Exercise 4

Vibration Analysis

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1 Introduction

Vibration analysis is a typical application of signal processing in which the path of a signal can be traced from a physical signal to a computer system. The latter often works without a user interface, connected to the object to be measured, so the analyzer works as an embedded system.

Vibration analysis is used for a variety of purposes. One common application is the diagnostics of various machines and equipment. In this case, the condition and possible failure of the measured mechanical system can be deduced from the power and spectrum of the vibration. Using a priori knowledge, advanced expert systems can also identify faulty parts, resulting in significant savings (e.g., aircraft engine testing). Vibrating mechanical systems often emit significant noise into their environment. Harmful noise sources can be localized with vibration analysis. Vibration analysis can also be used in the design of mechanical structures: by carrying out measurements on the finished prototype (e.g., car chassis), the elements exposed to dangerous vibration loads can be determined, and the construction can be modified. Vibration sensors are also used for acoustic measurements, e.g., for testing musical instruments.

2 Theoretical Summary

2.1 Vibration Sensors

The exhaustive description of vibration sensors exceeds the scope of a measurement exercise guide. The sensors do not detect the vibration as such, but the displacement, velocity, and acceleration of the vibrating body, and within these there are several solutions. These quantities are derivatives of each other, so they can be converted into each other. A few options are listed below without claiming to be exhaustive.

2.1.1 Displacement Sensors

They are inductive or capacitive sensors, that are rarely used to test vibrations, because a static (non-vibrating) unit relative to the system to be measured is needed. On the other hand, the laser interferometer has a notable role in accurate calibration of acceleration sensors.

2.1.2 Velocity Sensors

Here, we only mention the laser vibrometer, which measures the speed of vibration based on the Doppler effect. These are very expensive devices, but their advantage is that they do not interfere with the mechanical system and do not cause extra weight, which would otherwise lead to the system being out of tune. Another advantage is that measurements can be made at several points during one measurement, without changing the measurement setup. They usually include an optical cable, so when the otherwise relatively bulky laser is placed near the system to be measured, only the optical cable needs to be moved. However, its disadvantage is that the vibrating surface must reflect the light. An example of the application of the laser vibrometer is the testing of combustion engines. The laser beam reflected from the metal surface of the block has sufficient intensity for the measurement.

2.1.3 Acceleration Sensors

Acceleration sensors are the most often used type for vibration measurement. The sensors must be placed on the surface of the vibrating body. One or more seismic masses are placed inside the sensors, which, due to acceleration, create some signal depending on the given sensor. In the following, we will deal with acceleration sensors that work on the piezoelectric principle.

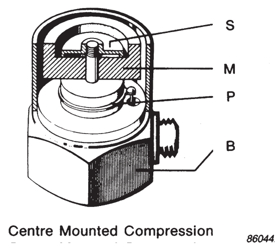

Figure 1: Conventional acceleration sensor. Base (B), seismic mass (M), piezoelectric crystal (P), bias spring (S)

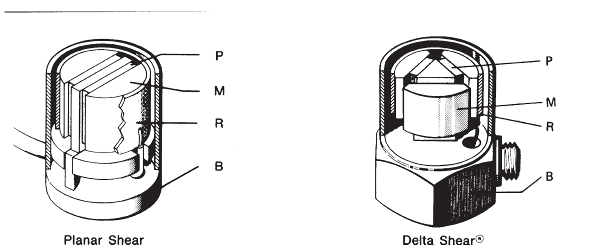

Figure 2: Improved acceleration sensors. Base (B), connector (R), seismic mass (M), piezoelectric crystal (P)

2.1.4 Microphones

Microphones detect sound pressure, not vibration, and can only be used indirectly to measure vibration, primarily for qualitative measurements.

2.2 Piezoelectric Acceleration Sensors

Figures 1 and 2 show the structure of the sensors of the Brüel & Kjær company, which is dominant in the given professional field. The more complicated geometry shown in Figure 2 can be achieve higher sensitivity.

The presented sensors all detect acceleration in the vertical direction, in the position shown in the figures. The sensor shown in the right side of Figure 2 is a more advanced construction. Compared to the one in the left, it has a higher resonance frequency and a higher sensitivity for a given seismic mass. We will use sensors of this design for the measurement.

As a result of deformation, charge accumulates on the surface of the piezoelectric material, which creates a small voltage on the sensor's capacitance. This "capacitor" discharges slowly due to leakage currents, so these acceleration sensors are not directly suitable for measuring constant acceleration (e.g., acceleration due to gravity). A typical curve of the transfer function of an acceleration sensor in the frequency domain can be seen in Figure 3. The smallest frequency included in the specification is usually 1 Hz, the largest depends on the resonance frequency, typically 10..20 kHz. The resonance frequency is the mechanical resonance frequency of the sensor, usually 20..50 kHz. In the specified range, the fluctuation of the magnitude is a few percent.

The sensitivity of the sensor is typically 10..100 pC/g. The charge (voltage) appearing at the output is not suitable for further processing, it must be amplified. In the first step, the so-called a charge amplifier is used, which converts the charge appearing on the crystal into a (loadable) voltage. A charge amplifier is basically an operational amplifier with negative feedback through a capacitor. Due to the small charge, an amplifier with a high input resistance must be used, and the sensor signal must also be connected to the input in such a way that there is as little leakage as possible. The signal of the operational amplifier can be further amplified with another voltage amplifier stage.

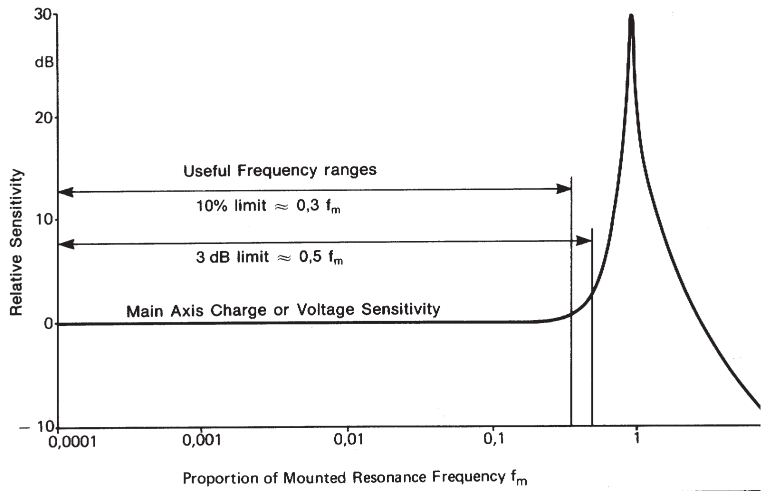

Figure 3: Magnitude response of an acceleration sensor

Accelerometers are supplied with an individual calibration record. It contains the following important data and characteristic curves:

- geometric, physical data,
- electronic parameters,
- sensitivity,
- transversal sensitivity (typically below 1%),
- amplitude and phase diagram,
- temperature sensitivity,
- resonance frequencies.

2.3 Calibration of Acceleration Sensors (Supplementary Material)

As we have seen, acceleration sensors detect acceleration only indirectly, so calibration is necessary. For this, known acceleration excitation must be used with high accuracy and the voltage appearing at the sensor output must be measured. The B&K company specifies the sensitivity with an accuracy of 10^{-3} in the calibration report for each sensor. It is worth considering that it is not a trivial task to excite the vibration sensor with such precisely known acceleration.

The manufacturer uses a laser interferometer for calibration, which measures the displacement of the sensor. A sinusoidal vibration with a precisely known frequency is produced as excitation, so the acceleration can be determined with the equations for harmonic motion. (In the case of g acceleration at 100 Hz, the deviation is 25 μm.) Accurate measurement of the voltage of the output signal is a simple task.

In common laboratories, comparative measurement is used for calibration: the sensor to be calibrated is compared to a more precisely known sensor. Hand-held excitation devices are used for quick calibration, which generate a given (e.g., g) acceleration with a moderate (a few %) accuracy.

2.4 Processing of Acceleration Signals

Here, we only describe in more detail the methods that we will use during the exercise, the other options are only mentioned. During the exercise, the basic equations of the discrete Fourier transformation are supposed to be known, especially the calculation of the resolution, the choice of the correct length, and the use of window functions.

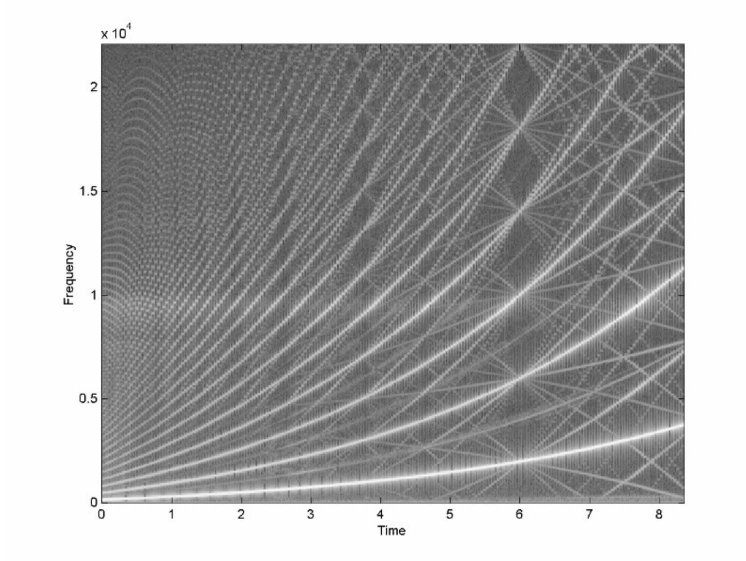

Figure 4: Spectrogram

The most common case is when the vibration signal is periodic, and the task is to analyze the components of the periodic signal. Non-periodic (stochastic) signals are analyzed, e.g., in the case of certain transfer function measurements, or in the case of signals from several sensors, when a cross-correlation or coherence function is calculated. If one is looking for low-power, noise-covered components of unknown frequency, one can calculate a cepstrum instead of a spectrum. However, in some cases, the location of the frequency components is well known, and the question is how they change. In this case, the spectrum is not scaled in frequency, but in the number of components. This is the so-called harmonic analysis (order analysis or order tracking). The Wavelet transform or the Wigner distribution is used to analyze time-varying but dominantly periodic signals. A special case of the latter is the short-time Fourier transformation, which we will also use.

2.4.1 Calculation of the Spectrogram

The spectrogram shows the change of the absolute value of the spectrum over time. Since three dimensions – time, frequency, amplitude – must be displayed, a spatial diagram or such a planar diagram is used, on which the color and intensity of a point on the plane carries the information. Figure 4 shows such a spectrogram, here in black and white.

The figure shows the acceleration response of a mechanical system to a logarithmically sweeping triangle signal. The horizontal axis is time, the vertical axis is frequency. The individual harmonics are clearly visible, as well as the fact that the higher harmonics quickly leave the analyzed range. You can also see components of lower intensity and decreasing frequency over time: these show that during the analysis we did not comply with the condition of the sampling theorem, because the aliasing components seem to be decreasing.

The spectrogram can be calculated as follows. The record must be divided into smaller sections and their discrete Fourier transform must be concatenated. The frequency resolution determines the number of points needed, so sometimes too long sections would be needed, during which the signal can change significantly. The solution is to transform overlapping sections, so the time step will be smaller. (An overlap of 75% is common.) It can be seen that increasing the time and frequency resolution contradict each other. This problem is improved by the mentioned Wigner distribution or the Wavelet transformation. Window functions can be used to correct the errors of the discrete transformation.

MATLAB supports spectrogram calculation with the `spectrogram` function.

2.4.2 Calculation of the Steady-State Spectrum

If the vibration signal changes only slightly, it is possible to calculate the spectrum based on a longer record. In this case, the record can also be cut into smaller sections, but its transformation is averaged in some way (most often linearly) (Welch method), as a result of which the variance of the calculated spectrum is reduced. If the overlap of the sections is not greater than 75%, the averaging is performed on the basis of nearly independent elements, and the variance is divided by N in the case of N elements. However, care must be taken not to use the complex spectrum obtained with the FFT, but its absolute value for this processing.

MATLAB supports the calculation of the steady-state spectrum with the `pwelch` function.

2.4.3 Measurement of Frequency Response Function (FRF)

Since we are examining signals from sensors, it is important to clarify what the system is whose transfer characteristics are being measured. In the case of vibration analysis, we examine a mechanical system, on which we place the vibration sensor at a specific point. Accordingly, the output of the system is displacement, velocity or acceleration; due to the sensors used in the measurement, we investigate acceleration signals. On the other hand, the input of the system can be of several types: any signal that results in mechanical excitation. A measuring hammer is often used, in which a force sensor is placed, the input signal in this case has a force dimension. Another stimulation option is the application of a so-called shaker, which creates force and acceleration proportional to the voltage signal applied to its input. Its construction resembles that of an electrodynamic loudspeaker without its diaphragm. The input can be the voltage of the shaker, but we can also place a force sensor at the output of the shaker, at the excitation point of the mechanical system. If the mechanical system itself contains an electromechanical converter, the voltage (current) given to the converter can also be the input of the system. If the system is characterized by a single FRF, it is also assumed that the system is linear, or the nonlinear behavior is neglected.

Thus the input and the output signals are given to determine the FRF. By definition, the FRF is the ratio of the Fourier transform of the two signals. However, the implementation of the operation also raises principal difficulties, but the discussion of this topic far exceeds the scope of an exercise guide. The Fourier transforms are estimated using a discrete Fourier transform (DFT) based on a finite record, which has a significant variance for theoretical reasons (see the previous section), but the signals can also be affected by significant measurement noise. It is also important that the excitation be persistent, i.e. there should be excitation available with a good signal-to-noise ratio at all frequencies where the FRF is to be determined. For example, if we want to analyze in the $20 \dots 2000$ Hz range and use a shaker with sinusoidal excitation, then the frequency of the excitation voltage must "cover" this range.

If the persistent excitation is given, the FRF is the ratio of the steady-state spectrum of the output and the input. In many cases, it is sufficient to determine the magnitude response, i.e. the absolute value of the FRF. If we know that the spectrum of the excitation signal is constant (white) in the given range, then it is sufficient to determine the steady-state spectrum of the output, its absolute value will be the magnitude response of the measured system. The "sweeping" sine signal (chirp signal) or random noise used in previous measurement exercises has a white spectrum. We will use the latter during the exercise, because it is easy to generate random noise in the MATLAB program.

2.4.4 Calculation of the Decay Time Constant, Signal Generation (Supplementary Material)

In the following, we consider the case where the parameters of a decaying periodic signal must be calculated. Figure 5 shows the first steps of processing.

The decaying periodic signal is shown in the diagram on the left. We select two sections of this for investigation: one section is located at the beginning of the record, where the signal level is still high, but the initial transients have already subsided; in the other section, the signal level is definitely lower, but has not yet reduced

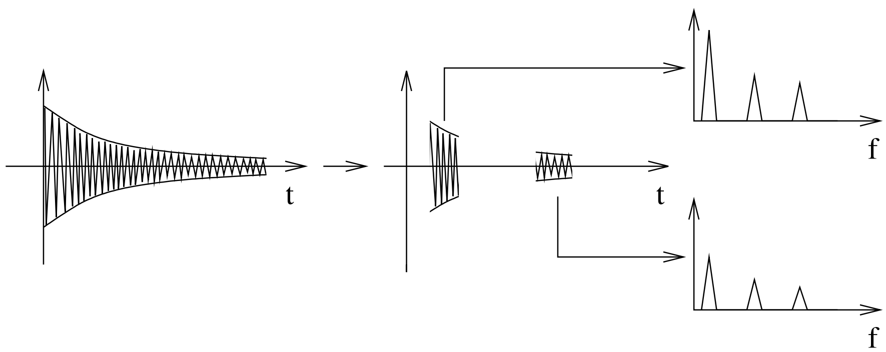

Figure 5: Decay time calculation

to the level of the noise in the record. The middle diagram shows the two selected sections. The diagram on the right shows the spectrum calculated for these two sections.

We assume that the system is linear and that each harmonic component decays exponentially with one time constant. The frequency of the individual components does not change, therefore the frequency of the peaks in the diagrams on the right does not change, only their amplitudes do. Since the decay is exponential and the signal is discrete, it is true that:

$$X_i(n+1) = \lambda_i X_i(n), \quad \lambda_i < 1 \tag{1}$$

where $X_i(n)$ is the amplitude of the i-th peak at the n-th time instant. Let the amplitude of the ith peak in the first spectrum be $X_{i,1}$ and $X_{i,2}$ in the second, and the number of samples between the beginning of the two sections be K. Then λ_i can be calculated as follows:

$$\lambda_i = \left(\frac{X_{i,2}}{X_{i,1}}\right)^{\frac{1}{K}} \tag{2}$$

The decaying periodic signal can be "assembled" from its components. A component can be characterized by its frequency (f_i), its initial magnitude ($X_{i,1}$), and its discrete time constant (λ_i). Then the N sample of the given component is:

$$x_i(n) = X_{i,1} \sin(2\pi \frac{f_i}{f_s} n) \lambda_i^n, \quad n = 0..N-1 \tag{3}$$

The original signal can therefore be approximated as follows:

$$x(n) \approx \hat{x}(n) = \sum_{i=1}^{I} x_i(n), \quad n = 0..N-1 \tag{4}$$

During the test and synthesis, we did not determine the phase of the periodic signal component. According to experience, this does not play a role at first approximation.

3 Measurement Setup

During the measurement, two devices must be tested. The sketch of the first object is shown in Figure 6.

We will investigate the vibrations of a fan and optionally a bell mounted on a wooden board. The fan is driven by a mains voltage asynchronous motor, the speed of which can be slightly changed with the help of a toroidal transformer. There are several threaded holes in the wooden plate, where the vibration sensor can be attached. The vibration sensor is a B&K 4384 type sensor.

The second object is a model of a rotating machine fixed in a room or larger structure, as shown in Figure 7. We placed a small speaker in a cardboard box, and a vibration sensor can be attached to the wall of the box

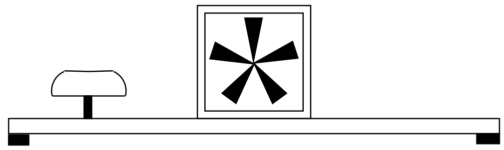

Figure 6: The first object to be measured

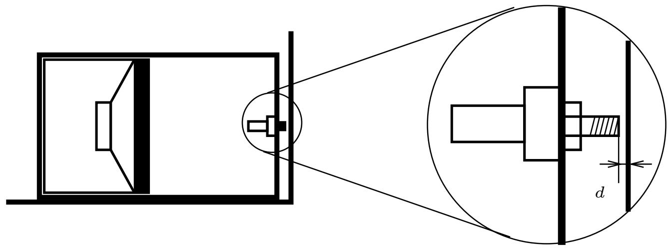

Figure 7: The second object to be measured

with a screw as shown in the figure. The rotating machine functions as a vibration generator, which is modeled by introducing periodic excitation into the loudspeaker. A periodic signal with variable (sweeping) frequency models an accelerating or decelerating rotation. The setup also includes a rigid plate bent in an "L" shape. By fine-tuning the air gap d, it can be achieved that if the wall of the box resonates, the screw hits the plate, and the hits result in pulses in the vibration signal. The phenomenon is clearly audible in the audio frequency range. This setup well models an error phenomenon that we also experience in practice, e.g. on a bus. Such a phenomenon can also lead to damage to the structure, but it is definitely unpleasant for the user. The phenomenon can also be identified from the vibration signal, so it is also suitable for monitoring the state of the mechanical system.

We use the same sensor for both objects. Take care on the correct connection of the cables to the sensors. Connect the cable only if the sensor is already in place. Never unscrew the sensor from its place with the cable still connected!

The acceleration sensor has a charge output, so its signal is amplified with a charge amplifier. Adjust the sensitivity on the charge amplifier based on the sensor's own datasheet. The signal of the charge amplifier can be connected to an oscilloscope or a sound card.

The properly conditioned signal is fed to the sound card of a PC for processing. You can make and play recordings with the `Audacity` program on the PC. When recording, be sure to select the sampling frequency and 16-bit mode. The recordings can be evaluated under MATLAB, the wav files can be read with the `audioread` function, and a MATLAB vector can be written to the wav file with the `audiowrite` function.

4 Measurement Tasks

1. Place the vibration sensor next to the fan. Start the fan at rated voltage. Record the signal with the program Audacity and analyze it in MATLAB! Determine the optimal sampling frequency. Based on the sensitivity of the acceleration sensor and the gain, determine the magnitude of the vibration signal in m/s^2 units as well!

2. By changing the output voltage of the toroidal transformer, change the speed and take more records! Identify the components that can be detected in the 0..500 Hz range! Determine the slip of the asynchronous motor at rated voltage. Be careful to choose the DFT length!

3. Place the vibration sensor on the side of the box and send a variable frequency sinusoidal signal to the loudspeaker. Adjust the "L" shaped plate to get it to "rattle" at certain frequencies. Without allowing rattling, measure the magnitude response between the input voltage and the acceleration signal! To do this, generate white noise of suitable duration in MATLAB! Interpret the final result, justify the most important breakpoints, maximum and minimum locations of the FRF!

4. What kind of excitation and analysis would you use to detect if the "L" shaped plate gets too close to the box, i.e. a harmful mechanical phenomenon occurs? During the measurement, only a measurement method capable of distinguishing between "correct" and "uncorrect" conditions must be developed and demonstrated, automatic recognition of the condition is not a task.

5. (Optional) Use an acceleration sensor to record the sound of the bell being struck. Determine the main components and their decay time constant.

6. (Optional) Using the results of the previous task, generate a "bell sound". Listen to the signal on a loudspeaker and evaluate the result.