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Brief Paper

Frequency response function measurements in the presence of nonlinear distortions☆

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Abstract

In this paper, we deal with the measurement of the frequency response function (FRF) of linear dynamic systems in the presence of nonlinear distortions. It is shown that it is possible to detect, qualify and quantify the nonlinear distortions during a broadband frequency response measurement. Advises are formulated how to get the best measurements under these conditions. All results are illustrated by experiments. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Identification; Frequency responses; Nonlinear distortions; Linear approximation; Detection

1. Introduction

The aim of this paper is to provide the reader with insight in the behaviour of nonlinear distortions of a dominantly linear system, and to study their impact on frequency response function measurements. The measured output $y(t)$ consists of a linear y_{L} plus a nonlinear y_{NL} contribution. We have two basic options: (1) the goal of the measurement is to get the FRF of the underlying linear system (if it exists!), minimizing the impact of the NLS (nonlinear system) on the measurements; (2) try to find the best linear approximation to the global system, including the NLS. The second choice is preferred if the nonlinear system is linearised around its operating point. This best approximation can still be made, even if the underlying linear system does not exist.

This paper is based on the theoretical framework setup by Schoukens, Dobrowiecki, and Pintelon (1998). Precise definitions and all the proofs of the theoretical results that we use in this paper can be found there. Here, we focus completely on the application of these results to FRF measurements to show the practising engineer how he can deal with nonlinear distortions.

2. Properties of FRF measurements in the presence of nonlinear distortions

2.1. Theoretic results

FRF measurements in the presence of nonlinear distortions strongly depend on the class of excitation signals. We focus on normalized random multisines $u_N \in E_N$:

$$u_N(t) = \sum_{k=-N}^{N} U_k e^{j2\pi f_{\max} kt/N}, \quad (1)$$

with $U_k = \bar{U}_{-k} = |U_k| e^{j\varphi_k}$, $\bar{U}$ the complex conjugate of U, φ_k random phases s.t. $\mathscr{E}\{e^{j\varphi_k}\} = 0$. The amplitudes are properly scaled such that the power of the signal behaves as an $O(N^0)$ for an increasing number of components. All the results that we present in this paper can be extended directly to normal distributed noise excitations with a user defined power spectrum.

For periodic excitations, the FRF is obtained by simple division of the output by the input spectrum $G(j\omega_k) = Y_k/U_k$ (Schoukens, Guillaume, & Pintelon, 1993), while correlation methods are classically used

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for noise excitations $\hat{G}(\mathrm{j}\omega) = S_{YU}(\omega)/S_{UU}(\omega)$ (Bendat & Piersol, 1980). The results we present here are independent from the selected method.

For normalized random excitations, and imposing some conditions on the considered class of systems S (that includes relays, quantizers and other discontinuous nonlinear systems), the FRF $G(\mathrm{j}\omega_k)$ at frequency $\omega_k = 2\pi f_{\max} k/N$ can be written as:

$$G(\mathrm{j}\omega_k) = G_{\mathrm{R}}(\mathrm{j}\omega_k) + G_{\mathrm{S}}(\mathrm{j}\omega_k) + N_{\mathrm{G}}(\mathrm{j}\omega_k), \tag{2}$$

where $G_{\mathrm{R}}(\mathrm{j}\omega_k)$ is called the related dynamic system, $G_{\mathrm{S}}(\mathrm{j}\omega_k)$ the stochastic nonlinear contributions, and $N_{\mathrm{G}}(\mathrm{j}\omega_k)$ are the errors due to the output noise.

- For the class of normally distributed signals (including random multisines and noise excitations) $G_{\mathrm{R}}(\mathrm{j}\omega_k)$ is the best linear approximation to the nonlinear system (Schoukens et al., 1998). It consists on its turn of two parts:

  $$G_{\mathrm{R}}(\mathrm{j}\omega_k) = G_0(\mathrm{j}\omega_k) + G_{\mathrm{B}}(\mathrm{j}\omega_k), \tag{3}$$

  with $G_0(\mathrm{j}\omega_k)$ the underlying linear system (if it exists) and $G_{\mathrm{B}}(\mathrm{j}\omega_k)$ the bias or systematic errors due to the nonlinear distortions. These do not depend on the actual phases of the random multisine, but they do depend on the applied power spectrum.
- $G_{\mathrm{S}}(\mathrm{j}\omega_k)$ is called a stochastic contribution since it behaves as uncorrelated (over the frequencies) noise. But the reader should be aware that it is not a random signal once the phases of the excitation signal are fixed. Because of this noisy behaviour, the presence of nonlinear distortions is often not recognized. Even for a very large number of frequencies and in the absence of disturbing noise, the FRF measurement is not smooth as a function of the frequency. It is scattered around its expected value (with respect to the random phases of the input) which equals G_{R}, and these deviations do not converge to zero.
- $N_{\mathrm{G}}(\mathrm{j}\omega_k)$ describes the impact of the disturbing noise on the FRF measurement. It is assumed to be zero mean, normally distributed noise, and is independent over the frequencies.

The different contributions to the FRF were studied for two situations. In the first case the FRF measurement is averaged for different realizations of the excitation with N fixed. The second case deals with the asymptotic behaviour if the number of harmonics $N \to \infty$.

Theorem 1. *For a system belonging to the system set S, excited with independent realizations of a random multisine $u_N \in E_N$, the expected value equals $\mathscr{E}\{G(\mathrm{j}\omega_l)\} = G_{\mathrm{R}}(\mathrm{j}\omega_l)$.*

Proof. See Schoukens et al. (1998). □

Only the odd nonlinear distortions contribute to G_{B}, the even distortions create only stochastic contributions with zero mean value.

The stochastic behaviour of $G_{\mathrm{S}}(\mathrm{j}\omega_l)$ can be further characterized, showing that its second order properties are completely similarly to those of the noise N_{G}, for excitations with a large number of harmonic components. This explains why it is difficult to distinguish between noise and nonlinear distortions. It is also the reason why nonlinear distortions are often not recognized.

Theorem 2. *For a system belonging to the system set S, excited with a random multisine $u_N \in E_N$ the following properties are valid*:

For $k, l \neq 0$

$$\begin{aligned}
&\mathscr{E}\{G_{\mathrm{S}}(\mathrm{j}\omega_l)\} = 0;\ \mathscr{E}\{G_{\mathrm{S}}(\mathrm{j}\omega_l)\bar{G}_{\mathrm{S}}(\mathrm{j}\omega_k)\} = O(N^{-1});\\
&\mathscr{E}\{G_{\mathrm{S}}(\mathrm{j}\omega_l)\,|\,G_{\mathrm{S}}(\mathrm{j}\omega_k)|^2\} = O(N^{-1})\\
&\mathscr{E}\{G_{\mathrm{S}}(\mathrm{j}\omega_k)|^2 - \sigma^2_{\mathrm{GS}}(k))(|G_{\mathrm{S}}(\mathrm{j}\omega_l)|^2 - \sigma^2_{\mathrm{GS}}(l))\} = O(N^{-1})
\end{aligned} \tag{4}$$

For $k = l$ these results become:

$$\begin{aligned}
&\mathscr{E}\{|G_{\mathrm{S}}(\mathrm{j}\omega_l)|^2\} \equiv \sigma^2_{\mathrm{GS}}(l) = O(N^0);\\
&\mathscr{E}\{(|G_{\mathrm{S}}(\mathrm{j}\omega_k)|^2 - \sigma^2_{\mathrm{GS}}(k))(|G_{\mathrm{S}}(\mathrm{j}\omega_l)|^2 - \sigma^2_{\mathrm{GS}}(l))\} = O(N^0)
\end{aligned}$$

Proof. See Schoukens et al. (1998). □

Remark. These observations are in agreement with the classical result, showing that the output of a nonlinear system can be split into two parts (Bendat, 1998; Forsell & Ljung, 1999): a first part that is linearly related with the input (in our case leading to G_{R}), and second part that is uncorrelated with the input (leading to G_{S}). Theorem 2 tells more about the second and higher order properties of the uncorrelated part.

2.2. Experimental illustration

The concept of best linear approximation G_{R}, bias contribution G_{B}, and stochastic nonlinear contributions G_{S}, are illustrated on a nonlinear electrical circuit that is ideally described by the following nonlinear 2nd order differential equation:

$$m\frac{\mathrm{d}^2}{\mathrm{d}t}y(t) + \mathrm{d}\frac{\mathrm{d}}{\mathrm{d}t}y(t) + ay(t) + by(t)^3 = u(t). \tag{5}$$

Of course the actual realized circuit is not in perfect agreement with (5), for example, we noticed in the measurements also the presence of a small quadratic term $y(t)^2$.

The underlying linear system is measured using a normalized random multisine ($f_k = kf_0$, $k = 1,3,5, \ldots,$

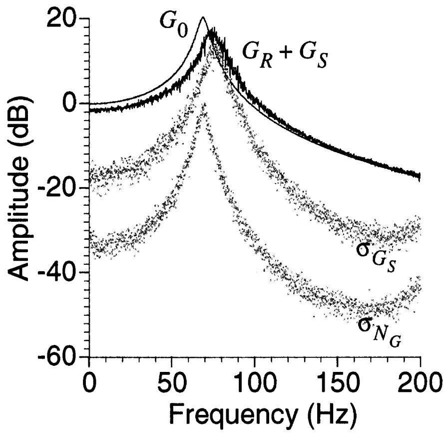

Fig. 1. Measurement of the related dynamic system (G_R), the underlying linear system, the stochastic nonlinearities (σ_{G_S}), and the noise level (σ_{N_G}).

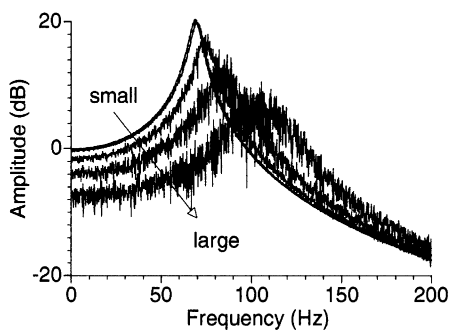

Fig. 2. Evolution of the related dynamic system for growing excitation levels: 34, 54, 127, 253, and 507 mV_{RMS}.

$2N-1$, $N = 1342$ and $f_0 \approx 0.0745$ Hz) with a small amplitude (34.2 mV_{RMS}). The standard deviation σ_{N_G} is calculated from 10 consecutive periods. The results are shown in Fig. 1 (G_0,σ_{N_G}).

The impact of the nonlinearity is made visible by increasing the excitation level of the normalized random multisine to 127 mV_{RMS}. The measurement was repeated for 10 different realizations of the excitation signal so that also σ_{G_S} could be measured. These measurement results are shown in Fig. 1. Note that the resonance frequency is shifted to the right, the peak value is decreased (G_B); and the measurement became more noisy (G_S). Changing the excitation level did not change the disturbing noise, but G_S became much larger. The standard deviation σ_{G_S} is obtained by measuring the FRF for different realizations of the normalized random multisine. For the small excitation level, it is completely dominated by the measurement noise σ_{N_G} while for the large excitations σ_{G_S} dominates. This is also illustrated in Fig. 2 where the evolution of the measured FRF is shown as a function of the excitation level. As can been seen, the stochastic contributions are growing with the level, while the measurement conditions (and hence the disturbing noise) remained the same. Again it is very hard to understand this result without the previous gained insight in the behaviour of nonlinear systems. This also suggests a first test to detect the presence of nonlinear distortions. The standard deviation σ_{N_G}, calculated from a set of consecutive periods (without changing the excitation signal), should be the same as that calculated from repeated measurements, using different realizations of the excitation signal. In the presence of nonlinear distortions, this will not be true.

3. Detection of nonlinear distortions

The ideal FRF-measurement method should not only provide the measured FRF, but at the same time the presence of nonlinear distortions should be detected, qualified (even or odd distortions) and quantified (what is the level of the distortions). Since the prime interest of these measurements is the FRF, it is unacceptable that most of the time would be spent on the detection of the nonlinear distortion at the cost of a reduced quality of the FRF-measurement. This excludes most existing methods that require a series of dedicated measurements to make the nonlinearity test. In general, it is impossible to realize this ideal, however when specially selected periodic excitations are applied, we can come close to it.

3.1. Detection of nonlinear distortions using periodic excitations

The sine test is the most simple test, characterizing directly the nonlinear behaviour, by verifying the generation of higher harmonics. However, it is not only very slow, but also does not measure the best linear approximation for random excitations, except for very small excitations. This is due to the fact that it is no random multisine excitation. This leads to the first conclusion that the excitation signals are restricted to broadband random multisines. The possibility to detect nonlinear distortions with these signals will be embedded by a careful selection of their amplitude spectrum; only a selected set of harmonics is excited. This idea was already suggested by Evans, Rees, and Jones (1994), Evans (1998), and McCormack, Godfrey, and Flower (1994). The odd-odd multisines, that excite the system at the frequencies $l_k f_0$ with $l_k = 1 + 4k, k = 0,1,\ldots,F$, are such a possibility. The linear system generates only an output at the excitation lines, while the nonlinear distortions also hit the nonexcited harmonics. This allows their detection and characterization: at lines $4k+1$: the output consist of the linear contribution + odd nonlinear distortions; at lines $4k+2$, $4k+4$: only the even nonlinear distortions appear; at lines $4k+3$: only odd nonlinear distortions appear. So it is possible to detect and separate the even and the odd nonlinearities. The level of the distortions is indicated by the level at the detection lines. This can be extrapolated with some care to the measurement lines, although significant differences can still occur, especially when the low harmonics are filtered before arriving at the nonlinearity (e.g. a high-pass input behaviour of the system). For that reason the results should be used as an indication and not as an absolute measure (Van hoenacker, Dobrowiecki, & Schoukens, 2000). The test can be made more robust against these problems by using a special odd multisine with components at $l_k = 1, 3, 9, 11, 17, 19, \ldots$ (Vanhoenacker & Schoukens, 1999). In that case the even nonlinearities are detected at the even lines, and the odd nonlinearities at the nonexcited odd lines.

In many applications the nonlinear distortions are of the same magnitude as the noise distortions and additional tools are needed to separate them from the noise. A first elegant method to distinguish between noise and distortions is to measure the 'harmonic' coherence (McCormack et al., 1994) at the nonexcited DFT frequencies. A second possibility is to calculate the sample variance from M measured periods of the periodic excitation (for a single realization of the excitation), and to compare directly the measured distortion levels with the noise levels. The advantage of this approach is that a full characterization of the 2nd order moments of the noise is available at the end of the measurement.

In practice some additional problems can occur during this test. The nonlinear interaction between generator and plant can also generate unwanted excitations at the detection frequencies, and it is no longer clear what part of the output should be assigned to the linear behaviour, and what part is due to the nonlinear distortions. In that case a first order correction can compensate the output: $\tilde{Y}(k) = Y(k) - \tilde{G}(j\omega_k)U(k)$. $\tilde{G}(j\omega_k)$ is obtained by linear interpolation of the FRF measurements at the excited frequencies (Vanhoenacker & Schoukens, 1999).

Conclusion. At the end of this simple experiment, the user gets broadband measurement of the FRF, a detection, qualification and rough quantification of the nonlinear distortions together with a noise analysis. The price to be paid is the loss in resolution caused by the nonexcited lines. This will be discussed in Section 4.

3.2. Illustration

The experimental test setup of Section 2.2 is used again. This time the system is excited with an odd–odd random multisine exciting the system at $(4k-3)f_0$, $k = 1,2,\ldots,128$ an $f_0 \approx 0.596$ Hz with an excitation level of 62.7 $\mathrm{mV_{RMS}}$. The measured output spectrum is shown in Fig. 3. This result shows that this test allows to measure in one experiment the FRF, the noise level, and the nonlinear distortions. In this case, it is clear that the latter are the dominating error mechanism acting on the setup, the odd nonlinear distortions are 20 dB larger than the noise. This is very valuable information for the rest of the modelling process.

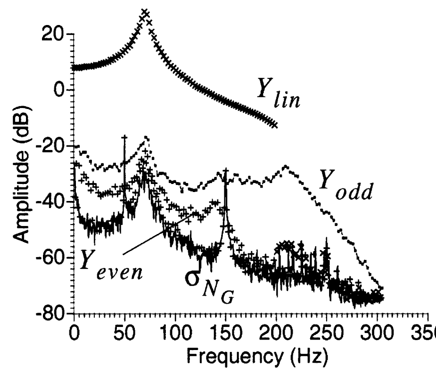

Fig. 3. Detection of nonlinear distortions at the output of the nonlinear circuit using an odd-odd multisine. x: linear + odd nonlinear contributions; + : even nonlinear contributions; • odd nonlinear contributions.

3.3. A short overview of other methods to detect nonlinear distortions

The literature describes a series of other methods, different from that presented before. Here we will touch only a few of them, an extended list of references is available in the paper of Natke, Juang, and Gawronski (1988). Also Haber (1985) gives a brief review of nonlinearity tests. The most simple method is to scale the input $u(t) \to \alpha u(t)$, and to verify if also the output scales with α after taking care for the offsets. In practice this method is less appealing. Two separate measurements are needed, and in many applications it is not that simple to impose a scaled input, due to the nonlinear load of the generator with the input impedance of the tested system. Moreover, the small nonlinearities have to be detected as the difference between two large measured signals, making the methods extremely sensitive to all possible measurement errors. Another popular test is to check the coherence. However, as pointed out before this method does not allow to separate noise disturbances from nonlinearity problems and it fails at all for periodic excitations. Extending the test to higher order spectra, probing directly for higher order correlations that are typical for nonlinear systems, eliminate these drawbacks. But these methods are very time consuming, especially for random excitations. Also Hilbert-transform tests are proposed (Tomlinson, 1987). Actually, these methods do not directly detect the nonlinear behaviour itself, but the noncausality in the impulse response of the linear approximation (FRF) that is induced by the nonlinearity. The method imposes significant constraints (e.g., only working on lowly damped systems) and a series of correction terms should be added because an FRF measurement can only be made in a restrictive frequency band. For these reasons we do not discuss these methods in detail and refer the reader to the available literature.

4. Minimizing the impact of nonlinear distortions on FRF measurements

For the clarity of the presentation we give first a set of general advises so that the reader keeps a maximum overview over the problem. Next, we illustrate them with experimental results.

4.1. Choices and advises

(A) Goal: measurement of the underlying linear system G_0

○ *First choice*: *odd-odd random multisine, keep the amplitude as small as possible*
Advantage: this allows to measure the FRF together with its standard deviation σ_{N_G}. Also the presence of nonlinear distortions is detected, qualified and quantified. The impact of the nonlinear distortion on the uncertainty σ_{G_S} is minimized. It is advised that the crest factor of the signal (ratio of peak amplitude to RMS-value) is minimized, initialising the crest factor minimization algorithm from random phases, in order to maintain the random behaviour.
Disadvantage: a loss in frequency resolution with a factor 4.

○ *Second choice*: *odd random multisine with minimized crest factor*
Advantage: this allows to measure the FRF with its standard deviation σ_{N_G} due to the disturbing noise. The impact of the nonlinear distortion on the uncertainty σ_{G_S} is minimized (the same quality as in choice 1) and the loss in frequency resolution is reduced to a factor 2.
Disadvantage: it is no longer possible to detect the presence of odd nonlinearities.

○ *Third choice*: *binary excitation*, *preferably with an odd spectrum*
Advantage: the impact of the distortions is minimized for a given RMS value of the excitation.
Disadvantage: almost no possibility to detect the presence of nonlinear distortions.

(B) Goal: measurement of the best linear approximation G_R

○ Advice: use test signals with the same power spectrum and the same amplitude distribution as those that will be applied later on to the system.

○ *First choice*: *use M_R different realizations of an odd-odd* (*or odd*) *random multisine and average the FRF over these experiments*.
Besides the advantages and disadvantages discussed under point A, the major advantage is that the stochastic contributions G_S are reduced in the averaging process: σ_{G_S} is reduced with $\sqrt{M_R}$. The major disadvantage is the increased measurement time to measure these different realizations.

○ *Second choice*: *use one realization of a very dense odd-odd* (*or odd*) *random multisine*.
Advantage: only 1 experiment is needed. It is still possible to smooth the FRF over small frequency bands.

4.2. Illustrations

(A) Goal: the underlying linear system G_0

Example. In order to visualize the impact of the crest factor, and the type of excitation (consecutive, odd and odd-odd multisines) on the nonlinear distortion a simulation was made. The FRF of a static nonlinear system $y = u + u^2/2.8 + u^3/15$ ($G_0 = 1$) is measured using three different excitation signals with a flat power spectrum: a random noise (zero mean normally distributed), an odd (50 frequencies) and a consecutive (100 frequencies) multisine excitation all with an RMS value of 1. In Fig. 4 the mean absolute error is plotted as a function of the crest factor for 1000 realizations of the studied excitations. It clearly shows that an odd multisine is doing significantly better than the consecutive one or the normally distributed noise excitation. The odd-odd multisine has a similar behaviour. The errors of the full multisine are also significantly smaller than those of the random excitation. The binary signal results in the smallest error, but all indications about the presence of a nonlinearity are lost in this case.

(B) Goal: the best linear approximation G_R
The impact of the excitation signal on the quality of the related dynamic system measurement is illustrated on

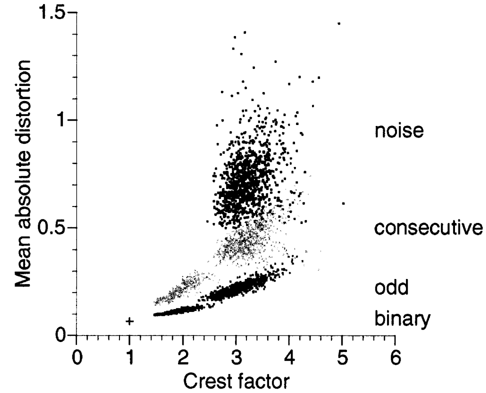

Fig. 4. Mean absolute distortion for different excitation signals.

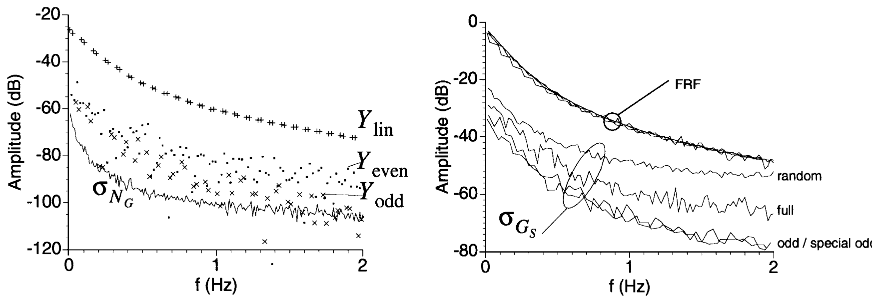

Fig. 5. Hair dryer experiment. Left side: detection of nonlinear distortions. Right side: impact of the excitation signal on the uncertainty (σ_{G_S}) due to stochastic nonlinear distortions.

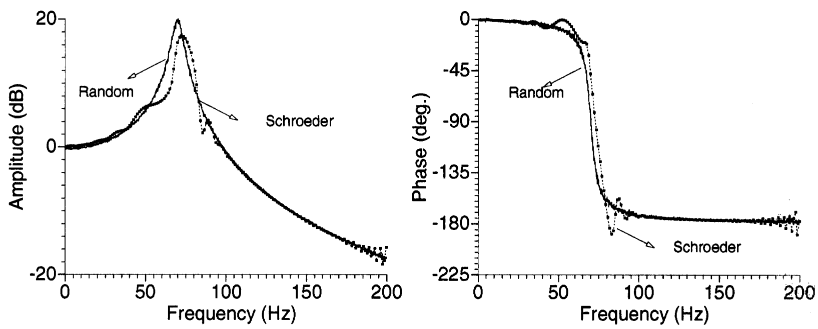

Fig. 6. Impact of the phase of the multisine on the measured FRF (RMS value of 54 mV). Random: odd random multisine with 1342 components. Schroeder: odd multisine with Schroeder phase. This signal acts like a swept sine.

a hair dryer system (Németh & Vargha, 1999). The output temperature is measured as a function of the fire angle α of the thyristors. The power of the heating element is proportional to $1 - \cos\alpha$, and nonlinear distortions appear. The system was operated with an excitation of 80% of the full range, with an offset of 20% of the full range. On the left side of Fig. 5, the level of the nonlinear distortions is detected. Next the FRF is measured using a noise excitation, a full, odd, and special odd random multisine. As can be seen, the FRF is about the same for all the excitations, but there are large differences between the standard deviations (almost completely dominated by the stochastic nonlinearities) on the measured FRF. The odd multisines result clearly in a superior quality.

(C) Dependency of the best linear approximation on the nature of the excitation signal

The FRF obtained for a random multisine (as advised) is compared with that of a swept sine like signal (a Schroeder multisine in this case, see Schoukens et al., 1993 or Ljung, 1999). The measurement results on the electrical circuit are shown in Fig. 6. While the random multisine still results in an FRF measurement that is very similar to the small signal results, the Schroeder multisine strongly deviates from it. Without prior knowledge no second order system is recognized any more. This illustrates again that in the presence of nonlinear distortions, the choice of the excitation signal is crucial. A random multisine combines the advantages of random excitations and periodic excitations, resulting in fast measurements of G_R, the best linear approximation.

The quality of the linear approximation, and its dependency on the excitation signal is finally illustrated in a last experiment. We checked if the linear models, identified on the previous experimental data using the random and Schroeder multisines, can simulate the plant output that is measured for a normally distributed random noise excitations. So we checked that a model obtained with a first class of excitations, can be used to predict the behaviour of the system for a second class of excitations. Three models were considered: the first one obtained using the Schroeder multisine, the second one with a crest factor minimized random multisine, and the last one with a random multisine without crest factor minimization. The models were obtained using a weighted output error method. In Fig. 7 the simulation errors are shown. The Schroeder multisine results are poor compared to the random phase results. The simulation errors of the minimized crest factor model (c) have a smaller standard deviation, but have more spikes than the random phase multisine (d). These spikes appear at those instances where the output makes a large excursion. It is clear that the application will have a strong impact on the final choice of the model.

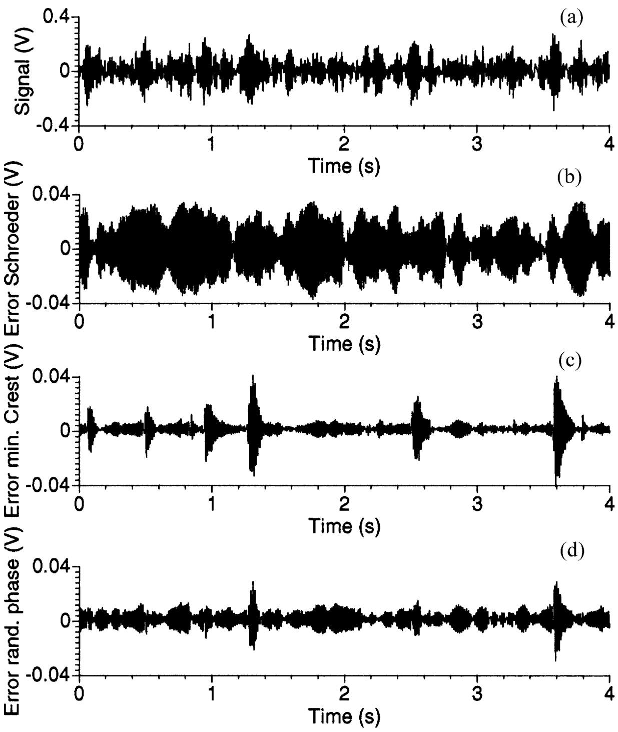

Fig. 7. Simulation error for the linear model obtained, respectively, with a Schroeder (b), random phase minimal crest factor multisine, (c) and random phase multisine (d), (a) shows the output signal of the circuit when driven with a normally distributed noise excitation.

5. Conclusion

In this paper, we have shown that it is possible to measure at the same time the FRF of a device, together with a detection, quantification and qualification of nonlinear distortions. The excitation should be selected in agreement with the final goal of the measurements. Advices are given to the user how to make this choice, and the results are illustrated on two nonlinear devices.

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