

OR:  $V(e, M(\theta))$  IF PARAMETRIC MODEL

(15)

OPTIMAL MODEL (OPTIMAL PARAMETER FOR ASSUMED STRUCTURE)

$$M_{\text{opt}} = \arg \min_M V(e, M)$$

$$\theta_{\text{opt}} = \arg \min_{\theta} V(e, M(\theta))$$

DUE TO FINITE DATA

$$\hat{\theta} = \dots$$

OPTIMAL PARAMETER ESTIMATE

OPTIMIZATION   
 CONVEX — ONE, GLOBAL MINIMUM   
 NON CONVEX — MANY, LOCAL MINIMA

CLOSED FORM SOLUTION   
 ITERATIVE   
 CONVEX, LINEAR-IN-PARAMETERS   
 OTHER

↳ GRADIENT-BASED

$$\hat{\theta}_{\text{NEW}} = \hat{\theta}_{\text{OLD}} - \eta \nabla_{\theta} V(e, M(\theta)) \Big|_{\theta = \hat{\theta}_{\text{OLD}}}$$

↑ GRADIENT OF CRITERION

SCALAR   
 — CONST   
 — VARIABLE

MATRIX ~ CURVATURE OF V

QUALITY OF THE ESTIMATE  $\hat{\theta} \sim$  KNOWLEDGE ABOUT  $e(t)$

$e(t)$  — DETERMINISTIC — BOUNDS

— RANDOM   
 — BIAS  $b(\hat{\theta})$    
 — VARIANCE  $\text{Var}(\hat{\theta})$    
 — MSE (MEAN SQUARED ERROR)

$$\text{MSE} = b^2 + \text{Var}$$

$$\text{MSE} = E\{(\hat{\theta} - \theta_0)^2\} = (E\{\hat{\theta}\} - \theta_0)^2 + E\{(\hat{\theta} - E\{\hat{\theta}\})^2\}$$

— FINITE INFORMATION  $\rightarrow \text{MSE} \neq 0$

— IDENTIFICATION METHOD  $\rightarrow$  BIAS/VARIANCE TRADE-OFF