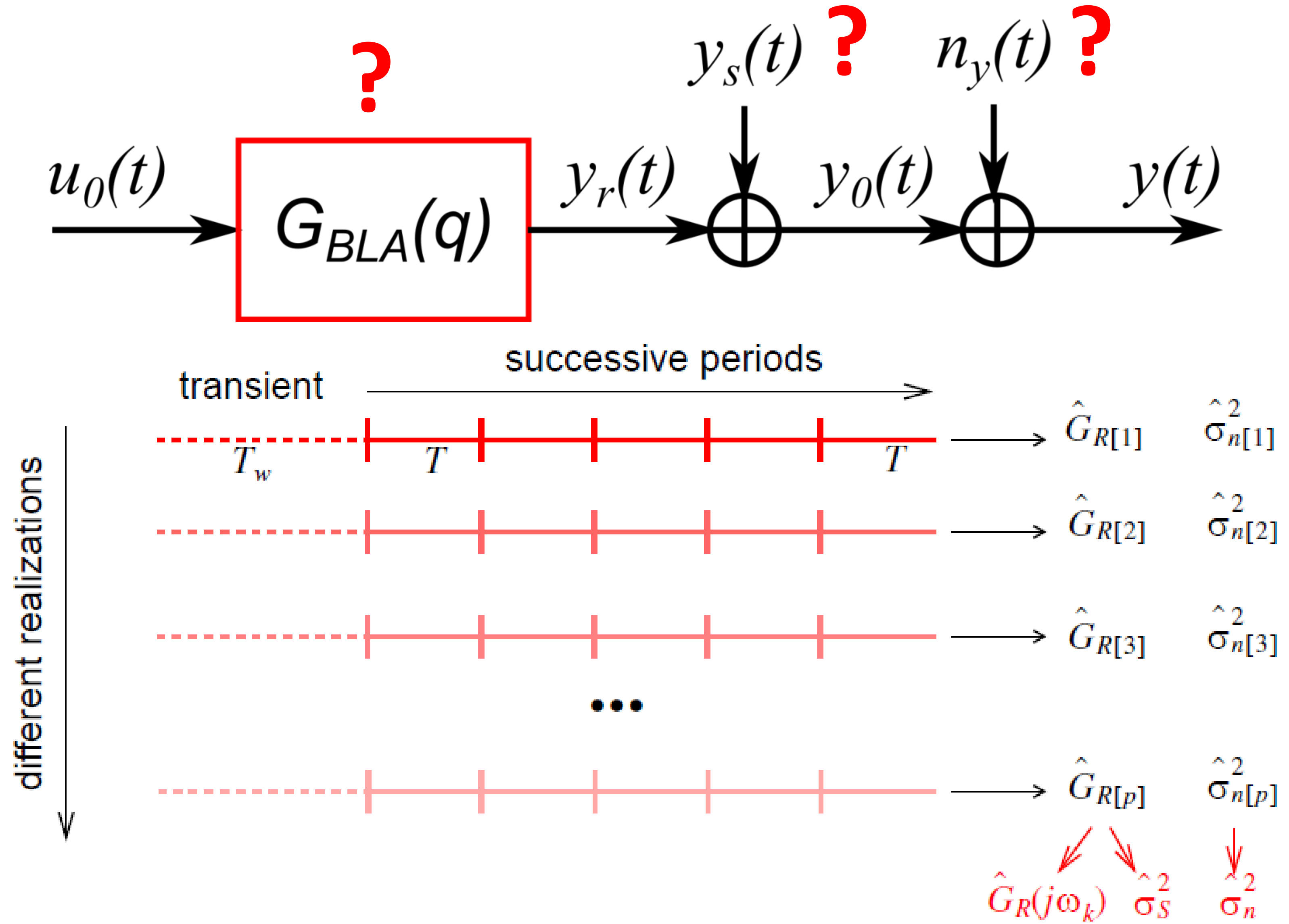
→ non-coherent

Best Linear Approximation

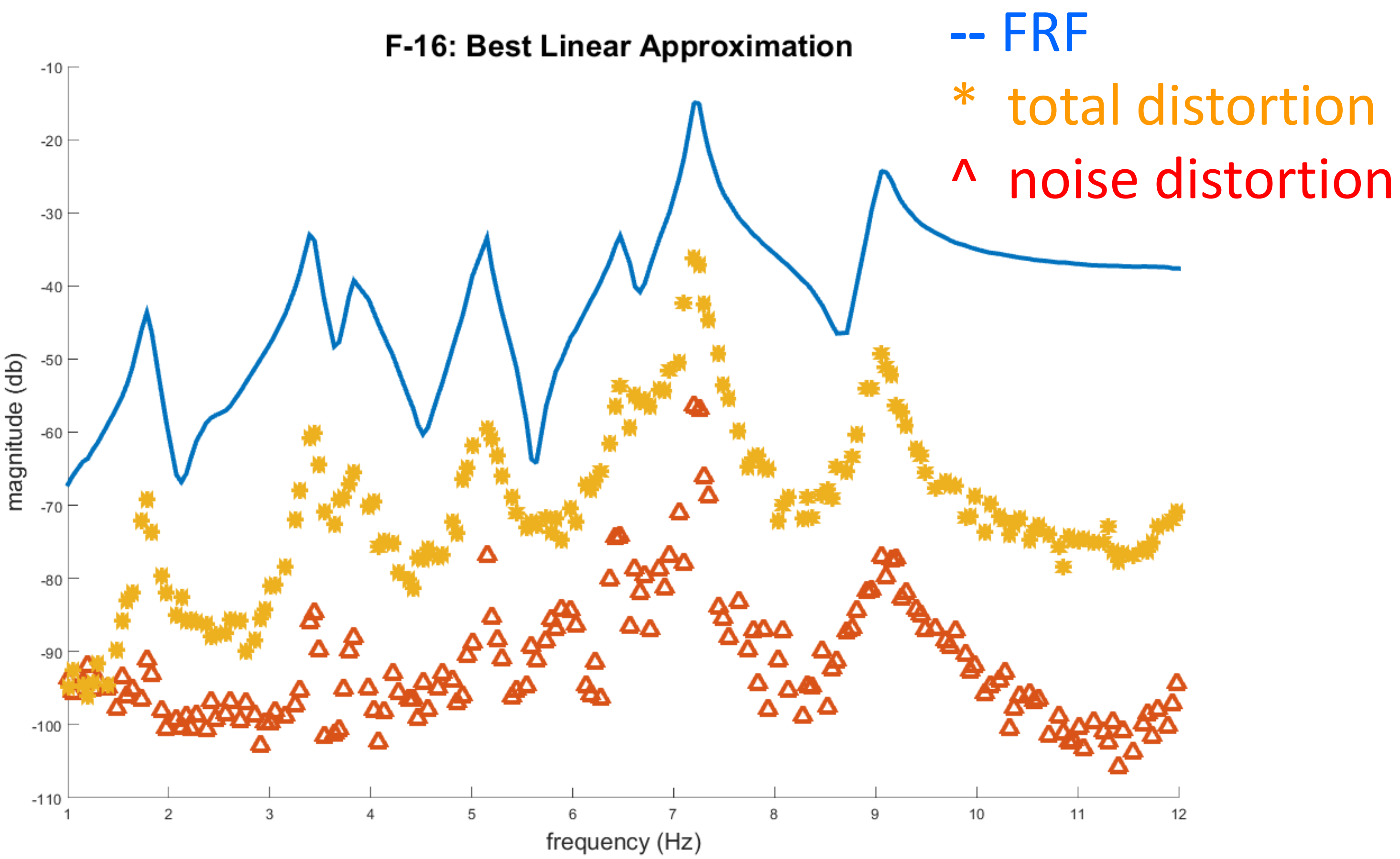


- $u_0(t)$: noiseless Gaussian input
- $y_s(t)$: non-coherent nonlinear contributions
- $n_y(t)$: output noise source

Robust implementation



BLA Example: F-16



Best Linear Approximation

- Nonlinear framework ✓
- Robust implementation ✓
- Extract the dynamics of a nonlinear system ✓
- Quantify the nonlinear distortions ✓

Best Linear Approximation

- Nonlinear framework
- Robust implementation
- Extract the dynamics of a nonlinear system
- Quantify the nonlinear distortions

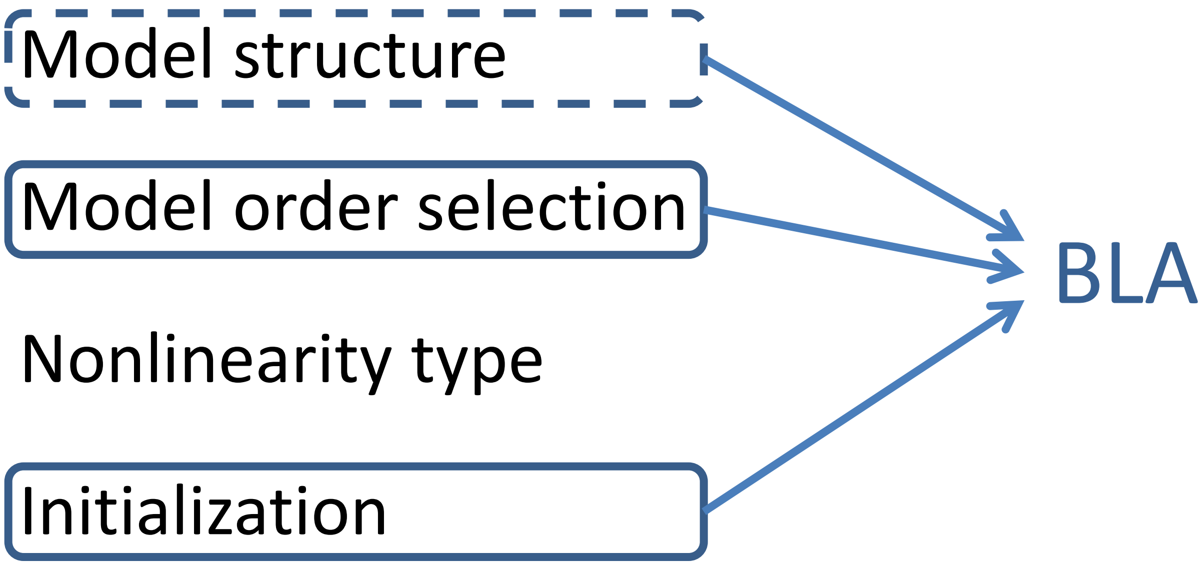
Why the BLA?

Nonlinear modelling is difficult:

- Model structure
- Model order selection
- Nonlinearity type
- Initialization

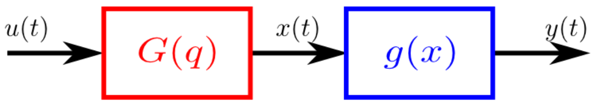
Why the BLA?

Nonlinear modelling is difficult:

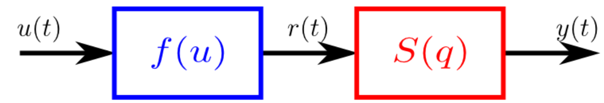


Block-oriented structures

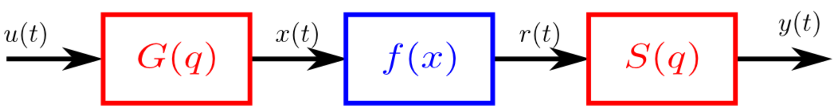
Single branch structures



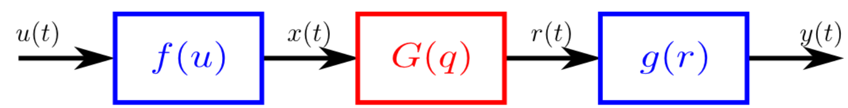
Wiener



Hammerstein

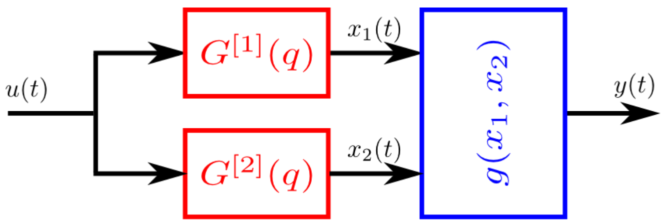


Wiener-Hammerstein

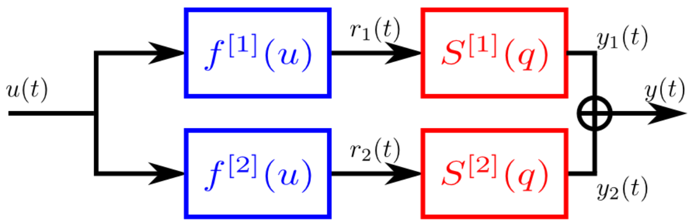


Hammerstein-Wiener

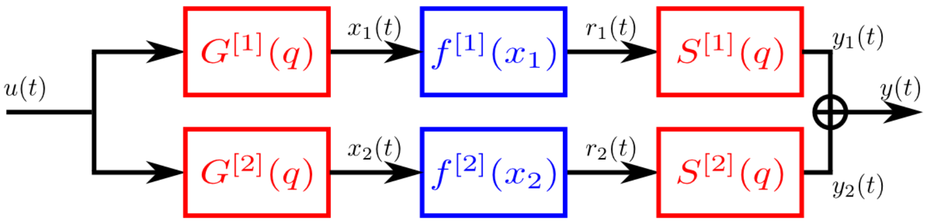
Parallel branch structures



Parallel Wiener

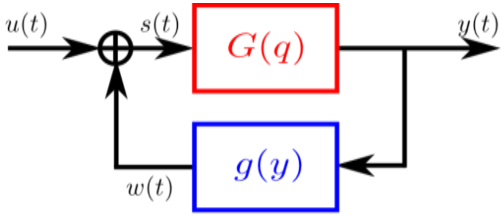


Parallel Hammerstein

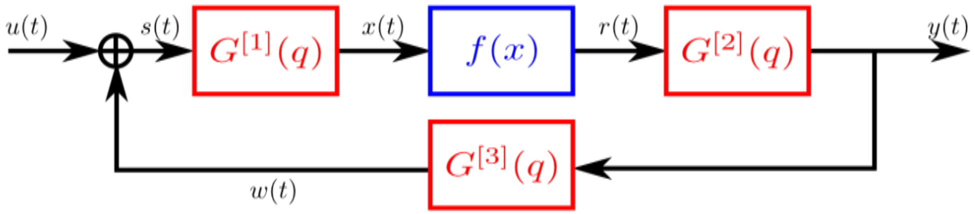


Parallel Wiener-Hammerstein

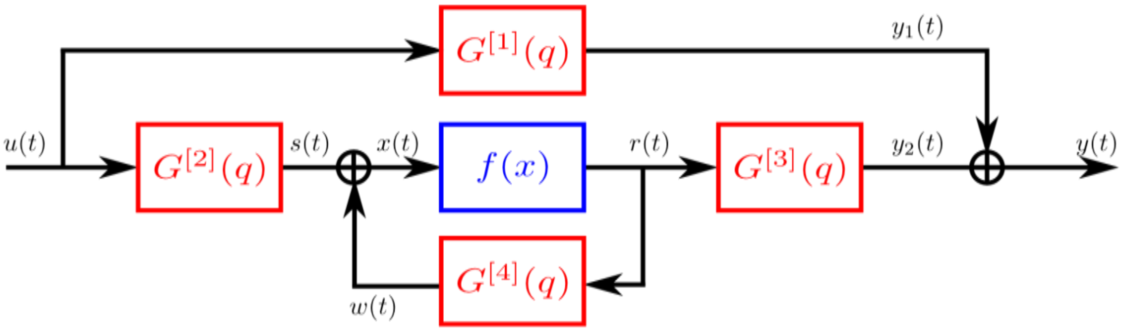
Feedback structures



Simple Feedback

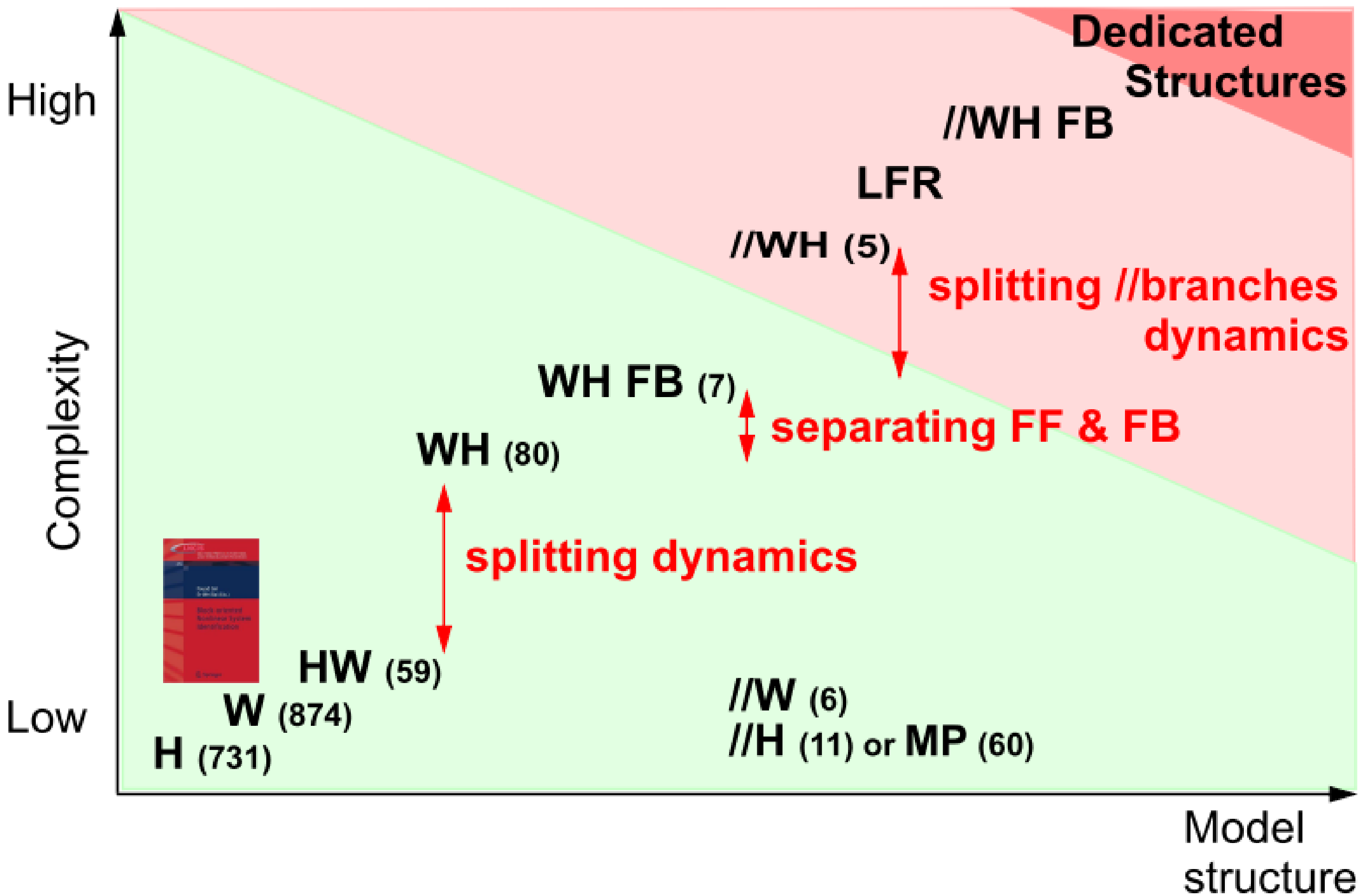


Wiener-Hammerstein Feedback



Linear Fractional Representation (LFR)

Identification Complexity vs Model Complexity



Structure detection

Structure detection

- Bussgang’s theorem
- ε – approximation
- Structure detection

Bussgang's Theorem

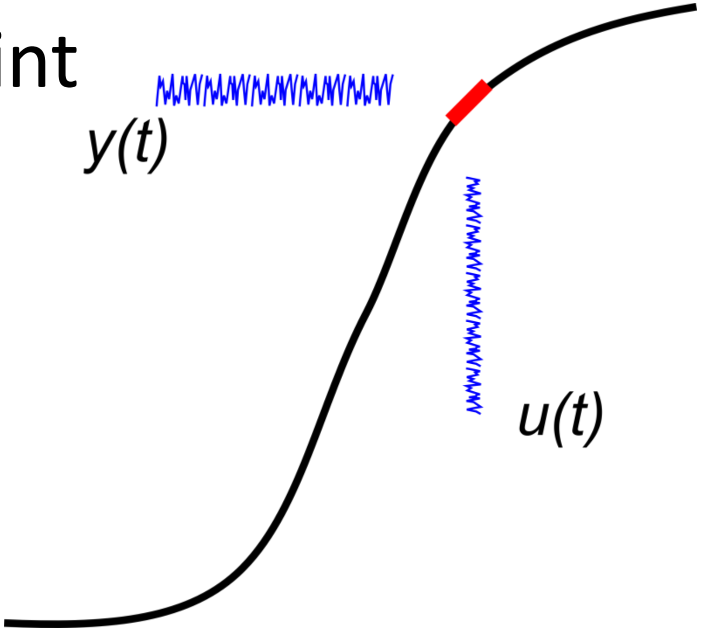
Stationary Gaussian input

→ Static nonlinearity ≈ static gain

$$f(u) = \gamma u$$

ε - Approximation

Small signal around a setpoint



Taylor approximation

→Static nonlinearity ≈ static gain

$$f(u) = \gamma u$$

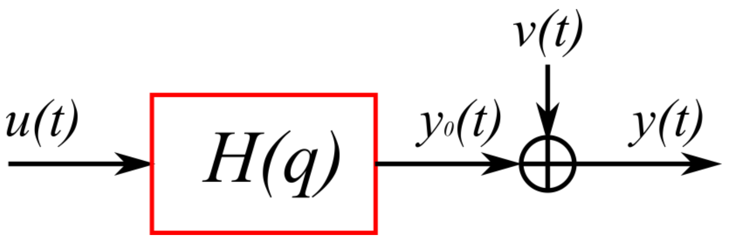
Structure detection

BLA, ε - Approximation @ different setpoints

- Change offset
- Change power spectrum

Structure detection

Linear-Time-Invariant (LTI)



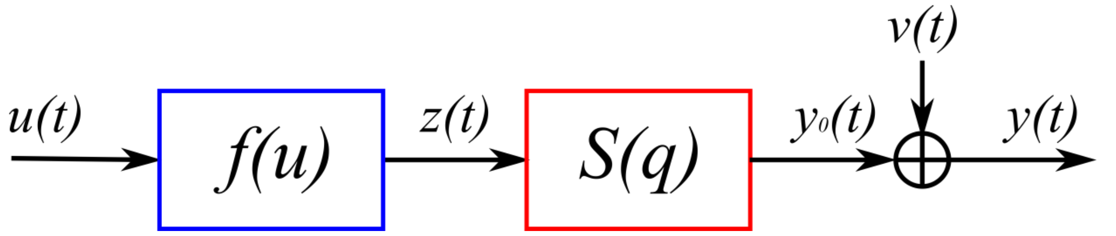
$$G_{bla}(q) = H(q)$$

➔ No changes

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein			
Wiener			
Wiener-Hammerstein			
Parallel WH			
Feedback			
LFR			

Structure detection

Hammerstein



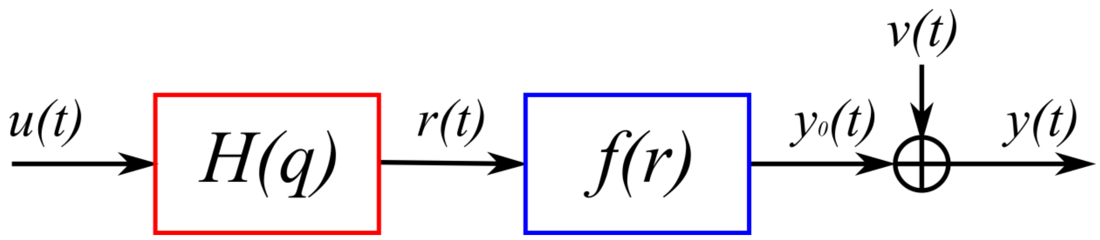
$$G_{bla}(q) = \gamma S(q)$$

➔ Only gain factor

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein	Variable	Fixed	Fixed
Wiener			
Wiener-Hammerstein			
Parallel WH			
Feedback			
LFR			

Structure detection

Wiener



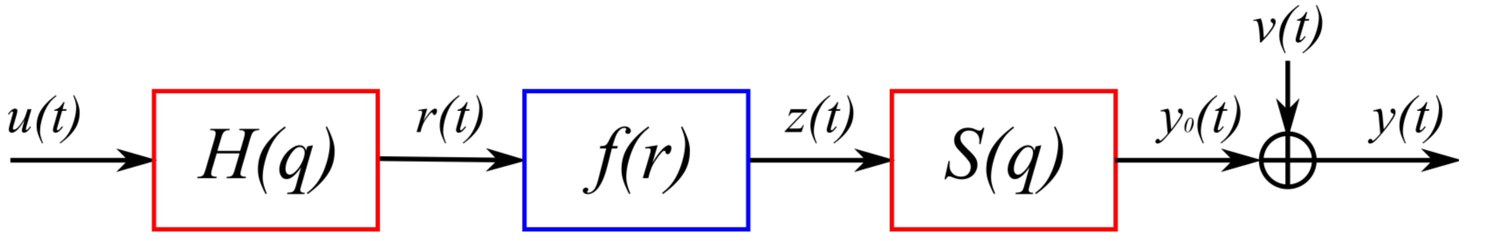
$$G_{bla}(q) = \gamma H(q)$$

➔ Only gain factor

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein	Variable	Fixed	Fixed
Wiener	Variable	Fixed	Fixed
Wiener-Hammerstein			
Parallel WH			
Feedback			
LFR			

Structure detection

Wiener-Hammerstein



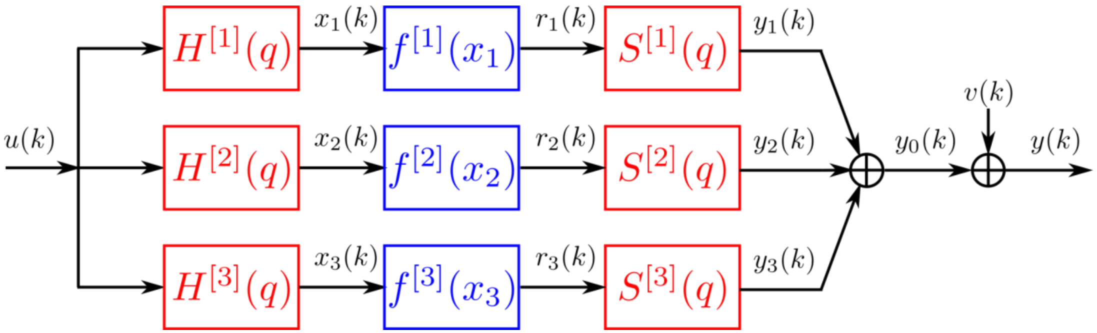
$$G_{bla}(q) = \gamma H(q)S(q)$$

➔ Only gain factor

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein	Variable	Fixed	Fixed
Wiener	Variable	Fixed	Fixed
Wiener-Hammerstein	Variable	Fixed	Fixed
Parallel WH			
Feedback			
LFR			

Structure detection

Parallel Wiener-Hammerstein



$$G_{bla}(q) = \sum_i \gamma_i H^{[i]}(q) S^{[i]}(q)$$

➔ Moving zeros, fixed poles

Structure detection

Moving zeros, fixed poles

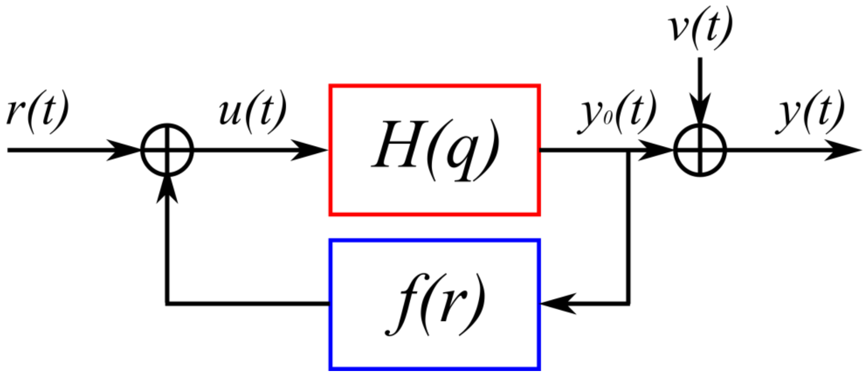
$$G_{BLA} = \gamma_1 \frac{B_1}{A_1} + \gamma_2 \frac{B_2}{A_2}$$

$$= \frac{\gamma_1 B_1 A_2 + \gamma_2 B_2 A_1}{A_1 A_2}$$

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein	Variable	Fixed	Fixed
Wiener	Variable	Fixed	Fixed
Wiener-Hammerstein	Variable	Fixed	Fixed
Parallel WH	Variable	Fixed	Variable
Feedback			
LFR			

Structure detection

Feedback system



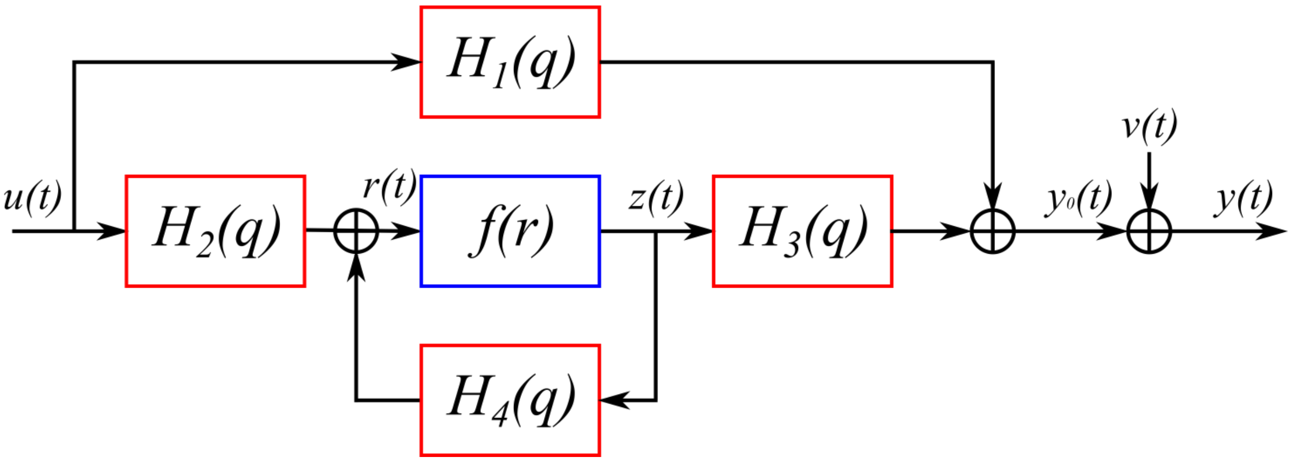
$$G_{\varepsilon}(q) = \frac{H(q)}{1+\gamma H(q)}$$

➔ Fixed zeros, moving poles

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein	Variable	Fixed	Fixed
Wiener	Variable	Fixed	Fixed
Wiener-Hammerstein	Variable	Fixed	Fixed
Parallel WH	Variable	Fixed	Variable
Feedback	Variable	Variable	Fixed
LFR			

Structure detection

LFR



$$G_{\varepsilon}(q) = H_1(q) + \frac{\gamma H_2(q) H_3(q)}{1 + \gamma H_4(q)}$$

➔ Moving zeros, moving poles

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein	Variable	Fixed	Fixed
Wiener	Variable	Fixed	Fixed
Wiener-Hammerstein	Variable	Fixed	Fixed
Parallel WH	Variable	Fixed	Variable
Feedback	Variable	Variable	Fixed
LFR	Variable	Variable	Variable

Structure detection

BLA, ε – approximation @ ≠ setpoints

Only gain change

- Hammerstein, Wiener, Wiener-Hammerstein, …

Zeros shift

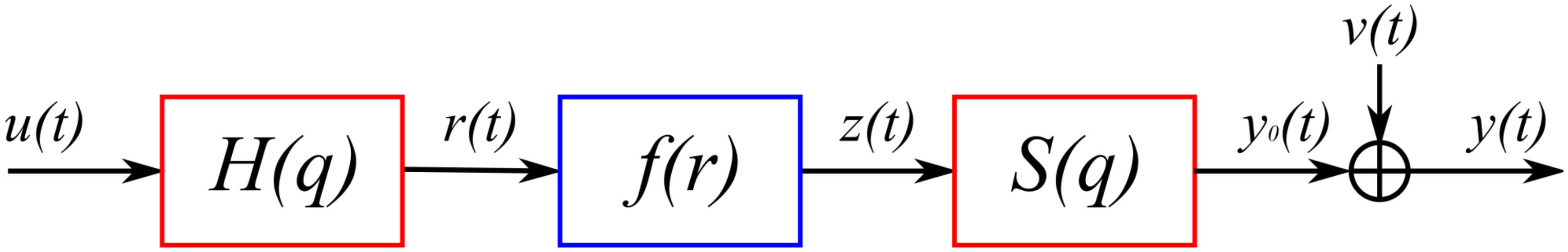
- Parallel feed-forward structure

Poles shift

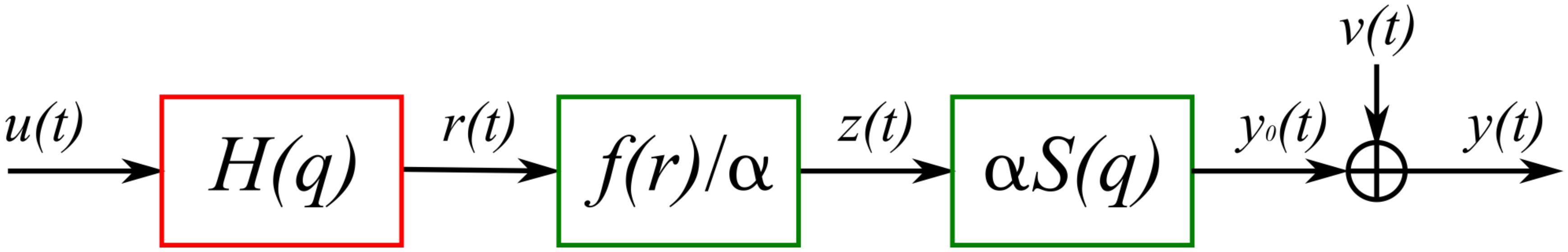
- Feedback present

Data-driven modeling

Wiener-Hammerstein

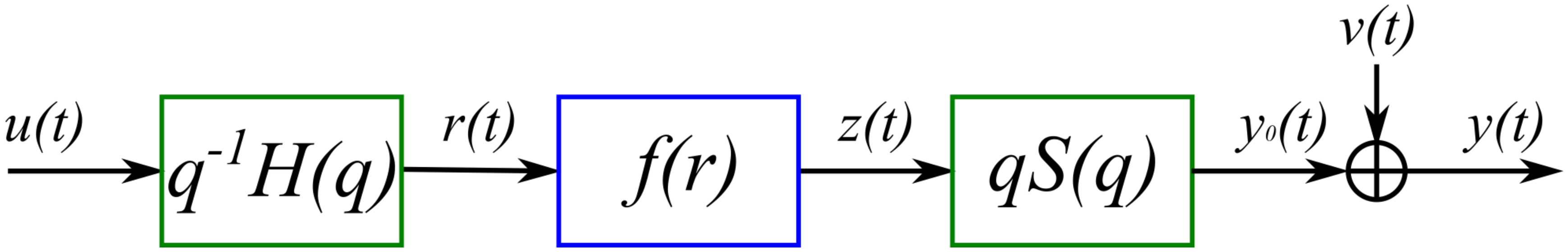


Identifiability



Gain exchange

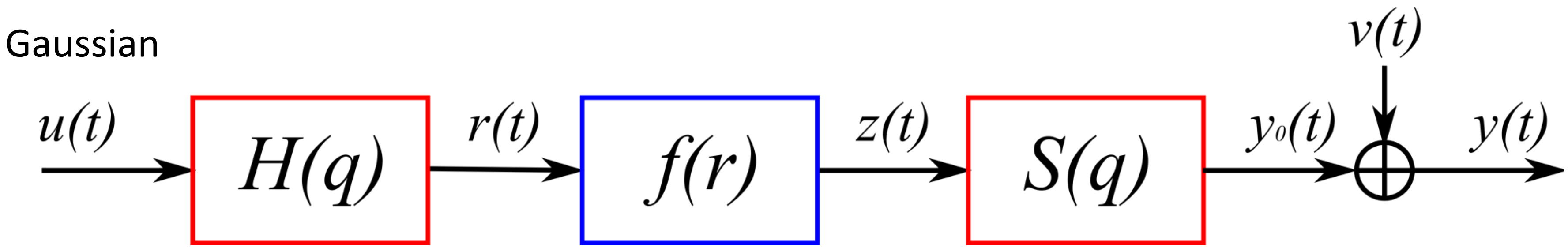
Identifiability



Gain exchange

Delay exchange

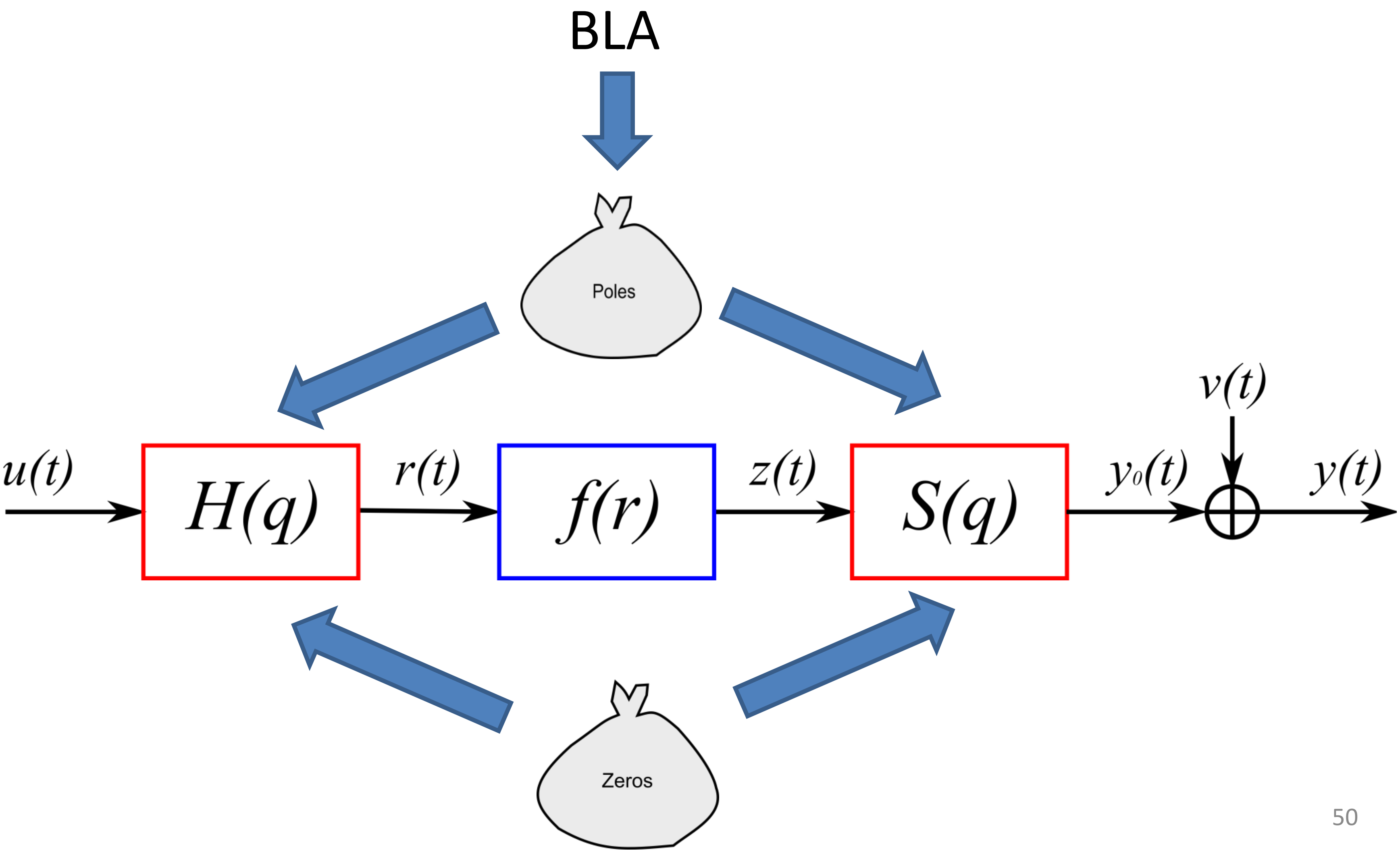
Best Linear Approximation



$$G_{bla}(q) = \gamma H(q) S(q)$$

➔ poles, zeros BLA = poles, zeros system

Partition the Dynamics



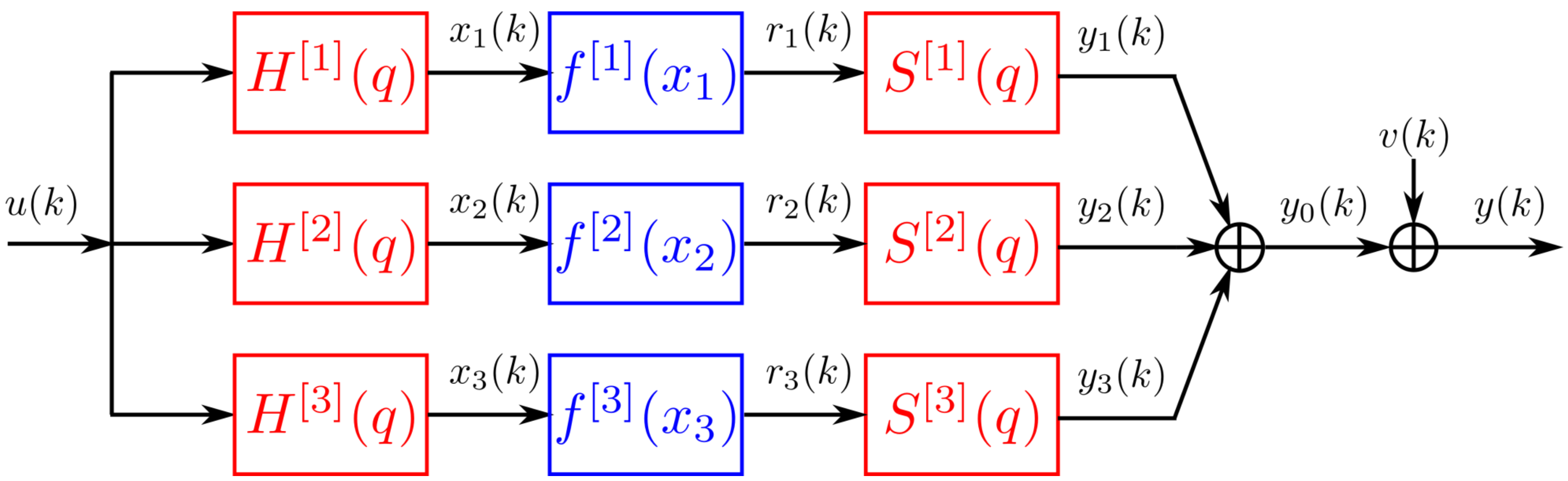
Nonlinear optimization

Initial parameter values

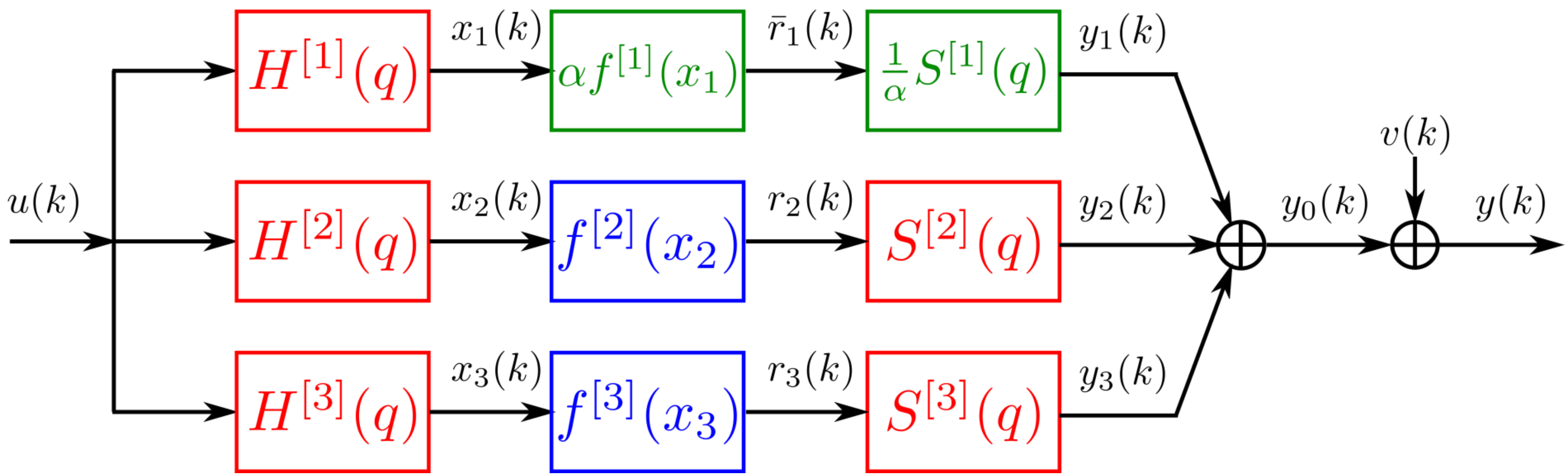
→ Optimization of all parameters together

→ Levenberg-Marquardt algorithm

Parallel Wiener-Hammerstein

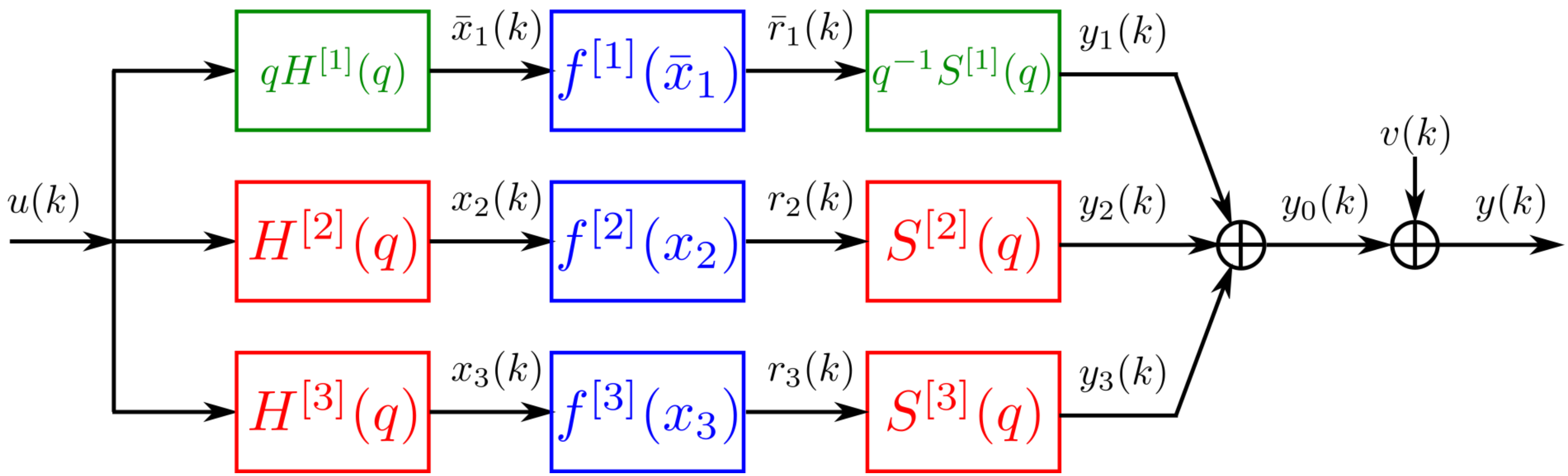


Identifiability



Gain exchange

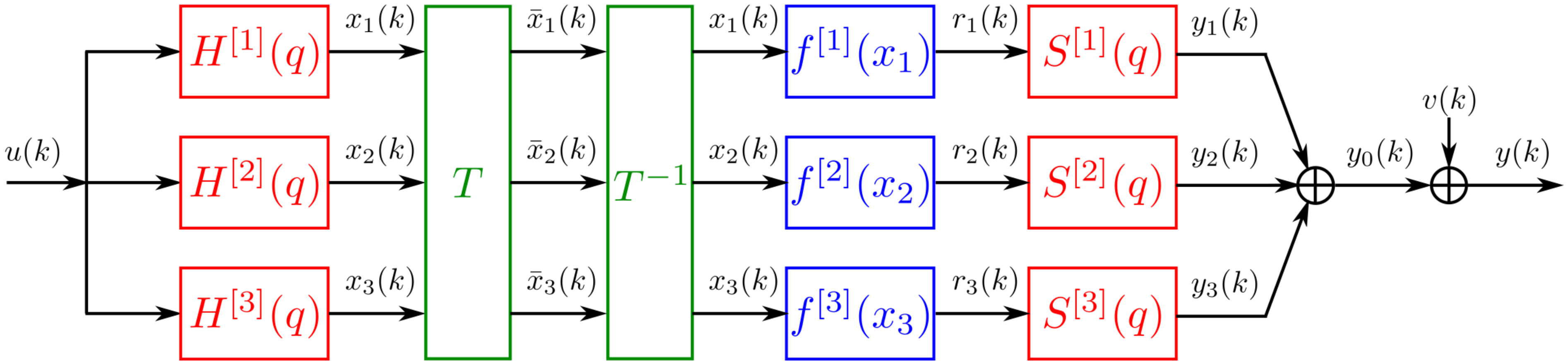
Identifiability



Gain exchange

Delay exchange

Identifiability

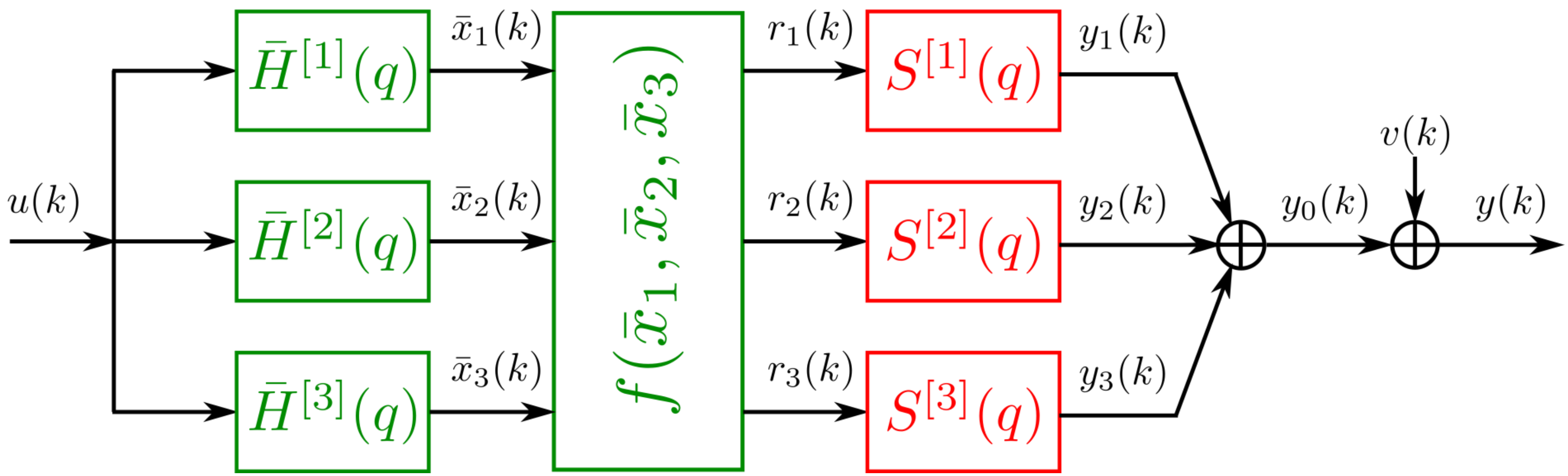


Gain exchange

Delay exchange

Full rank linear transform

Identifiability

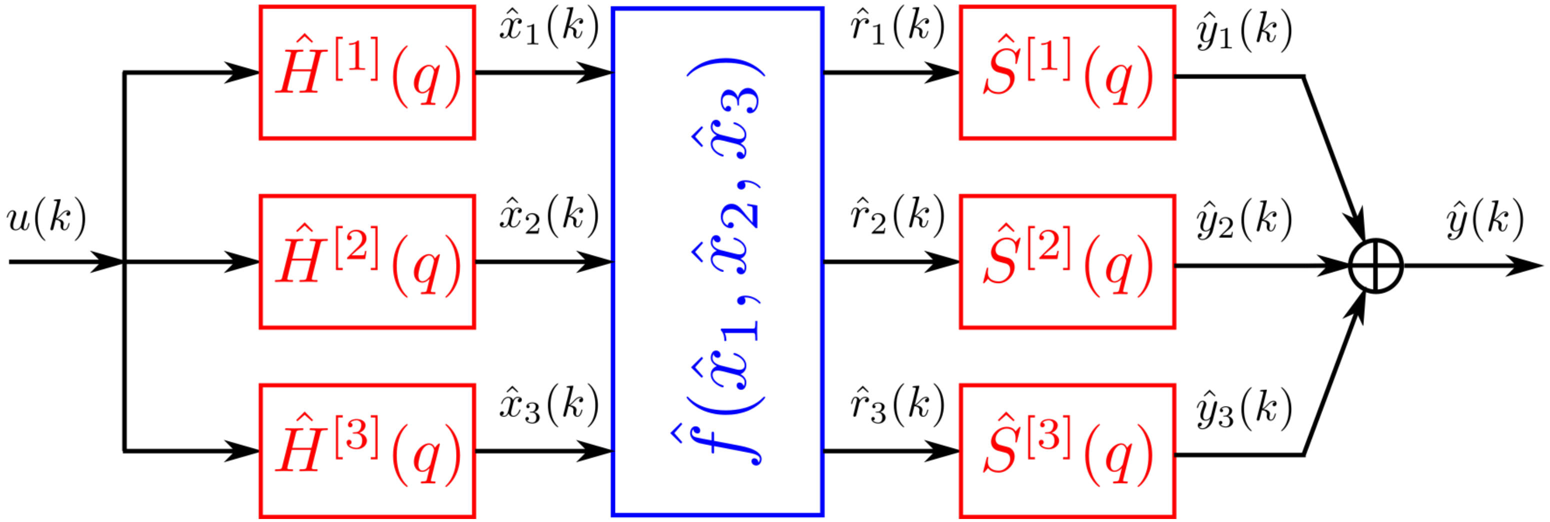


Gain exchange

Delay exchange

Full rank linear transform

Model structure

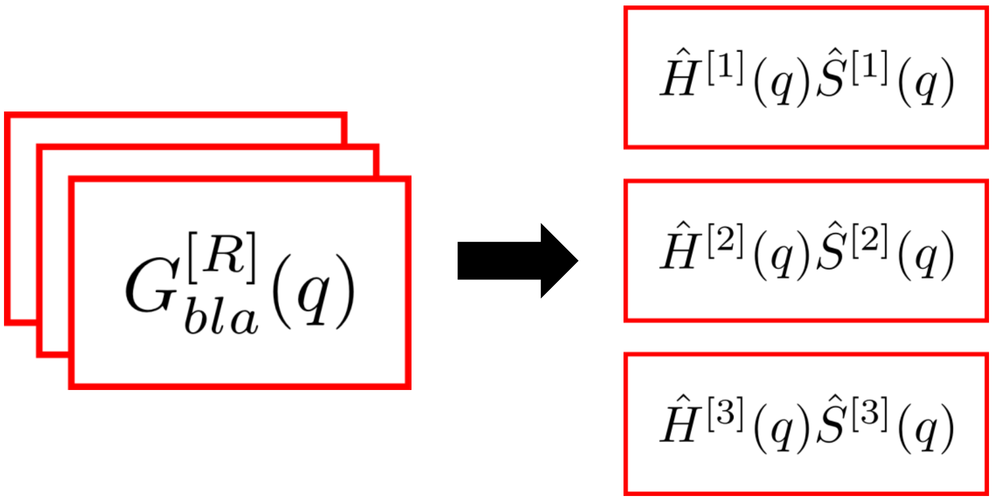


Identification approach

$$G_{bla}^{[R]}(q)$$

Estimate overall dynamics

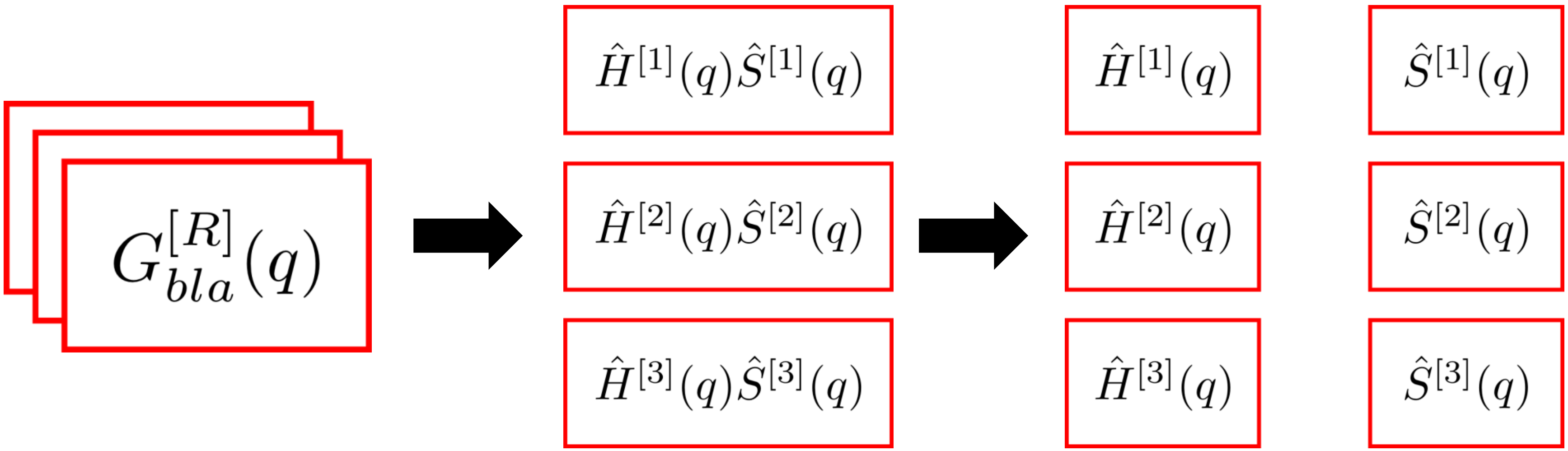
Identification approach



Estimate overall dynamics

Decompose the dynamics over the parallel branches

Identification approach



Estimate overall dynamics

Decompose the dynamics over the parallel branches

Partition the dynamics to the front and back

Identification approach

Identifying the overall dynamics

➔ Best Linear Approximation (BLA)

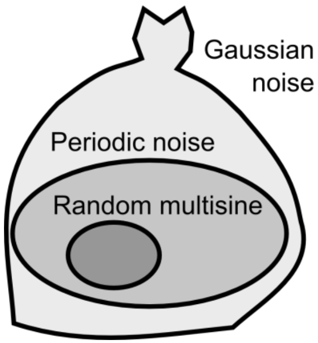
Decomposing the dynamics

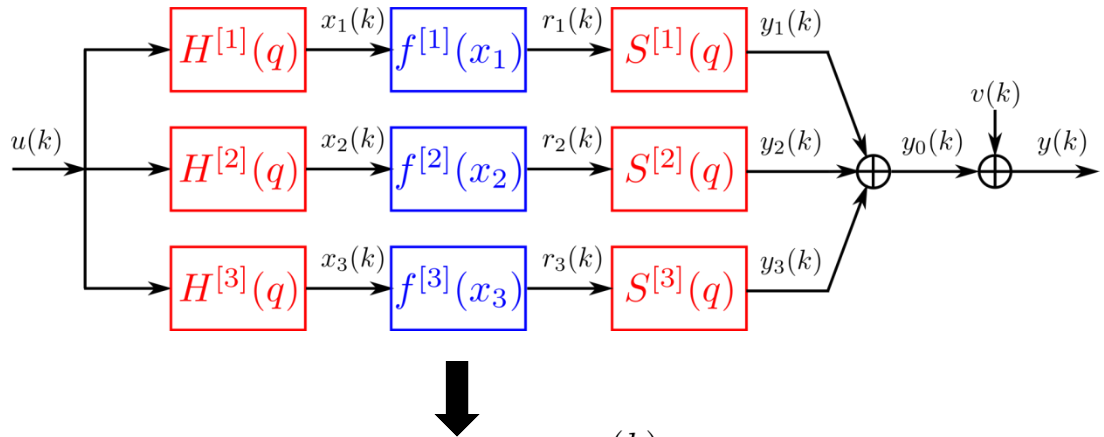
➔ Singular Value Decomposition (SVD) of the BLAs

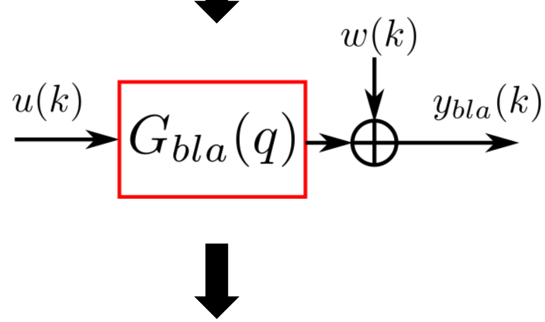
Partition the dynamics

➔ Pole and zero allocation scan

Best Linear Approximation



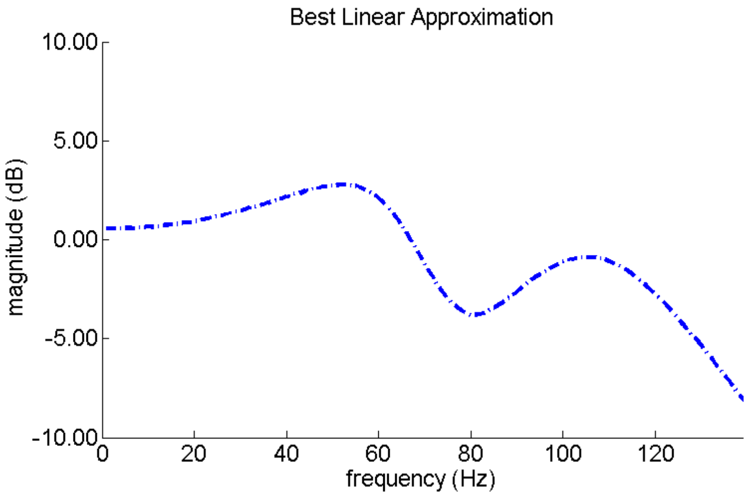


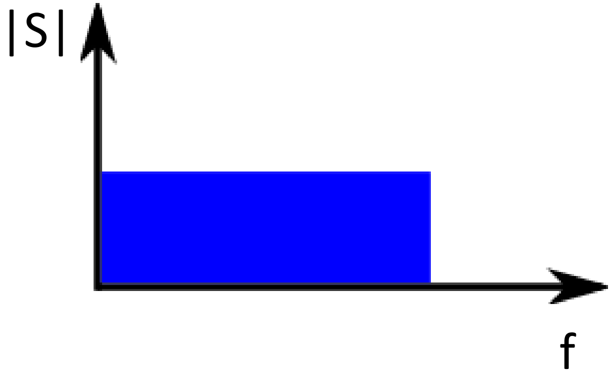


Combination of dynamics!

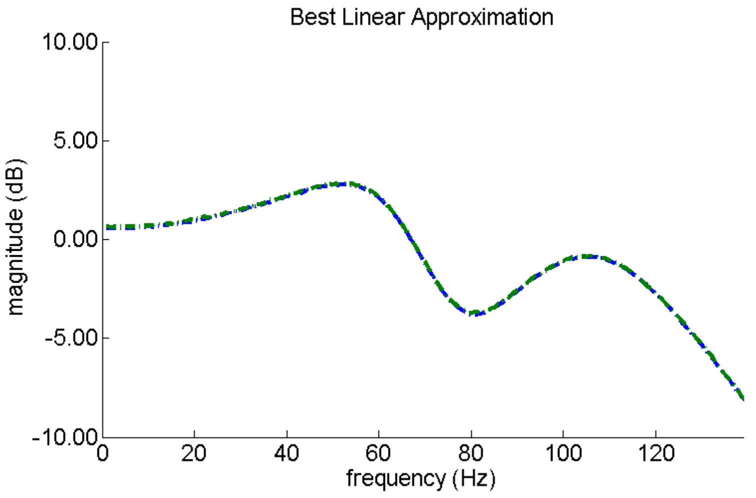
$$G_{bla}(j\omega) = \sum_i \gamma_i H^{[i]}(j\omega) S^{[i]}(j\omega)$$

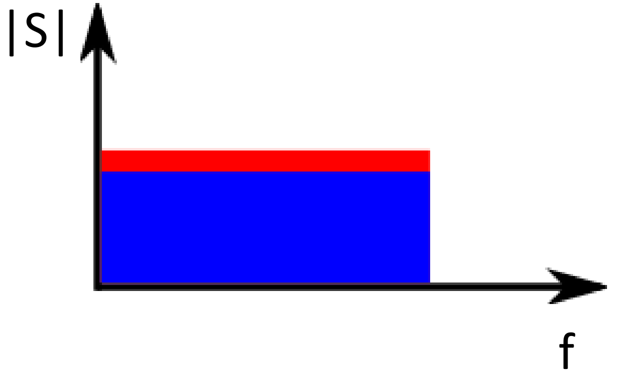
Best Linear Approximation



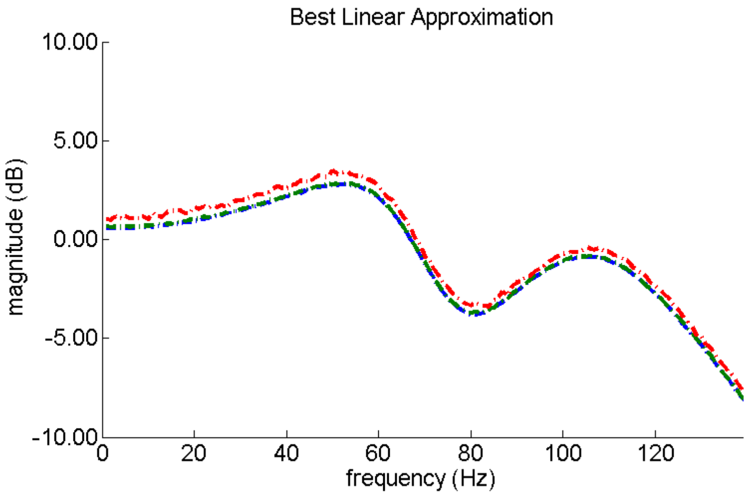


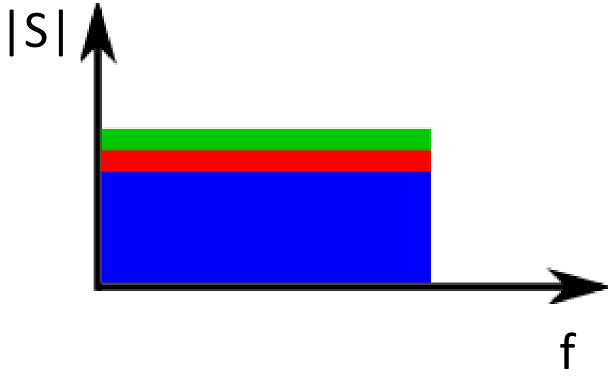
Best Linear Approximation



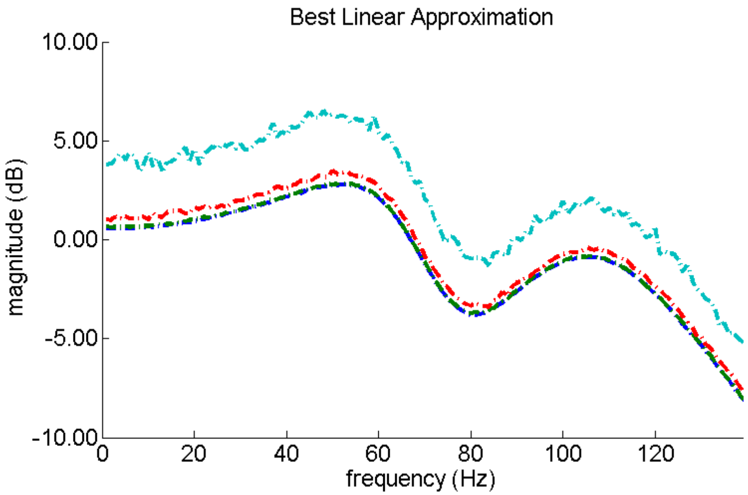


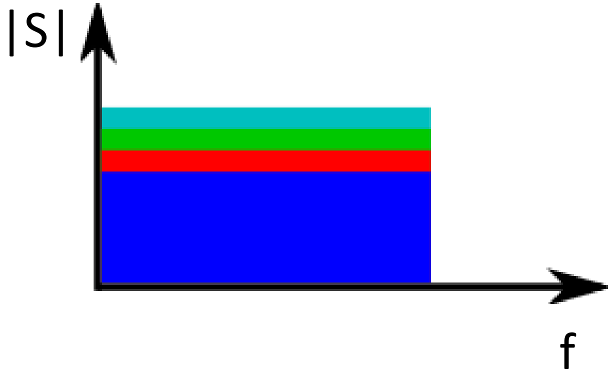
Best Linear Approximation



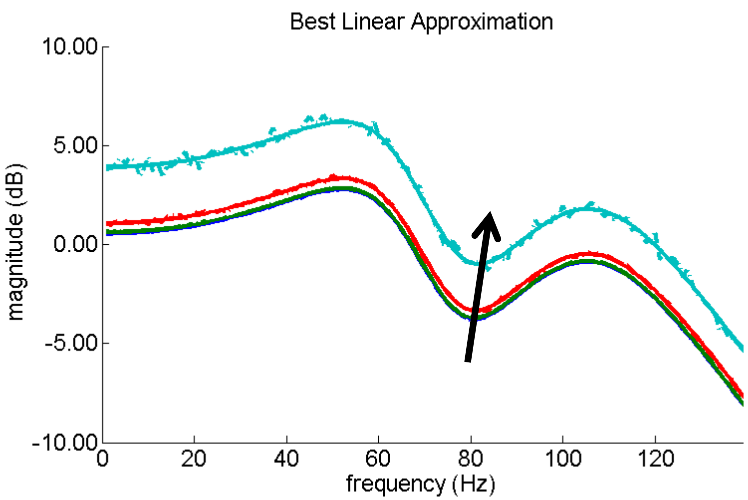


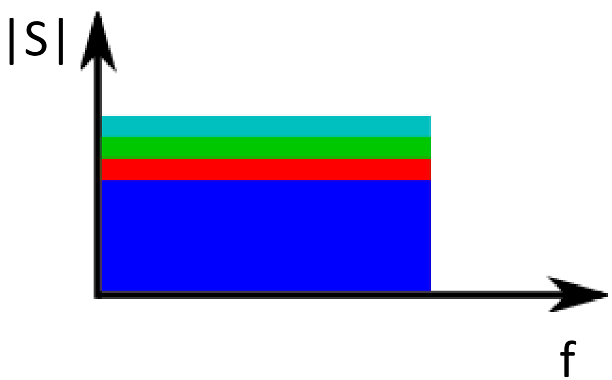
Best Linear Approximation





Best Linear Approximation



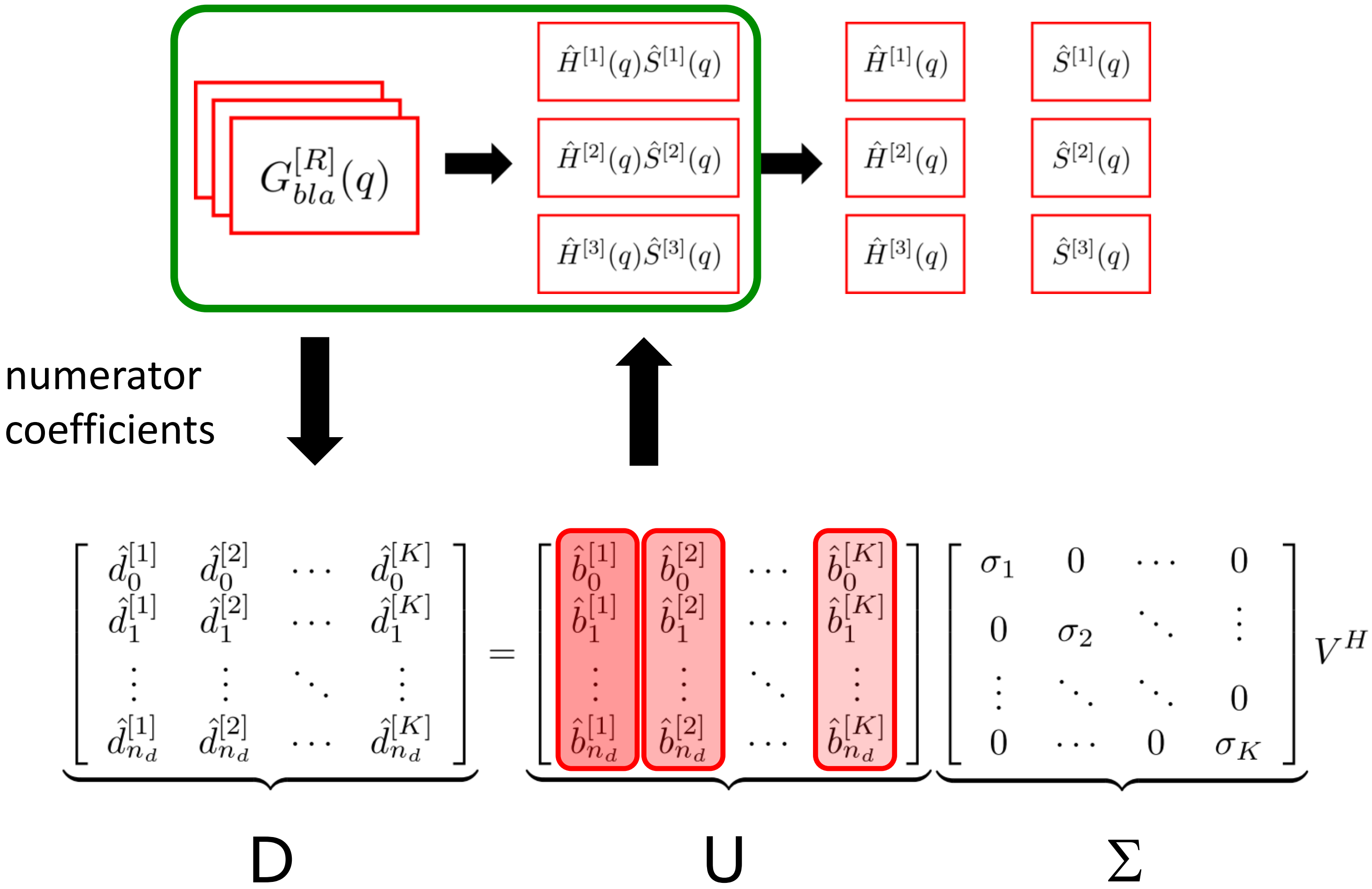


$$G_{bla}(j\omega) = \sum_i \gamma_i H^{[i]}(j\omega) S^{[i]}(j\omega)$$

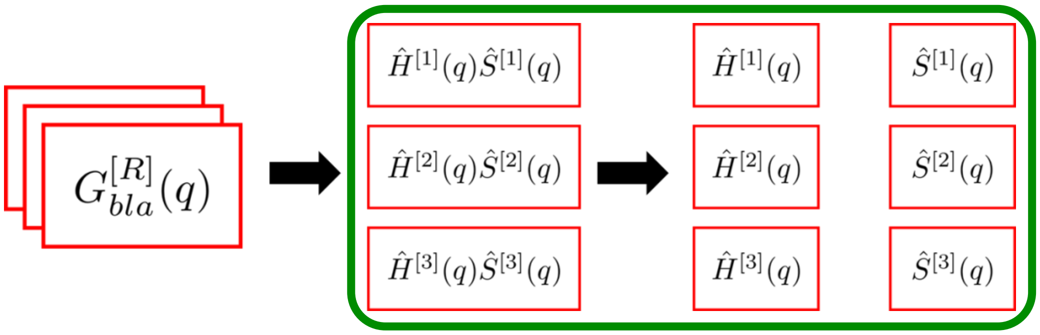
$$\hat{G}_{bla}^{[i]} = \frac{\hat{d}_0^{[i]} + \hat{d}_1^{[i]} q^{-1} + \ldots + \hat{d}_{n_d}^{[i]} q^{-n_d}}{\hat{c}_0 + \hat{c}_1 q^{-1} + \ldots + \hat{c}_{n_c} q^{-n_c}}$$

- Common denominator
  - Fixed poles
  - Moving zeros

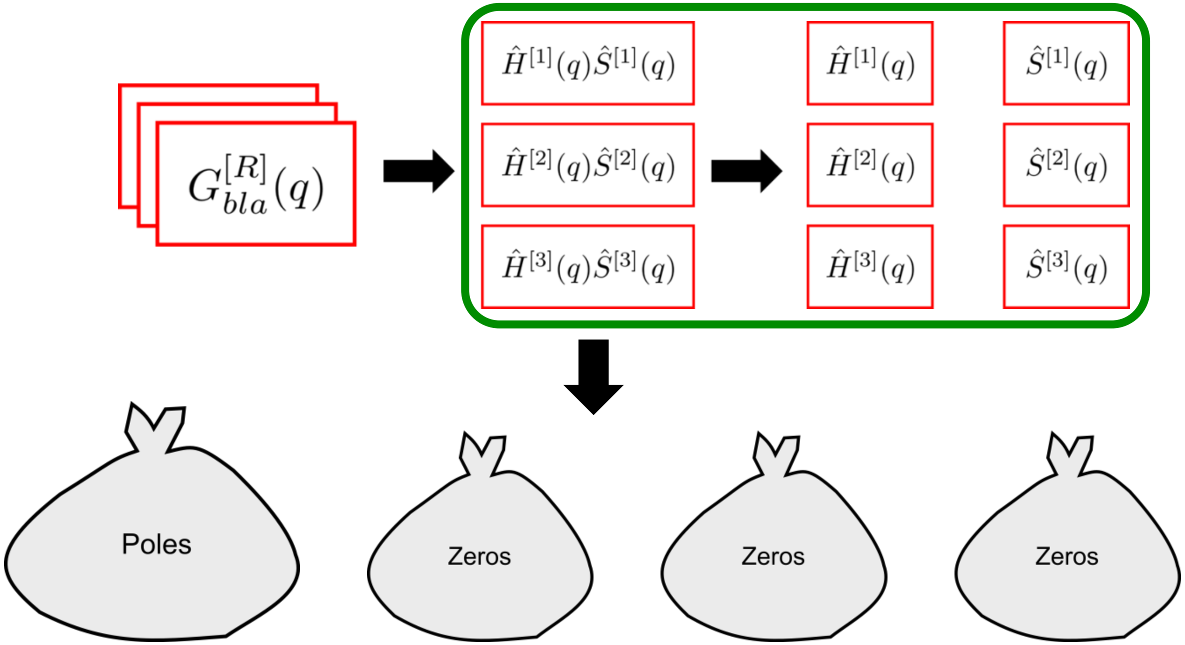
Decomposing the dynamics



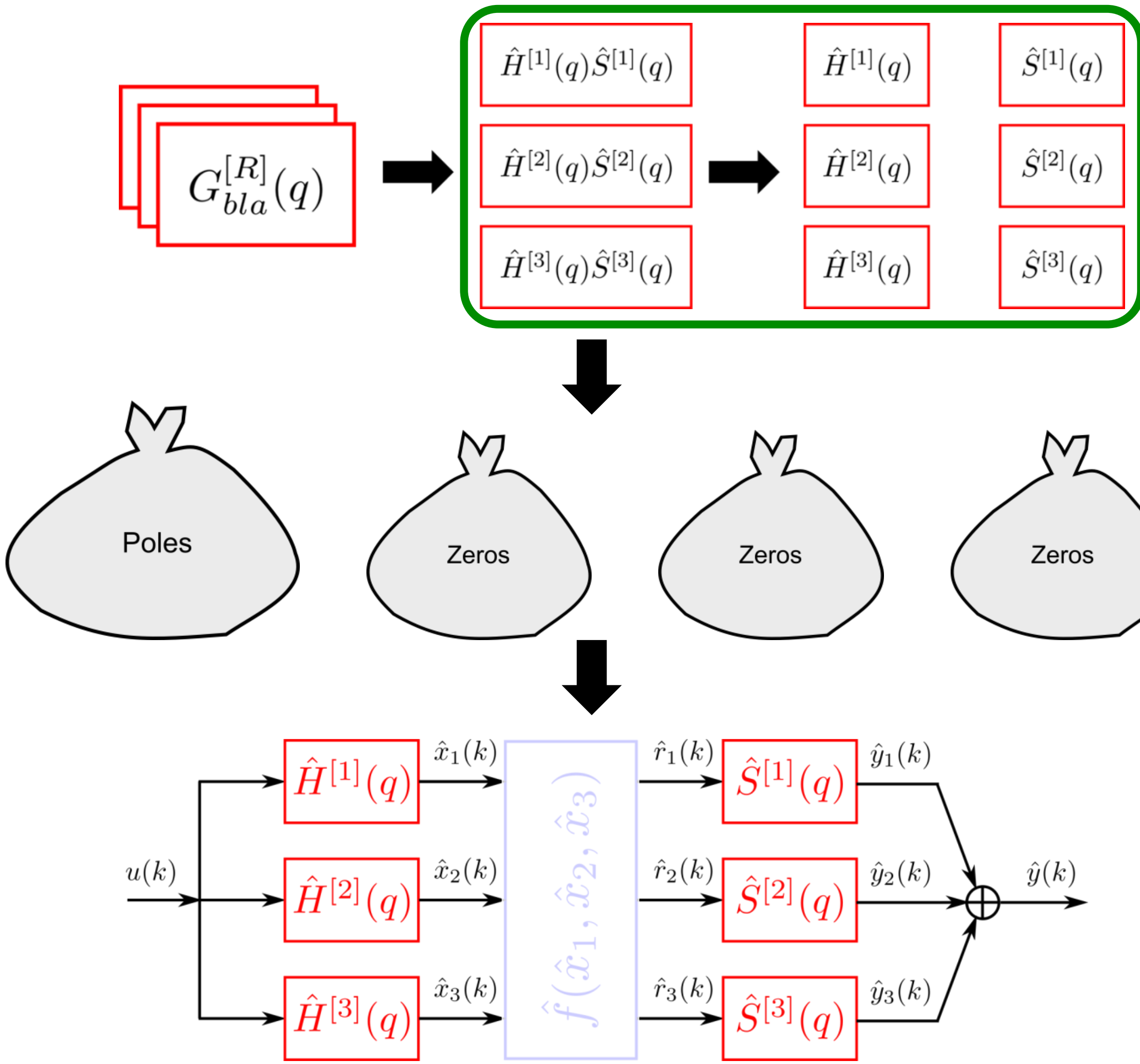
Partition the dynamics



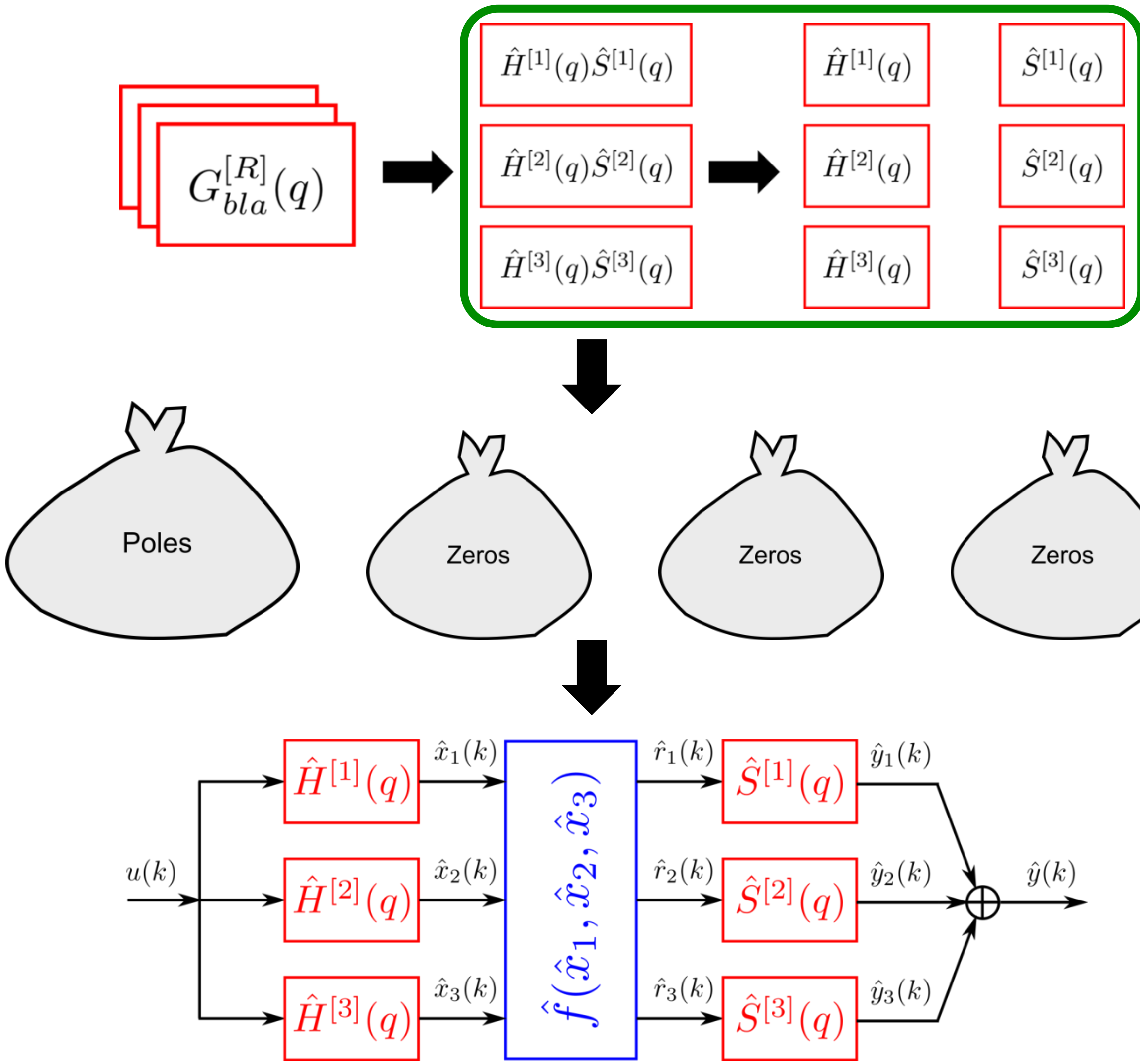
Partition the dynamics



Partition the dynamics



Partition the dynamics



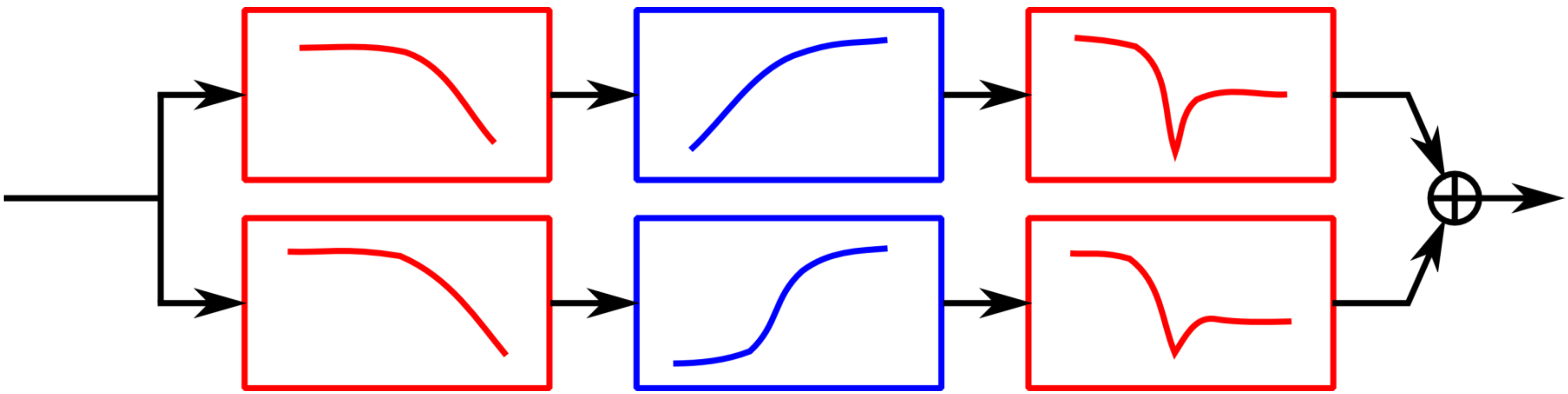
Nonlinear optimization

Initial parameter values

→ Optimization of all parameters together

→ Levenberg-Marquardt algorithm

Example: test system

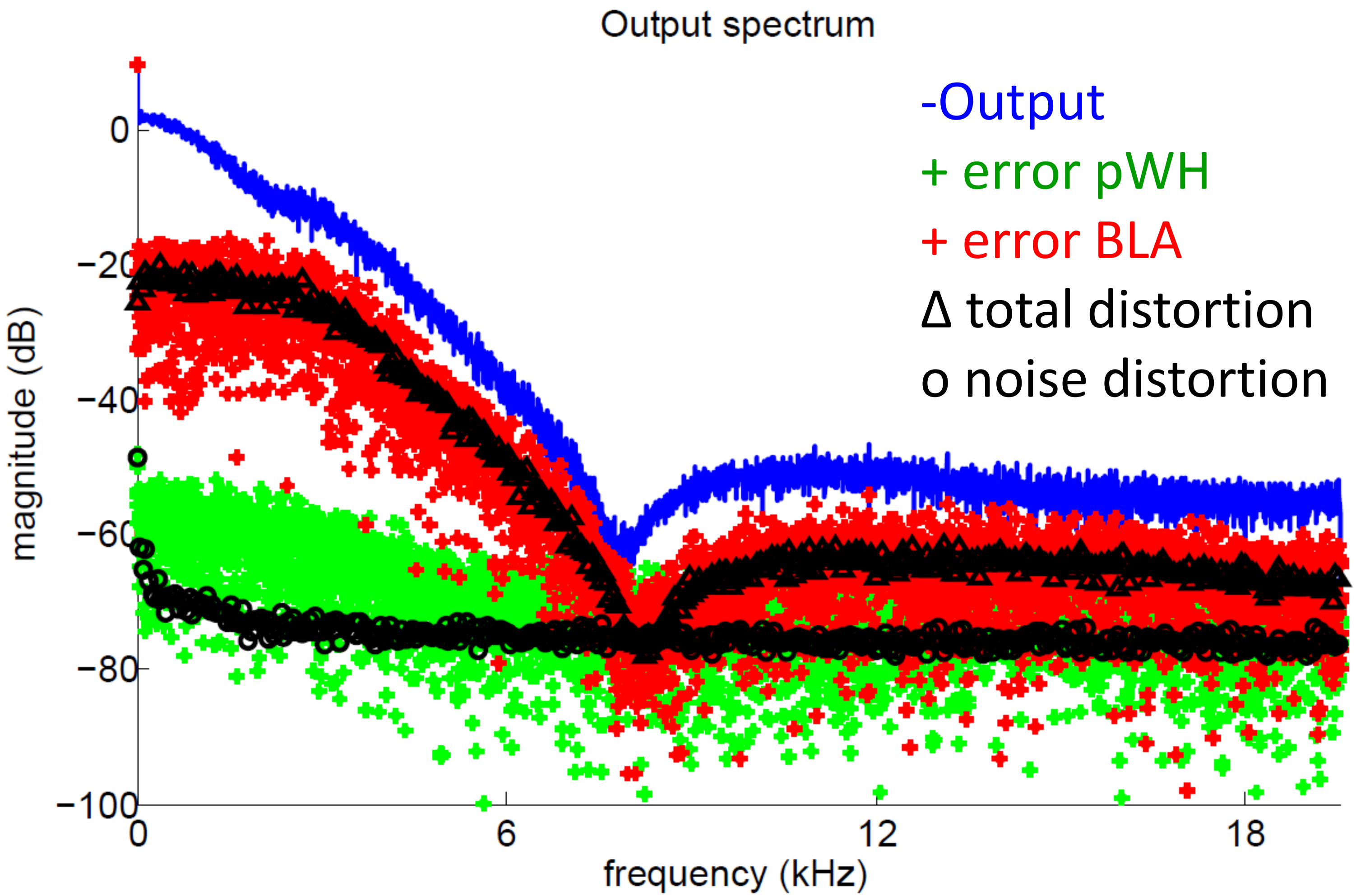


Multisine input:
5 amplitudes
20 realizations
2 periods
16384 samples

System:
Custom built circuit
12^{th} order dynamics
Diode-resistor NL

Model:
2 branches
10 neurons nn NL

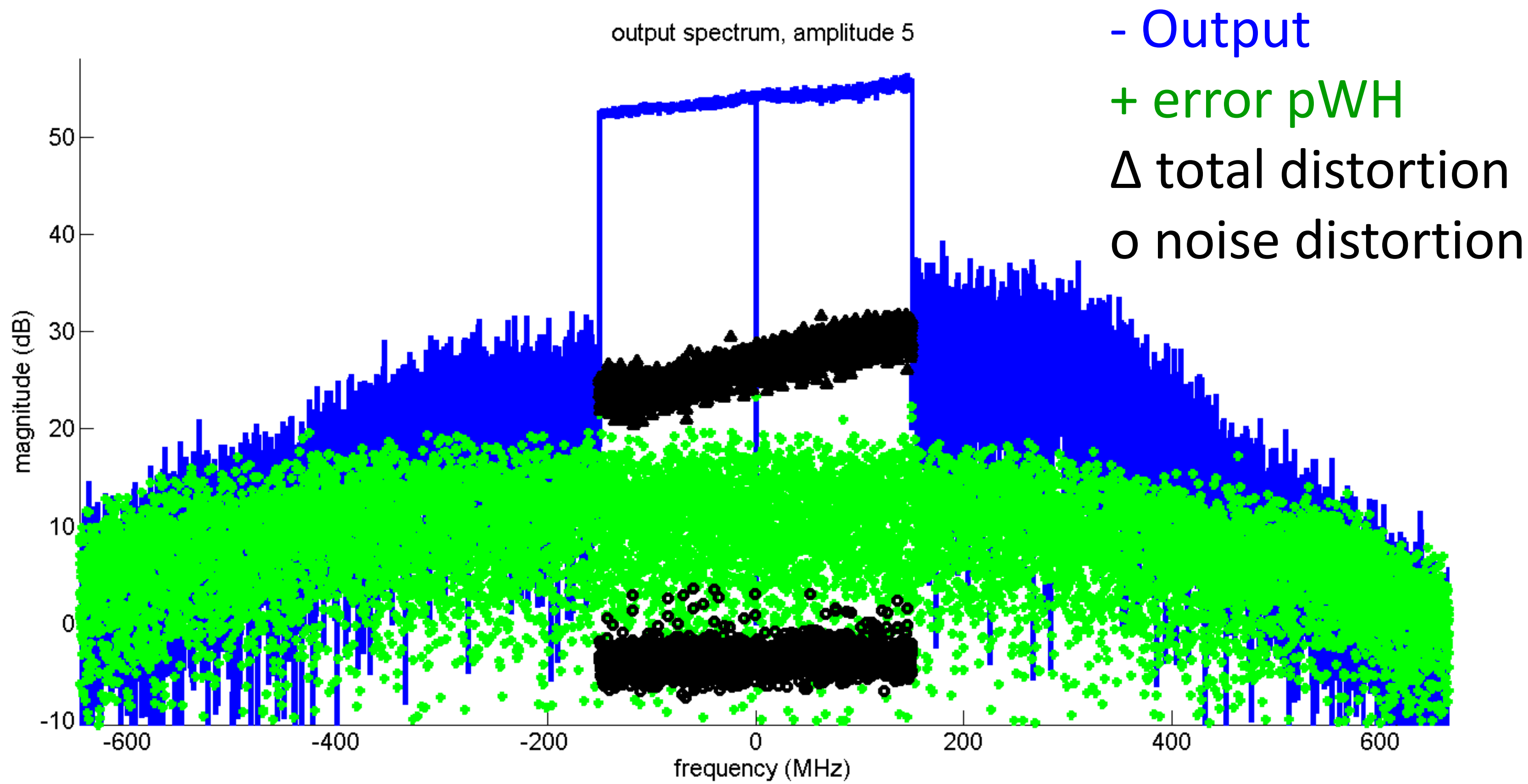
Example: test system



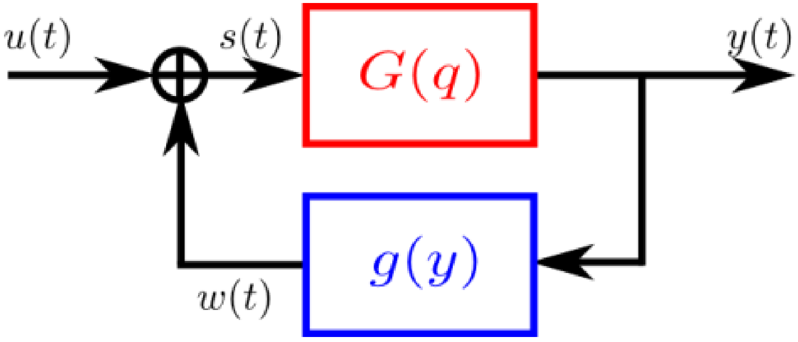
Example: Doherty PA

- Doherty PA
- Input:
  - Multitone, 5 amplitudes, 20 realizations
  - Bandwidth: 300MHz @ 3.45GHz
- Model
  - 2 branches
  - 10 tap FIR BLA
  - 7^{th} order NL

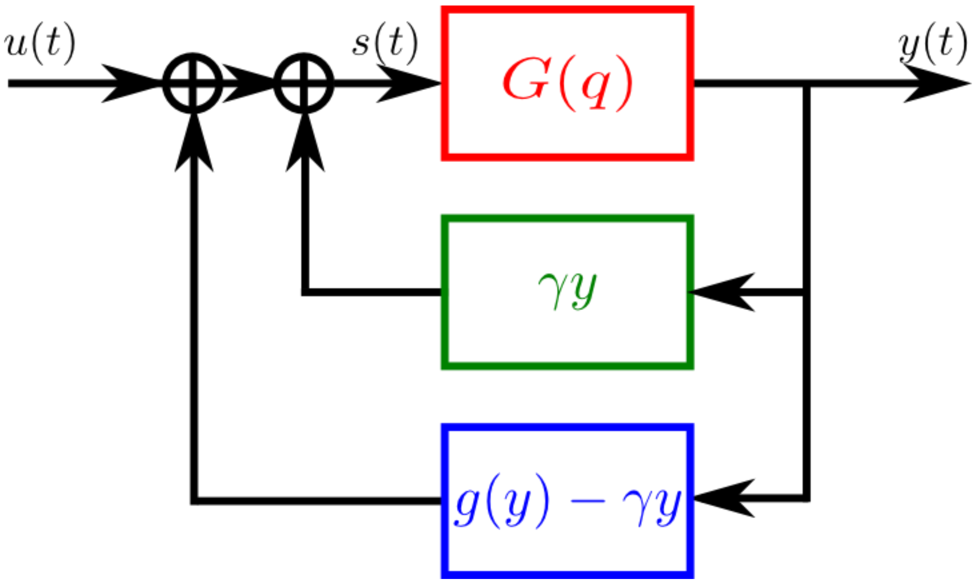
Example: Doherty PA



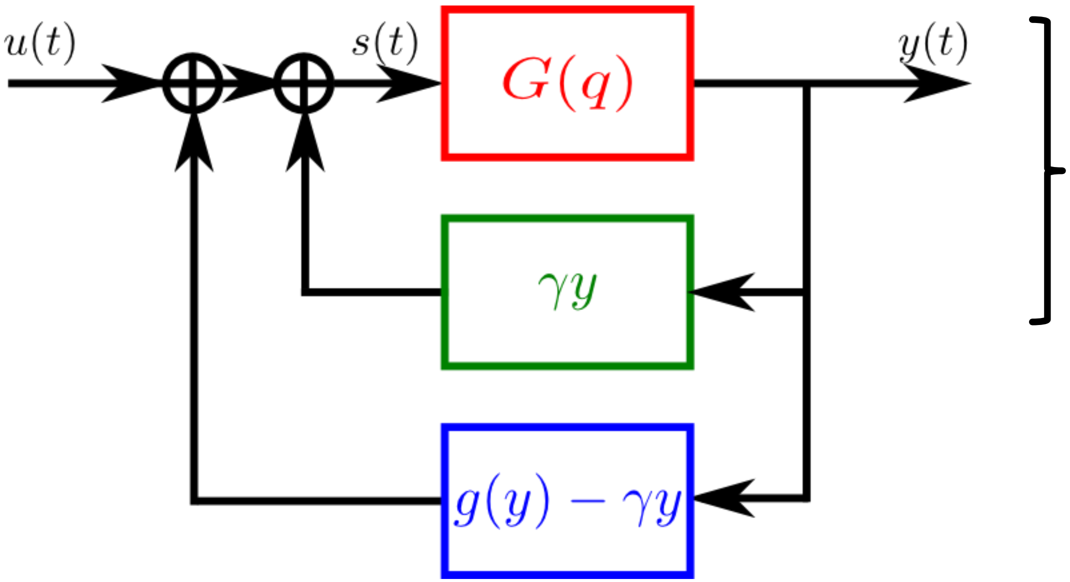
Simple feedback structure



Identifiability

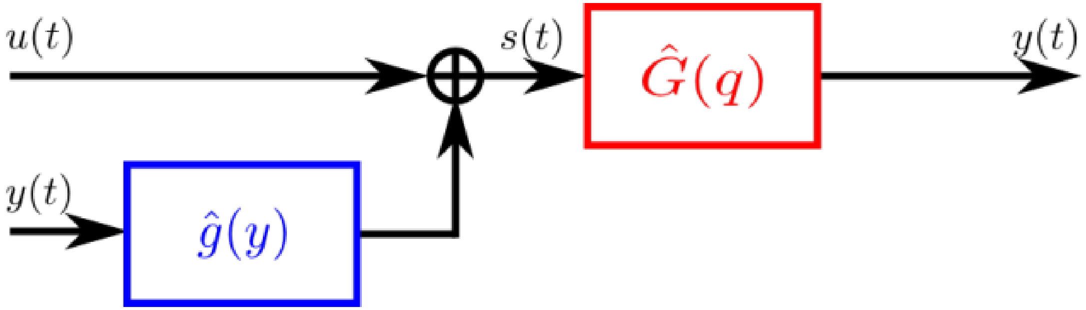


Identifiability



$$\frac{G(q)}{1+\gamma G(q)}$$

Identification



Nonlinear optimization

Initial parameter values

→ Optimization of all parameters together

→ Levenberg-Marquardt algorithm

Conclusion

BLA for nonlinearity analysis

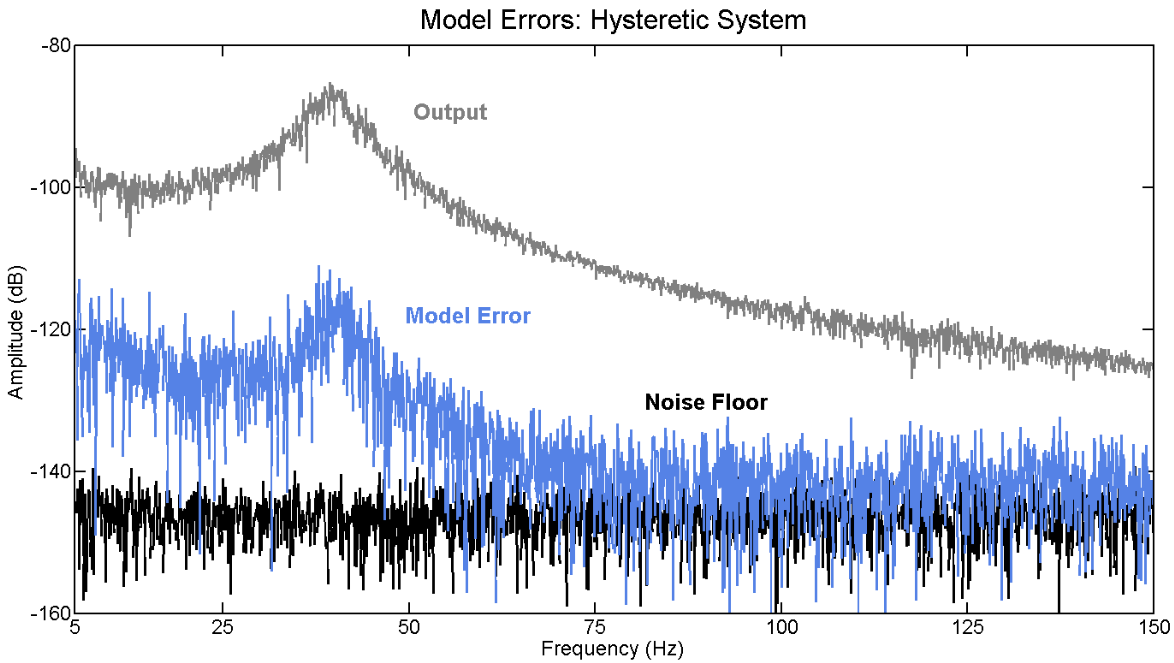
BLA for structure detection

BLA for modeling

- Single branch
- Parallel branch
- Feedback

Future perspectives

Model errors

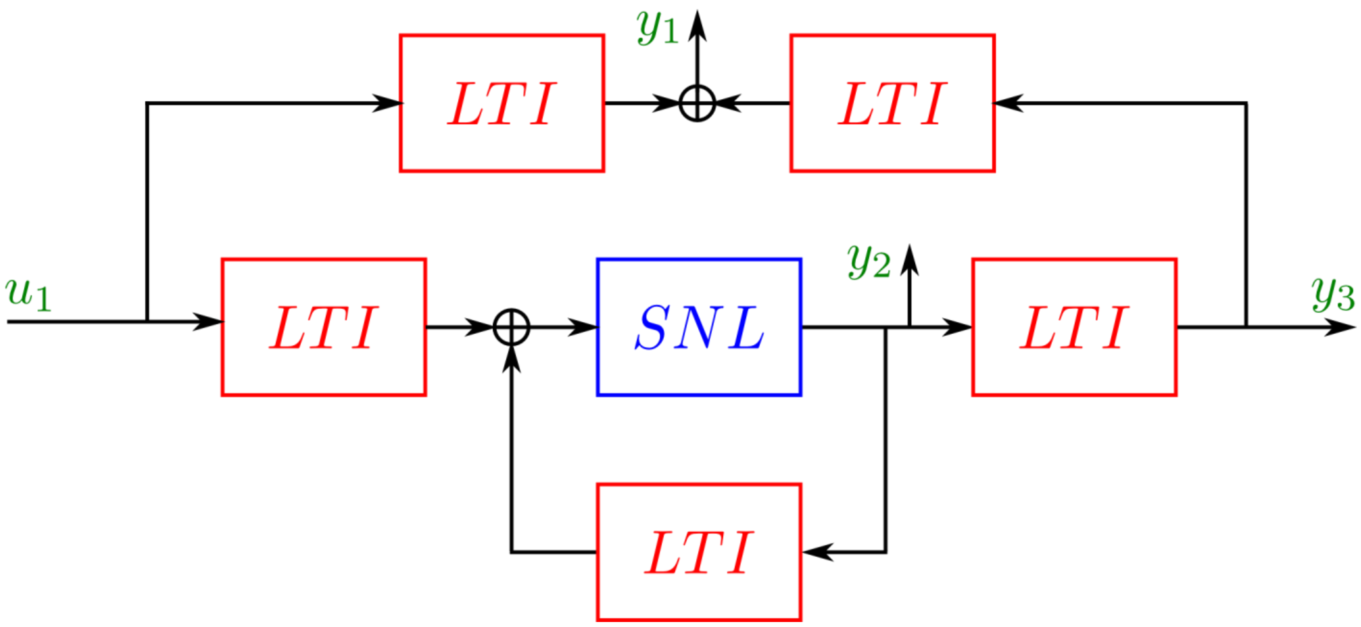


Dominant model errors

Statistical framework?

Best nonlinear approximation?

Modeling networked systems

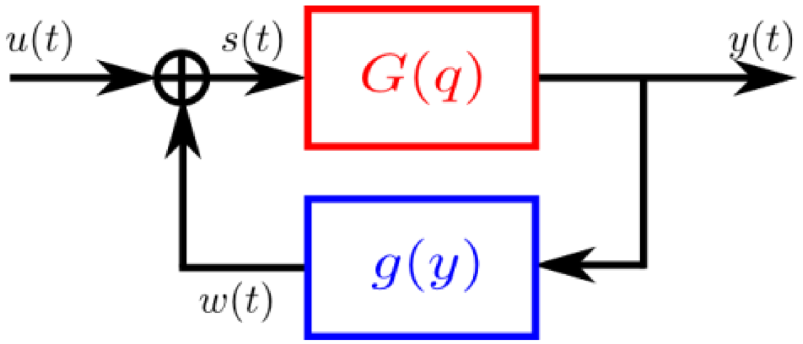


Nonlinear model for everything?

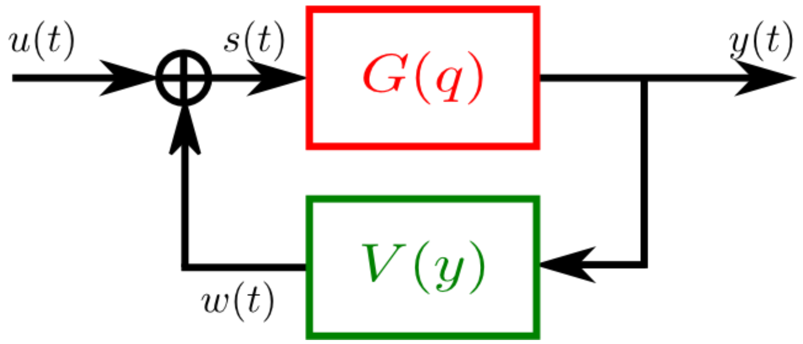
Nonlinearity detection?

Structure detection?

Volterra nonlinearities



Simple Feedback Structure



Volterra Feedback Structure

Introduce dynamic nonlinear block

Increase model flexibility

Keep model simplicity

Block-oriented data-driven modeling starting from the best linear approximation

Maarten Schoukens

Vrije Universiteit Brussel