NATURAL SELECTION OF GAME PLAYING AGENTS

Daniel L. Kovacs

Budapest University of Technology and Economics
Department of Measurement and Information Systems
H-1117, Budapest, Magyar tudosok korutja 2, building I, wing E, level 4
Hungary
dkovacs@mit.bme.hu

Abstract: *In this article we present an evolutionary simulation framework for the natural selection of game theoretic player agents. The framework is inspired by evolutionary game theory and the work of Robert Axelrod. Our incentive was to develop an intuitive, realistic implementation of these models where agents are represented individually, each having its own lifecycle, resources, properties and decision mechanism, and where selection emerges from their mutual interaction. Thus selection is not pre-programmed, but emerges naturally (as agents interact, win or lose, are born or die). Interaction among agents is modeled with general n-player games. The framework can be used to model real world multi-agent scenarios to predict the fittest agent programs responsible for selecting agents' interaction strategies. Several experiments were conducted with generalized Hawk-Dove and Tragedy of the Commons games. Agents playing constantly, randomly, according to Nash equilibrium and a generalized Tit-for-Tat program were studied. Experiments indicate that some fundamental evolutionary game theoretic assumptions might be questioned.*

Keywords: *multi, agent, simulation, evolutionary, game, theory, replicator, dynamics, natural, selection*

1 Introduction

Evolutionary game theory (EGT) [1] is a branch of game theory (GT) [2] which considers large, usually infinite populations of player agents being engaged in iterated game theoretic interactions. Its main question is: *how will an initial population evolve over time?* The population consists of different subpopulations of agents having different programs for selecting their strategy in the game being played. The probability that two agents meet to play a game in a given round is defined by the actual proportion of their subpopulations (agents are chosen according to uniform distribution). Subpopulations may have average utilities which are also important since their proportion may change according to them. A dynamics (e.g. replicator dynamics) is said to govern the proportion of subpopulations over time.

Although many useful insights were obtained by this theory, its analytic nature is an obstacle in the way to more complex investigations. Only simple (usually constant) strategy selecting programs can be examined in relatively simple games and populations with explicitly given dynamics.

In this article **we propose a multi-agent simulation model extending the above EGT model** by handling arbitrarily complex agent programs, N-player games and finite populations without the need to explicitly define a governing dynamics. The latter emerges from the interaction among individual agents – the level of detail which is hidden by EGT's level of abstraction. Thus our simulation model is an important, intuitive special case of EGT.

EGT must have been a significant inspiration for Robert Axelrod, who conducted similar, well known experiments [3][4]. The goal of those experiments was to find a program which efficiently plays the repeated Prisoner's Dilemma (PD) game [5], see Table 1.

Table 1. The Prisoner's Dilemma game

Player 1 \ Player 2	Defect	Cooperate
Defect	-2; -2	3; -3
Cooperate	-3; 3	2; 2

This is a 2-player normal form game, i.e. it has two roles: "Player 1" and "Player 2". It is symmetric, which means that it is not important what role an agent assumes when playing this game, it faces the same situation. Namely both players have the same set of pure strategies: "Defect" and "Cooperate", and they must simultaneously decide which one to choose. The above bimatrix specifies their outcome in every case, i.e. when both cooperate, both defect, and when one defects while the other cooperates. For example, if the agent in role "Player 1" plays according to "Defect" while the other agent, whose role is "Player 2" chooses "Cooperate", then the payoff of the former agent is 3, while the payoff of the latter is -3. The game is called a dilemma because its only Nash equilibrium (NE) [6] – *an outcome where neither player has the incentive to change its strategy unilaterally* – is the sub-optimal Defect-Defect outcome (vs. optimal Cooperate-Cooperate). This game is an abstract model of many typical real world scenarios involving two parties.

A numerically slightly different, but essentially equivalent PD game defined the strategic interactions among agent programs in Axelrod's experiments. At first 15, then 62 programs were chosen to be compared pair wise. Every program played against each other a fixed number of PD rounds. So they had to choose between the two PD strategies (cooperate or defect) in every round. Tit-for-Tat (TFT) [7], a simple program, which *initially cooperates and then repeats the previous choice of its opponent* won both tournaments by collecting the most at the end.

Axelrod concluded that because of the importance of PD as a model of social interaction, the core characteristics of cooperation in general must be those found in TFT. He then conducted other experiments to confirm the success of TFT, called *ecological* and *evolutionary analysis*. In the former he examined the dynamics of a population composed of programs submitted for the second tournament (all having an equal proportion of the initial population). The proportion of a program changed according to the proportion-weighted average of its scores against other programs in the second tournament (like in EGT). The results of this experiment again underpinned the success of TFT. Its proportion was the largest, and it grew steadily. Moreover, Axelrod predicted the extinction of invading defectors in case of a homogenous TFT population. In the latter experiment (evolutionary analysis) Axelrod used genetic algorithms [8] to evolve programs represented as bit strings that could play against the programs submitted for the second tournament. Their fitness was equal to their average payoff. The algorithm produced programs that played effectively against the tournament programs, and resembled TFT.

What can we deduce from these experiments? It is well known: TFT should be an ideal option in case of the PD game, at least. Nonetheless our current experiments – among others – show that in our setup, which we claim to be plausible, this is not completely true. Thus in the current article we take sides with those who question Axelrod's claims concerning the rationality of TFT (e.g. [9]), and we support our statements with carefully described and interpreted experimental evidence. However we do not question the concrete output of Axelrod's experiments, just his conclusions. For example in the two tournaments TFT cumulated the most at the end. It is plausible that against the given set of opponents TFT was the most successful. Nonetheless in the ecological analysis using the average of the scores achieved in the 2nd tournament leads trivially to the success of TFT (since it was the most successful in that tournament, so its average score is the highest, and so its proportion will grow the most according to the dynamics).

Our experiments show also that the central role of replicator dynamics as a model of evolutionary dynamics may also be questioned although some consider it even to be a fundamental theorem of natural selection [10]-[13]. These results will be explained later in detail. The rest of the paper is structured as follows: in Section 2 we describe the simulation framework and its implementation; in Section 3 we detail experiments; in Section 4 we conclude the article and outline future research possibilities.

2 Simulation framework

The current simulation framework is a continuation of a previously started research [14], where the main concepts were laid for the 2-player case. Here we present an extension of that framework which is able to simulate agent interaction according to arbitrary N-player games. Such an extension has many pitfalls (finding relevant N-player games, describing and editing them efficiently, interpreting cooperation and defection in them, generalizing programs like Tit-for-Tat for the N-player case, calculating Nash equilibria of such general N-player games, etc), but luckily we have overcome all of them. The concept of the framework is shown in Figure 1.

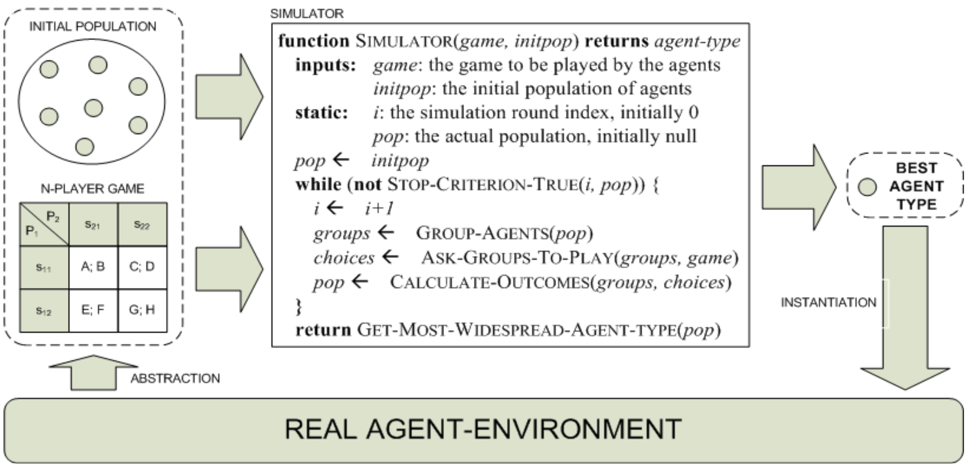

Fig. 1. Concept of proposed evolutionary simulation framework

The basis of the framework is an intuitive combination and extension of the ideas discussed in Section 1 about EGT and Axelrod's experiments. Simulation is divided into rounds. There is a finite number of agents in the population, that are randomly grouped in every round (according to uniform distribution, like in EGT) to play a given N-player game in the role of one of the players (less than N agents cannot play; agents' role is chosen randomly according to uniform distribution). Every agent of the population plays the same type of game in every round of a run (e.g. just PD), and each of them has a *program for selecting its strategy* in these plays (every agent plays once per round). We examined mainly the following:

- **PURE**: plays according to a given pure strategy (e.g. always cooperate in PD).
- **MIXED**: plays according to a given mixed strategy, i.e. *chooses its pure strategy according to a given probability distribution over its pure strategies*.
- **TFT**: plays according to a generalized version of 2-player TFT (see. below).
- **NASH**: plays according to a mixed strategy prescribed to it by a given Nash equilibrium of the game.

After a group of agents finished to play in a given round, the respective (possibly negative) payoffs are added to their individually cumulated utility. If that utility gets below a level (e.g. below zero), then the agent dies, i.e. it instantly disappears from the population, and won't play in the following rounds of the run; otherwise it may reproduce. Currently only asexual proliferation, i.e. *replication without change* is considered (like in EGT). Agents can have only a *limited number of offsprings* in their lifetime. They replicate, if their utility exceeds a given limit (*limit of replication*). After replication, their utility is decreased with the *cost of replication* (which is usually equal to the limit of replication). Offsprings start with the same utility, program and features as their parents originally (i.e. same *initial utility*, *lower limit of utility* necessary for survival, limit and cost of replication, and limit on the number of offsprings). The rationale behind these design choices originates in the following: biological agents (e.g. bacteria) can reproduce only a limited number of times during their lifetime, their replication is expensive, their offsprings usually develop from a lower (immature, initial) level and the level of "maturity" where they can reproduce is usually constant and inherited from their parents. After every agent finished a given round (died, survived or replicated), comes the next round until stop criteria are satisfied (the maximum number of agents or the maximum number of rounds was reached during the run).

It can be seen, that this framework corresponds to EGT and Axelrod's ecological analysis, but there is no explicit selection mechanism controlling the proportion of agents in the population (like replicator dynamics) – it is an emergent phenomena. Every agent has its own lifecycle (they are born with given features, interact with each other, change, maybe even reproduce and then possibly die), and only those survive, whose features make them able to do so in the ever actual state of the environment. In our view this process is *natural selection*. Of course there are many other more or less similar definitions in literature originating mostly from Darwin [15]. For instance, it is usual to say that "natural selection is a process that affects the frequency distributions of heritable traits of a population" [16] (page 16), and that "heritable traits that increase the fitness of their bearers increase in frequency in a population" [17] (page 821). These statements do not contradict our views, but they aren't specific enough in the sense, that they allow even the selection behind a genetic algorithm to be called "natural". The reason for that is that these statements stem from biology and genetics, which are concerned with natural systems [18], [19]. This is why we use a "new" definition.

2.1 Generalized TFT

Since agents are now engaged in N-player games we had to generalize TFT from the 2-player case. The following pseudo-code describes our generalization precisely.

```
IF cooperation can be interpreted (i.e. there is a cooperative strategy for every
player) in the game THEN
   FOR every opponent we actually play against DO
          IF the game is symmetric THEN
                 IF we have met the given opponent THEN
                        Choose its previous action against it
                 ELSE
                        Choose to cooperate with it
                 END IF
          ELSE
                 IF we have met the given opponent in the same role THEN
                        IF it cooperated the last time we met it THEN
                               Choose to cooperate against it
                        ELSE
                               Choose a random pure strategy against it
                        END IF
                 ELSE
                        Choose to cooperate against it
                 END IF
          END IF
   END FOR
   IF we chose to cooperate with majority of the opponents above THEN
          Play cooperatively (execute the cooperative strategy)
   ELSE
          Play according to the response which was selected for most of the opponents
   END IF
ELSE
   Play randomly (execute a pure strategy at random)
END IF
```

It can be seen that in case of 2-player, symmetric games where cooperation can be interpreted (e.g. PD) we get the original TFT. If cooperation can be interpreted, we make a separate decision for every opponent and then aggregate those decisions. Conversely we could aggregate our opponents first and then decide what to do against them, but this would have its pitfalls. In a larger population there is a small chance that we meet the same group of agents all in the same role again and so a decision would be even less trivial. Such considerations led us to the above generalization of TFT and the use of only symmetric games where cooperation can be interpreted in our experiments for now.

2.2 Playing according to Nash equilibria

Another program for strategy selection can be the game theoretic solution concept of Nash equilibrium. Such an equilibrium (or solution) practically always exists [6]. Nonetheless there may be more (even infinitely many) such equilibria and their computation is nearly not trivial (in 2-player games it is easy in comparison to the case of N-player games). An equilibrium prescribes a mixed strategy (a probability distribution over pure strategies) to every player. It is an equilibrium because if all the players play according to it, then none can gain more by deviating from it unilaterally (and since we consider only non-cooperative games, coalitions are not modeled at this level of interaction).

Several methods exist to compute Nash equilibria for N-player games [20][21]. Taking implementation aspects into account we chose *enumerating all extreme Nash equilibria* by solving polynomial systems [22] and *approximating a given equilibrium* by using a simplicial subdivision approach [23]. The former algorithm is essentially much slower than the latter since it tries to compute all the equilibria of a game (which may soon become infeasible above a given size, i.e. number of players and strategies). Moreover there is the problem of *selecting an equilibrium* [24]. If there are multiple equilibria *coordination* may become necessary, since when every player chooses its strategy according to an individually selected equilibrium (e.g. that maximizes its expected profit) then those equilibria may be different, and thus unexpected outcomes (even worst, irrational cases) may be realized. So in current experiments we decided to use simplicial subdivision to compute Nash equilibria. This solves the problem of finding and selecting a coordinated Nash equilibrium at the same time.

2.3 Implementation details

The proposed evolutionary simulation framework was implemented in **JADE** (**J**ava **A**gent **DE**velopment framework) [25] and **JADEX** [26] using **Eclipse IDE** (**I**ntegrated **D**evelopment **E**nvironment) [27]. JADE and JADEX are open-source, Java-based, platform independent, distributed middle-wares complying with the **FIPA** (**F**oundation for **I**ntelligent **P**hysical **A**gents) standards [28]. They enable relatively fast and easy implementation of physically distributed, asynchronous, high level multi-agent systems.

The implemented software architecture was aimed to be as simple, as possible. It consists of only two agents: a *GameAgent* (JADE) and a *PlayerAgent* (JADEX). GameAgent is responsible for conducting the runs of the simulation and orchestrating PlayerAgents, while PlayerAgents are the actual members of the population, who are grouped by the GameAgent and asked to play N-player games. Games are created, modified and analyzed manually with a stand-alone application called **GAMBIT** [29], but NE computation is done at application-level by using appropriate libraries.

Each agent had a variety of (mostly optional) startup parameters, which in case of a GameAgent set the type of the game to be played (e.g. PD), the maximal number of agents in the population and the maximal number of rounds in the run. The OR-relation of the latter two defined the termination criteria of a run. The startup parameters of a PlayerAgent set it’s program, initial utility, lower limit of utility, limit and cost of replication, limit on the number of offsprings and memory capacity. The latter was needed because each PlayerAgent had to be able to use its percept history in order to decide upon the strategy to be played in a given round. The percept history of an agent relates a series of events (information about past plays) to opponents. There is a limit on the length of these series and the number of opponents stored. If any of these limits is exceeded then the oldest element is overwritten.

The simulation went as follows: first a given number of PlayerAgents (constituting the initial population) was started on a JADE agent platform followed by a GameAgent, who first searched the platform for available PlayerAgents at the beginning of every round (because there may be newly born agents or some agents terminated later on), made a grouping of them (randomly dividing them into groups of size N and associating different roles to each of them according to uniform distribution). After that the GameAgent informed these groups about the game to be played in the given round (who plays with whom and in what role). The groups then replied to the GameAgent with a chosen strategy ID respectively (which was selected by their program). The GameAgent then calculated their payoffs and informed them about it. This was repeated until the termination criteria was satisfied.

3 Experiments

Experiments consisted of running the above simulation with several different initial populations and games to observe the changes in quantity, proportion and average utility of the different types of agent programs. Each experiment had its own settings, but a part of them was the same in every case. The maximal number of agents in the population was 1000-1200; the maximal number of rounds was 250; the maximal number of offsprings was 3; the limit and the cost of reproduction was 20; the lower limit of agents’ utility and their initial utility was 0; the maximal number of percept histories (about different opponents) was 1000; and the limit on the length of such a percept history was 4 for every agent in every experiment. Every PlayerAgent was playing in every round (except when the number of agents was not divisible by N). Three N-player games were examined: generalized Prisoner's Dilemma (PD), Chicken Game (CG) and Tragedy of the Commons (TOTC) [30]. The former two are special cases of the Hawk-Dove (HD) game [31].

Table 2. Payoff matrix of the 2-player Hawk-Dove game

Player 1 \ Player 2	Hawk	Dove
Hawk	(V-C)/2; (V-C)/2	V; 0
Dove	0; V	V/2; V/2

Table 2 shows the HD game which is widely studied in EGT. The original story is about two players who are competing for a resource of value $V>0$. They may be Hawks (aggressive) or Doves (peaceful). When two hawks meet, they fight, which costs C, and so they get $(V-C)/2$ per head. When two doves meet, they divide the resource equally between each other (they get $V/2$ per head). When a hawk meets a dove, then the hawk takes the resource (gets V), while the dove is plundered (gets 0). This game corresponds to PD game when $V>C$, otherwise when $C>V$ it is just like the CG game. In PD the best outcome is to be defective when the other player cooperates, and it is the worst outcome for the other. The only NE of this game is: (Hawk, Hawk). In CG the worst outcome is when both players defect (both play "Hawk"), and the best is to be non-cooperative (Hawk) against a cooperating (Dove) player. The CG game is a mixed motivation game since it has two equal, symmetric NE: (Dove, Hawk) and (Hawk, Dove). Both games are symmetric.

3.1 Generalized HD game

The above 2-player HD game was generalized for the sake of the simulation as follows:

- When everyone plays Dove, then their payoff is V/N respectively;
- When there are Hawks, then Doves get 0 payoff;
- When there is only one Hawk, then the Hawk's payoff is V;
- When there are $K>1$ Hawks, then their payoff is $(V-C)/K$.

It can be seen that in the special case of $N=2$ we get the 2-player HD game as shown in Table 2. Figure 2 shows a 3-player HD game with V=6 and C=3. When $V>C$ then the game has only one NE: the sub-optimal outcome where everyone plays Hawk. When $C>V$ then it has 2^N-1 different NE.

		Hawk			Dove		
Hawk	Hawk	1.0	1.0	1.0	1.5	1.5	0
	Dove	1.5	0	1.5	6.0	0	0
Dove	Hawk	0	1.5	1.5	0	6.0	0
	Dove	0	0	6.0	2.0	2.0	2.0

Fig. 2. 3-player Hawk-Dove game (V=6, C=3)

3.2 Tragedy of the Commons

TOTC is the other N-player game we used in our experiments. The original story is about $N=10$ herders (or commons) who share a pasture and initially have 1 cow per head. Their cows yield them $N=10$ liters of milk per head initially. The goal of the herders is to maximize their yield, so they may consider sending another cow onto the pasture. If $K>0$ herders decide so, then there will be $N+K$ cows on the pasture which leaves less grass per cow, and so cows yield less, exactly N-K liters of milk. In case of K defective herders the payoff of cooperating herders (who decided to have only one cow) is N-K liters per head, while the defectors get $2*(N-K)$ liters per head. The payoff matrix of an N=3 player TOTC game is shown in Figure 3. TOTC, similarly to the generalized HD game where $C>V$, has 2^N-1 different NE.

		D			C		
D	D	0	0	0	2	2	1
	C	2	1	2	4	2	2
C	D	1	2	2	2	4	2
	C	2	2	4	3	3	3

Fig. 3. 3-player Tragedy of the Commons game

The dilemma of this game is similar to the PD or HD game when $V>C$. Almost every player has the incentive to defect and thus a significantly sub-optimal outcome is realized (instead of the optimal "everyone cooperates"). This similarity and non-triviality is why we chose to investigate these $N>2$ player games in our evolutionary simulation framework. The most important 2-player dilemmas were already studied in our previous work on the subject [14]. Nonetheless for the sake of completeness let us first check Axelrod's prediction about the extinction of invading defectors in a homogenous population of TFTs in case of the 2-player PD game.

3.3 Experiment 1: invading Defectors in a homogenous population of TFTs in case of the PD game

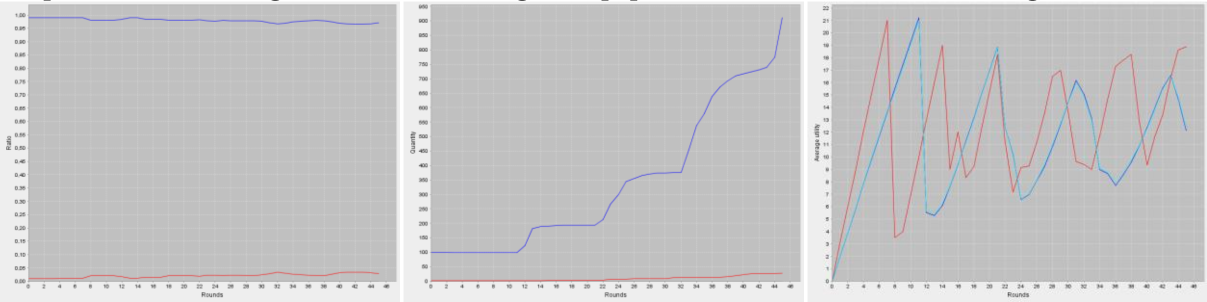

Fig. 4. 1 Defector (red) vs. 100 TFTs (blue) in 2-player PD games

Figure 4 shows a typical run of the experiment where the initial population consisted of 1 invading Always-Defect agent and 100 TFT agents. *Change of proportion*, *quantity* and *average utility* of the corresponding subpopulations can be seen over rounds (from left to right respectively). It can be seen that the proportion of TFT soars high above Defectors, but still invaders don’t vanish, but survive and reproduce (because they can gather enough utility by playing against initially cooperative TFTs, like “parasites”). Reproduction can be observed where the average utility of a subpopulation suddenly falls. I.e. Defectors reproduce regularly and even with a higher frequency than TFTs. These results contradict Axelrod’s predictions based on his ecological analysis, but correspond to our previous results [14]. The payoff matrix of the 2-player PD game agents were playing during the run is shown in Table 1.

3.4 Experiment 2: Hawks overwhelming Doves in case of the HD (PD) game

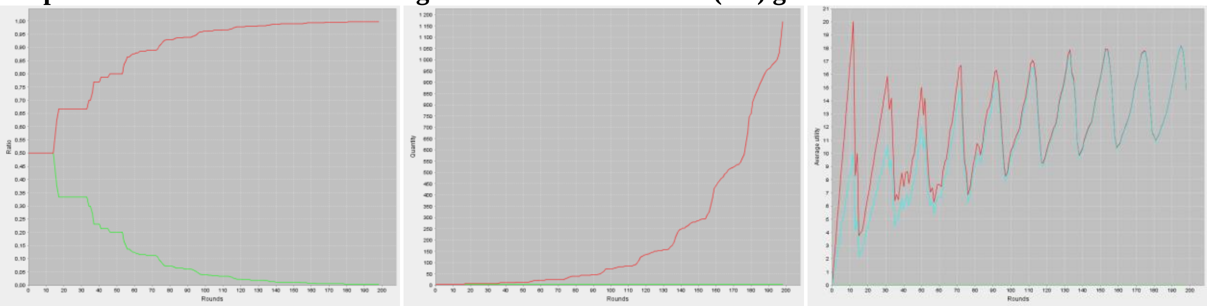

Fig. 5. 3 Hawks (red) vs. 3 Doves (green) in 5-player HD games (V=10, C=5)

Figure 5 shows the results of a typical run where the initial population consisted of 3 Hawks and 3 Doves. It can be seen that Hawks totally extinguish Doves. These results concerning the change of proportion of subpopulations correspond to the predictions of EGT [31].

3.5 Experiment 3: Random agents overwhelming TFTs in case of the HD (PD) game

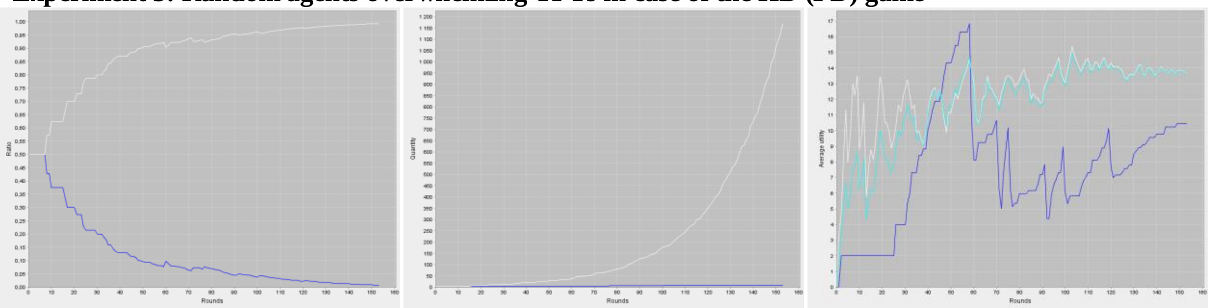

Fig. 6. 3 Random (white) vs. 3 TFTs (blue) in 5-player HD games (V=10, C=5)

Figure 6 shows an experiment where the initial population consisted of 3 Random and 3 TFT agents. Agents played 5-player HD games corresponding to PD games (since $V=10>C=5$). Surprising is that Random agents overwhelmed TFT agents, but this is not a coincidence – we’ve got the same result every time in this configuration, and similar results were achieved in many other configurations too. TFT seems to be not as fit (not even in the N-player PD game) as Axelrod claims.

3.6 Experiment 4: Hawks exploiting Doves and TFTs in case of the HD (CG) game

Figure 7 shows the typical result of an interesting experiment, where 50 Hawks, Doves and TFTs were put together in an initial population to play 5-player HD games corresponding to the CG game (since $V=5<C=10$). In this case mutual defection is the worst, so it is not surprising that the proportion of Hawks is the smallest.

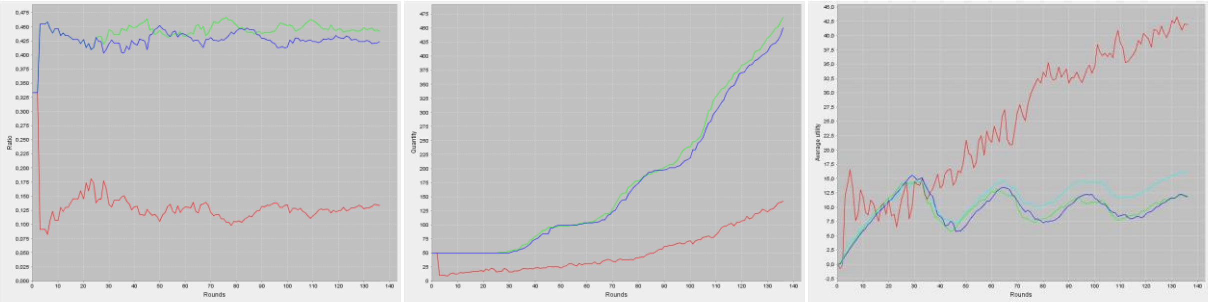

Fig. 7. 50 Hawks (red) vs. 50 Doves (green) vs. 50 TFTs (blue) in 5-player HD games (V=5, C=10)

Doves seem to be the fittest and TFTs are close to them (both in terms of proportion/quantity and average utility). It can be seen that Hawks nearly exterminated themselves at the beginning, but survived “on the back” of other cooperative players. Moreover, their average utility is way above the others’ and the average utility of the whole population (light blue line). This seems to contradict the predictions of replicator dynamics, which has a central role in EGT. It says that the change of subpopulations’ proportion is in direct proportionality with the ratio of their average utility and the average utility of the whole population (see. (1)).

$$p_H^{i+1} = p_H^i \frac{W_H^i}{W^i} \text{ and } p_D^{i+1} = p_D^i \frac{W_D^i}{W^i} \qquad (1)$$

Equation (1) defines discrete replicator dynamics: p_H^i and p_D^i is the proportion of Hawks and Doves in round i, W_H^i and W_D^i is their average utility in round i, and W^i is the average utility of the whole population in round i. This dynamics predicts that the proportion of Hawks should grow the fastest in the above scenario, but in Figure 7 we can see that it is stagnating and that the quantity of Doves and TFTs is increasing much faster. According to that replicator dynamics seems to be unable to model such scenarios properly (which aren’t rare in the real world, e.g. when a smaller group of agents exploits a much larger group of other agents and so lives much better, but can’t grow much larger because of this since it would eradicate itself on its own). Thus, in our view, the assumptions behind replicator dynamics are unrealistic and do not describe natural selection in general.

3.7 Experiment 5: Random, Nash, TFT, Hawk and Dove agents competing in case of the HD (PD) game

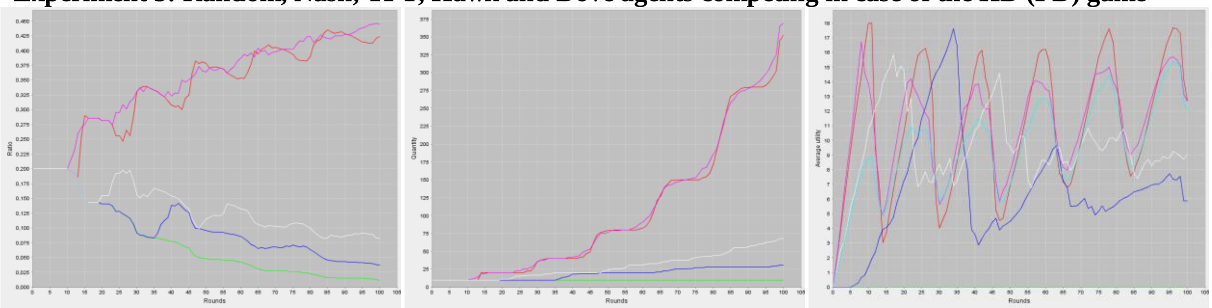

Fig. 8. 10 Randoms (white), Nashes (purple), TFTs (blue), Hawks (red), Doves (green) in 10-player HD (V=20, C=10)

Figure 8 shows a typical experiment with a 10-player HD (PD) game. It seems that the non-cooperative Nash and defective Hawk players dominate all the others (Randoms, TFTs and Doves). Only Doves are worse than TFTs.

3.8 Experiment 6: Random, Nash, TFT, Defective and Cooperative agents competing in case of the TOTC game

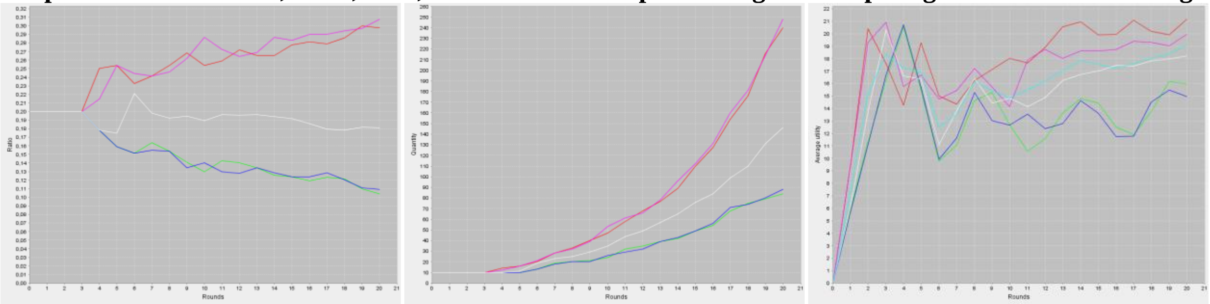

Fig. 9. 10 Randoms (white), Nashes (purple), TFTs (blue), Defectors (red), Cooperators (green) in 10-player TOTC

Figure 9 shows the results of a typical run with initially 10 Random, Nash, TFT, Defective and Cooperative agents in case of 10-player TOTC games. We can see that the performance of the different agent types is similar to the previous experiment (see. Figure 8), but the tendencies are much calmer now because of the differences in payoffs.

The differences between payoffs of adjacent outcomes (where only one player plays differently) are smaller in TOTC than in the HD (PD) game. It is not surprising to see coordinated Nash players and Defectors triumph, but it is disappointing to see that TFT agents' performance is the worst when playing this game of fundamental importance.

4 Conclusion

In this paper we presented an evolutionary simulation framework for the natural selection of programs choosing strategies in general N-person games. Our goal was (1) to reproduce some decisive EGT experiments in a more realistic scenario, (2) to simulate natural selection, and (3) to create a framework enabling prediction of agents' fitness. Experiments showed that the assumptions behind replicator dynamics are inherently unrealistic and that the fitness of TFT is much worse than expected according to the experiments of Axelrod. There are many possible ways to continue this research. For instance more complex programs could be examined. We could introduce sexual reproduction instead of the current asexual replication. A genetic representation of programs could be given to enable birth of new variations. Finally, the limits on the cardinality of the population could implicitly depend on environmental resource limitations.

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